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Summary of Ph.D. THESIS

MIXED SYMMETRY TENSOR FIELDS

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Contents

SUMMARY

Keywords: Quantum Field Theory, local Becchi-Rouet-Stora-Tyutin (BRST) cohomology, mathematical Physics.

Tensor fields with mixed symmetry transform in irreducible representations of $\mathrm{GL}(D, \mathbb{R})$ corresponding to Young diagrams with at least two columns ('exotic' representations of the Lorentz group). This class of fields appears in the context of many physically interesting theories, like superstrings, supergravities or supersymmetric high spin theories. One of the most important matters related to mixed symmetry-type tensor fields is the study of their consistent interactions, among themselves, as well as with higher-spin gauge theories. Beyond any doubt, the best approach to this problem is the cohomological one, based on the deformation of the solution to the classical master equation. This method is based on the reformulation of the problem of consistent deformations in gauge theories as a deformation problem of the solution to the classical master equation and further on solving the deformation equations withe the help of the local Becchi-Rouet-Stora-Tyutin (BRST) cohomology of the free theory.

In this work we approach the problem of constructing certain classes of interacting gauge theories with mixed symmetry tensors by using the deformation of the solution to the classical master equation combined with specific cohomological techniques. The main hypotheses used at the construction of the previously mentioned interactions are on space-time locality, Poincaré invariance, smoothness of the deformations in the coupling constant, and the conservation of the number of derivatives on each field.

The main subjects developed in this work can be synthesized into: 1) analysis of self-interactions for precise classes of gauge theories with mixed symmetry tensors; 2) study of the couplings between several models with mixed symmetry tensors and arbitrary matter fields; 3) investigation of the existence of interactions between various classes of gauge theories with mixed symmetry tensors and the Pauli-Fierz field; 4) construction of the couplings between two types of tensor fields with mixed symmetries and p-forms, for specific values of p. These results are contained in our papers [1]-[9].

The present work is organize into eight chapters. The first chapter is

introductory, while the second one exhibits a monographic character, its role being that of introducing the basic notions, general methods, and essential results from the literature that are further used in the thesis. Thus, various aspects related to gauge theories and associated antifield-BRST symmetry, cohomologies of exterior space-time and BRST differentials, reformulation of the problem of consistent interactions as a problem of deformation of the solution to the master equation as well as the general setting of describing mixed symmetry tensors from the point of view of generalized differential complexes are briefly reviewed.

The next five chapters expose the original contributions of the author to the subject of the thesis. More precisely, Chapters 3 and 4 deal with the non-massive tensor field with the mixed symmetry (3,1), Chapters 5 and 6 with the non-massive field with the mixed symmetry of the Riemann tensor, and Chapter 7 generalizes some aspects from Chapters 5 and 6 to the case of non-massive tensors with the mixed symmetry corresponding to a rectangular, two-column Young diagram.

Along this line, in Chapter 3 we give the Lagrangian description of the free theory with a non-massive tensor with the mixed symmetry (3,1). A key point of this chapter is represented by the interpretation of the free model in terms of a third-order nilpotent operator that acts on the vector space of tensor fields with mixed symmetries corresponding to a maximal sequence of Young diagrams with two columns. Further, we construct the free BRST symmetry and the basic cohomologies that compose the free, local BRST cohomology, namely, the cohomology of the exterior longitudinal derivative and the relevant local cohomology of the Koszul-Tate differential (also known as the characteristic cohomology), including the invariant characteristic cohomology. This chapter ends with a thorough study of the general properties of the co-cycles from the local BRST cohomology for the free model under study. Chapter 4 is dedicated to the analysis of consistent interactions in theories whose field spectrum contains a single non-massive tensor with the mixed symmetry (3,1). In view of this we gradually expose the study of self-interactions and of couplings to the Pauli-Fierz model, to an arbitrary matter theory, and respectively to a vector gauge field. The main results on the study of these couplings state that under the general hypotheses mentioned in the above there are neither consistent self-interactions nor consistent couplings to either the Pauli-Fierz model or matter theories. In turn, there appear non-trivial, five-dimensional (only) coupling terms in the case of a vector gauge field, manifested via the appearance of first- and second-order mixing components and through the deformation of the gauge transformations of the vector field at order one in the coupling constant with terms containing gauge parameters from the (3,1) sector. The results from these two chapters are contained in our papers [1], [7].

Chapters 5 and 6 follow closely the general line of Chapters 2 and respectively 3, but in relation to a free, non-massive tensor with the mixed symmetry (2,2). The following results, valid in the context of the general working hypotheses mentioned previously, are proved: (i) the self-interactions of the tensor field with the mixed symmetry of the Riemann tensor do not modify either the original gauge algebra or the gauge transformations and, in fact, reduce to a cosmological-like term; (ii) there are no consistent cross-interactions between such a tensor field and the Pauli-Fierz model; (iii) there are no couplings with purely matter theories such that the matter fields become endowed with gauge transformations; (iv) we cannot add consistent interaction terms to the action describing a non-massive tensor field with the mixed symmetry of the Riemann tensor and Abelian p-forms for $p = 1$ or $p = 2$. The content of these two chapters is included into our papers [2]-[3], [8]-[9].

Chapter 7 generalizes some aspects from Chapters 5 and 6 in the sense that the basic ingredients involved in the structure of co-cycles from the local BRST cohomology for a single non-massive tensor with the mixed symmetry (k, k) are investigated. In view of this, we firstly give the Lagrangian formulation of such a mixed symmetry tensor field from the general principle of gauge invariance and then systematically analyze this formulation in terms of the generalized differential complex of tensor fields with mixed symmetries corresponding to a maximal sequence of Young diagrams with two columns, defined on a pseudo-Riemannian manifold $\mathcal{M}$ of dimension D. Secondly, we compute the associated free antifield-BRST symmetry s, which is found to split as the sum between the Koszul-Tate differential and the exterior longitudinal derivative only, $s = \delta + \gamma$. Thirdly, we pass to the cohomological approach to this model and prove the following results: (a) the cohomology of the exterior longitudinal derivative $H(\gamma)$ is non-trivial only in pure ghost numbers of the type kl, with l any non-negative integer; (b) both the cohomologies of the exterior space-time differential d in the space of invariant polynomials and in $H(\gamma)$ are trivial in strictly positive antighost number and in form degree strictly less than D; (c) there is no non-trivial descent for $H(\gamma|d)$ in strictly positive antighost number; (d) the invariant characteristic cohomology $H^{\text{inv}}(\delta|d)$ is trivial in antighost numbers strictly greater than

$(k+1)$; (e) any co-cycle from the local BRST cohomology $H(s|d)$ of definite ghost number and in form degree D can be made to stop at a maximum value of the antighost number equal to either k or $(k+1)$ by trivial redefinitions only; (f) the non-trivial piece of highest antighost number from any such co-cycle can always be taken to belong to $H(\gamma)$, with some coefficients that are non-trivial elements from $H^{\text{inv}}(\delta|d)$. It is important to note that these cohomological properties can be used at the determination of the consistent couplings between the free, massless tensor field with the mixed symmetry (k,k) and other matter and gauge fields. This chapter is based on our papers [4]-[6]. The last chapter exposes the main conclusions of the present thesis.

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