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Test particles and test modes in plasma turbulence

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Abstract

Test particle statistics in turbulent plasmas is presented as a method to study the transport based on experimental measurements of the characteristics of the turbulence. The trapping or eddying appearing in the ExB stochastic motion in turbulent confined plasmas generates non-standard statistical behavior of the trajectories: memory effects, non-Gaussian probability and coherence. The memory effect produces anomalous diffusion regimes in the presence of weak perturbations. The diffusion coefficients produced by multiple decorrelation mechanisms (collisions, poloidal rotation, parallel motion) are determined here. Then we show that trajectory quasi-coherence has a strong influence of the evolution of the turbulence in the strong nonlinear regime. It provides the physical mechanism for the inverse cascade in drift turbulence.
Keywords: plasma turbulence, statistical approaches, test particle transport, Lagrangian methods

1 Introduction

The main aim of the studies of turbulence is to determine the diffusion coefficients. The statistics of test particle trajectories provides the transport coefficients in turbulent plasmas without approaching the very complicated problem of self-consistent turbulence that explains the detailed mechanism of generation and saturation of the turbulent potential. The statistical characteristics of the potential are considered known from experimental studies or numerical simulations. Alternatively they are described by general models and the dependence of the diffusion coefficients on the parameters of the models is determined.

The test particle studies of turbulent transport become in the last decade an active field essentially due to the constant interest of R. Balescu and to its important results. An excellent review of this approach and of the main results can be found in the recent book of R. Balescu *Aspects of Anomalous Transport in Plasmas* [1]. The aim of this paper is to present some recent developments of these results.

A component of test particle motion in magnetized plasmas is the stochastic electric drift produced by the electric field of the turbulence and by the confining magnetic field. This drift determines a trapping effect or eddy motion in turbulence with slow time variation. This is a nonlinear process that was analytically studied only in the last decade by developing new statistical methods [2], [3]. It was shown that trapping generates non-standard statistical behavior of the trajectories: memory effects, non-Gaussian probability and coherence. The memory effect is represented by long tail of the correlation of the Lagrangian velocity. It is shown that this tail determines non-Gaussian distribution of a scalar (density) advected by the stochastic ExB drift [4]. This tail of the Lagrangian velocity correlation is shown to strongly influence the transport coefficients and to produce anomalous diffusion regimes in the presence of a weak decorrelation mechanism [5]. The effects of several decorrelation mechanisms (collisions, average flows, parallel motion) were studied in a series of papers [6]-[8] but considered separately. In realistic plasma all these processes act together and their influence on the transport coefficients is expected to be complex in the presence of trapping when the nonlinear effects are strong.

We present in the first part of this paper (Section 2) a study of the turbulent transport in the presence of multiple decorrelation mechanisms: particle collisions and plasma rotation. We show that a different scaling of the diffusion coefficient in the parameters of the turbulence is obtained. It does not represent a smooth transition between the scaling resulting from only one of the decorrelation mechanisms. Another aspect of the statistics of the trajectories is the quasi-coherent behavior that characterizes trapped particles (Section 3). The second part of this paper extends the idea of test particle motion in turbulence to test mode evolution in a turbulent state of the plasma. We develop a Lagrangian method that takes into account the trapping of the trajectories. We show that the growth rate of the drift modes strongly depends on the characteristics of the background turbulence. When the latter has large Kubo number the unstable range of the mode is shifted toward small wave numbers and large potential cells appear (Section 4). The conclusions are summarized in Section 5.

2 Transport coefficients

Test particle studies connected with experimental measurements of the statistical properties of the turbulence provide the transport coefficients with the condition that there is space-time scale separation between the fluctuations and the average quantities. As shown by R. Balescu [4], particle density advected by the stochastic ExB drift in turbulent plasmas leads in these conditions to a diffusion equation for the average density. Recent numerical simulations [9] confirm a close agreement between the diffusion coefficient obtained from the density flux and the test particle diffusion coefficient. Experiment based studies of test particle transport permit to strongly simplify the complicated self-consistent problem of turbulence and to model the transport coefficients by means of test particle stochastic advection. The running diffusion coefficient $D(t)$ is defined as the time derivative of the mean square displacement of test particles. It was shown [10] that $D(t)$ is the time integral of the Lagrangian velocity correlation (LVC). Thus, test particle approach is based on the evaluation of the LVC for given Eulerian correlation (EC) of the fluctuating potential. The latter is the Fourier transform of the spectrum.

Important progress was obtained during the last decade in the study of test particle transport. It was based on the development of a semi-analytical statistical approach, *the decorrelation trajectory method* [2]. The turbulent transport in magnetized plasmas is a strongly nonlinear process. It is characterized by the trapping of the trajectories, which can determine a strong influence on the transport coefficient and on the statistical characteristics of the trajectories. The transport induced by the ExB stochastic drift in electrostatic turbulence [11] (including effects of collisions [6], average flows [7], motion along magnetic field [8], effect of magnetic shear [12]), the transport in magnetic turbulence [13], [14] and the Lorentz transport for arbitrary Larmor radius [15], [16] were studied in a series of papers using the decorrelation trajectory method. This statistical method was developed for the study of complex processes as the zonal flow generation [17], [18]. The results of all these studies are rather unexpected when the nonlinear effects are strong. The diffusion coefficients are completely different of those obtained in quasilinear conditions and the dependence on the specific parameters is reversed. A detailed presentation of the decorrelation trajectory method and of its relevance for transport studies in turbulent plasmas can be find in the recent book of R. Balescu [1].

An important conclusion obtained from the above studies is that all the components of particle motion (parallel motion, collisions, average flows, etc.) have to be taken into account. In the non-linear regimes characterized by the presence of trajectory trapping, these components have strong influence on the diffusion coefficients even if they are small. The reason for this behavior is the shape of the correlation of the Lagrangian velocity for particles moving by the ExB drift in a static potential. In the absence of trapping, the typical LVC for a static field is a function that decay to zero in a time of the order $\tau_{fl} = \lambda_c/V$, where λ_c is the correlation length of the stochastic potential and V is the amplitude of the stochastic velocity. This leads to Bohm type asymptotic diffusion coefficients $D_B = V^2\tau_{fl} = V\lambda_c$. Only a constant c is influenced by the EC of the stochastic field and the diffusion coefficient is $D = cD_B$ for all EC's. In the case of the ExB drift, due to trajectory trapping a completely different shape of the LVC is obtained for static potentials. A typical example of the LVC is presented in Fig. 1. This function decays to zero in a time of the order τ_{fl} but at later times it becomes negative, it reaches a minimum and then it decays to zero having a long, negative tail. The tail has power law decay with an exponent that depends on the EC of the potential [11]. The positive and negative parts compensate such that the integral of $L(t)$, the running diffusion coefficient $D(t)$, decays to zero. The transport in such 2-dimensional potential is thus subdiffusive. The long time tail of the LVC shows that the stochastic trajectories in static potential have a long time memory.

This stochastic process is unstable in the sense that any weak perturbation produces a strong influence on the transport [5]. A perturbation represents a decorrelation mechanism and its strength is characterized by a decorrelation time τ_d. The weak perturbations correspond to long decorrelation times, $\tau_d > \tau_{fl}$. In the absence of trapping, such a weak perturbation does not produce a modification of the diffusion coefficient because the LVC is zero at $t > \tau_{fl}$. In the presence of trapping, which is characterized by long time LVC as in Fig. 1, such perturbation influences the tail of the LVC and destroys the equilibrium between the positive and the negative parts. Consequently, the diffusion coefficient is a decreasing function of τ_d. It means that when the decorrelation mechanism becomes stronger (τ_d decreases) the transport increases. This is a consequence of the fact that the long time LVC is negative. This behavior is completely different of that obtained in stochastic fields that do not produce trapping. In this case, the transport is stable to the weak perturbations. An influence of the decorrelation can appear only when the later is strong such that $\tau_d < \tau_{fl}$ and it determines the increase of the diffusion coefficient with the increase of τ_d. This inverse behavior appearing in the presence of trapping is determined by the fact that a stronger perturbation (with smaller τ_d) liberates a larger

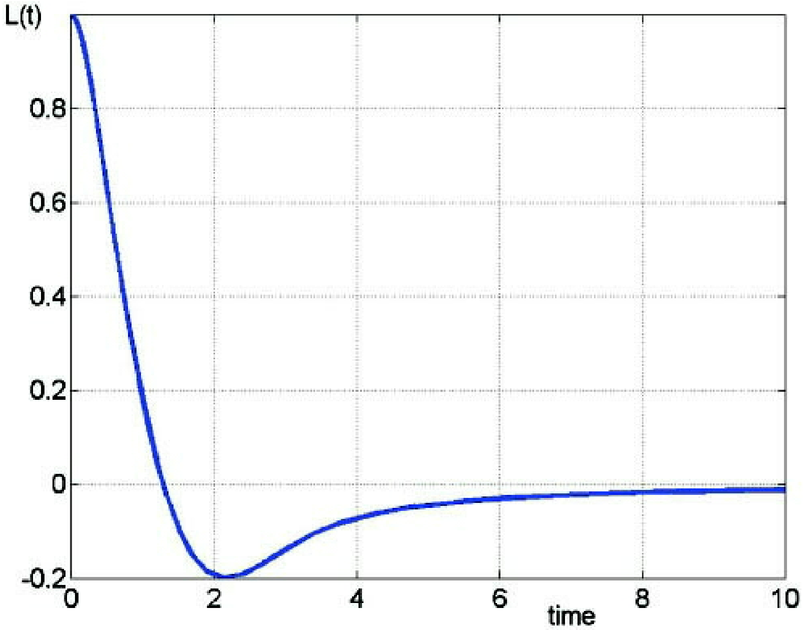

Figure 1: Typical Lagrangian velocity correlation in static potential.

number of trajectories, which contribute to the diffusion.

The decorrelation can be produced for instance by the time variation of the stochastic potential, which produces the decay of both Eulerian and Lagrangian correlations after the correlation time τ_c. The decorrelation time in this case is τ_c and it is usually represented by a dimensionless parameter, the Kubo number defined by $K = \tau_c/\tau_{fl}$. The transport becomes diffusive with an asymptotic diffusion coefficient that scales as $D_{tr} = cV\lambda_c K^{\gamma}$, with γ in the interval [-1, 0] (trapping scaling [11]), and thus it is a decreasing function of τ_c. For other types of perturbations, their interaction with the trapping process produces more complicated nonlinear effects. For instance, particle collisions lead to the generation of a positive bump on the tail of the LVC [6] due to the property of the 2-dimensional Brownian motion of returning in the already visited places. Other decorrelation mechanisms appearing in plasmas are average component of the velocity like poloidal rotation [7] or the parallel motion that determines decorrelation when the potential has a finite correlation length along the confining magnetic field.

We have developed a model that includes, besides the ExB stochastic drift, particle collisions, an average flow (poloidal rotation) and the parallel motion. The diffusion coefficients are determined using the decorrelation trajectory method. The later is based on a set of smooth trajectories determined by the Eulerian correlation of the turbulence and by all the other components of particle motion. A computer code was developed for determining the diffusion coefficients for given EC of the turbulent plasma.

The diffusion regimes are analyzed and the conditions when they appear are identified. A rich class of anomalous diffusion regimes appears when trajectory trapping is effective, i.e. when the combined action of the decorrelation mechanisms is weak enough. Trapping influences not only the values of the diffusion coefficients but also their scaling laws. A

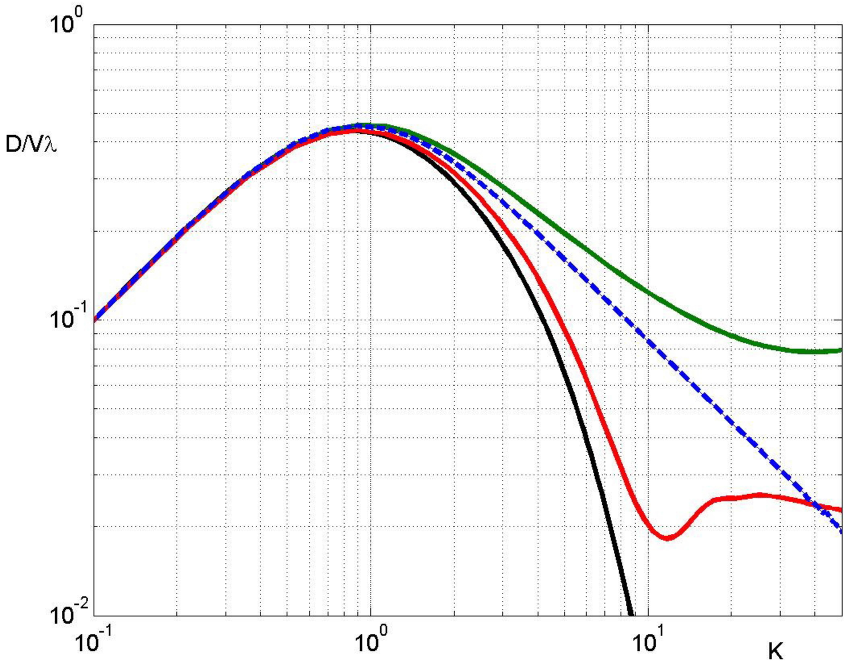

Figure 2: The diffusion coefficient as function of K for the values of V_p/V, $\chi/V\lambda_c$: 0, 0 (blue); 0.4, 0 (black); 0, 0.02 (green); 0.4, 0.02 (red).

systematic analyze of these regimes and of their specific conditions can be done only when one of the decorrelation mechanisms dominates. This corresponds to the case when one of the characteristic decorrelation times is much smaller than the others. When they have comparable values, the diffusion coefficient is a complicated function of all these characteristic times. The code we have developed provides a tool for the experimental studies of transport.

An example of the strong influence produced by the decorrelation mechanisms is presented in Fig. 2. The diffusion coefficient D_0 due only to the ExB drift for a time dependent potential is represented as function of the Kubo number by the dotted line. The diffusion coefficient is sensibly increased if particle collisions with a small collisional diffusivity $\chi \ll V\lambda_c$ are considered (dashed line). We note that the direct contribution of collisions χ is negligible in Fig. 2. A weak poloidal rotation with a velocity V_p that is smaller than the amplitude of the ExB drift determines a strong decrease of the radial diffusion (dashed-dotted line). When poloidal rotation and collisions act together, the diffusion coefficient (continuous line) can be smaller or larger than D_0, depending on the values of K, χ and V_p. Figure 2 also shows that at small Kubo number the diffusion coefficients are not changed by these small perturbations due to collisions and/or poloidal rotation.

3 Trajectory trapping and statistical coherence

Detailed statistical information about particle trajectories was obtained using *the nested subensemble method* [3]. This method determines the statistics of the trajectories that start in points with given values of the potential. This permits to evidence the high degree of coherence of the trapped trajectories.

The trapped trajectories correspond to large absolute values of the initial potential while the trajectories starting from points with the potential close to zero perform long displacements. These two types of trajectories have completely different statistical characteristics [3]. The trapped trajectories have a quasi-coherent behavior. Their average displacement, dispersion and probability distribution function saturate in a time τ_s. The time evolution of the square distance between two trajectories is very slow showing that neighboring particles have a coherent motion for a long time, much longer than τ_s. They are characterized by a strong clump effect with the increase of the average square distance that is slower than the Richardson law. These trajectories form structures, which are similar with fluid vortices and represent eddying regions. The statistical parameters of these structures (size, build-up time, dispersion) are determined. The dispersion of the trajectories in such a structure is of the order of its size. The size and the built-up time depend on the value of the initial potential. Trajectory structures appear with all sizes, but their characteristic formation time increases with the size. These structures or eddying regions are permanent in static stochastic potentials. The saturation time τ_s represents the average time necessary for the formation of the structure. In time dependent potentials the structures with $\tau_s > \tau_c$ are destroyed and the corresponding trajectories contribute to the diffusion process. These free trajectories have a continuously growing average displacement and dispersion. They have incoherent behavior and the clump effect is absent. The probability distribution functions for both types of trajectories are non-Gaussian.

The average size of the structures $S(K)$ in a time dependent potential is plotted in Figure 3. One can see that for $K < 1$ the structures are absent ($S \cong 0$) and that they appear for $K > 1$ and continuously grow as K increases. The dependence on K is a power low with the exponent dependent on the EC of the potential. The exponent is 0.19 for the Gaussian EC and 0.35 for a large EC that decays as $1/r^2$.

4 Test modes on drift turbulence

Test particle trajectories are strongly related to plasma turbulence. The dynamics of the plasma basically results from the Vlasov-Maxwell system of equations, which represents the conservation laws for the distribution functions along particle trajectories. Studies of plasma turbulence based on trajectories were initiated by Dupree [19], [20] and developed especially in the years seventies (see the review paper [21] and references there in). These methods do not account for trajectory trapping and thus they apply to the quasilinear regime or to unmagnetized plasmas. A very important problem that has to be understood is the effect of this non-standard statistical behavior of the test particle trajectories on the evolution of the instabilities and of turbulence in magnetized plasmas.

We extend the Lagrangian methods of the type of [20], [22] to the nonlinear regime characterized by trapping. We study linear modes on turbulent plasma with the statistical

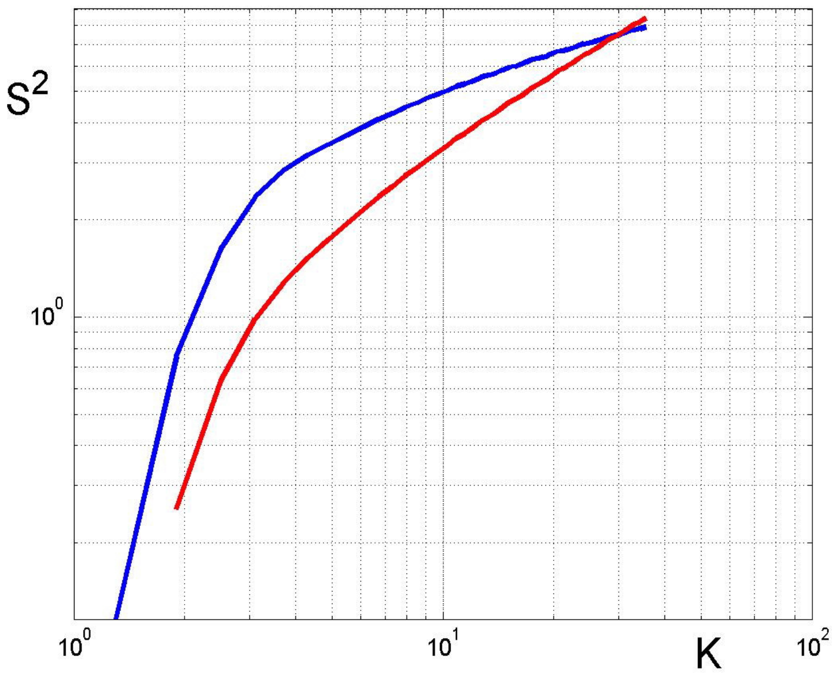

Figure 3: The average size of the trajectory structures for Gaussian EC (blue line) and for an EC that decays as $1/r^2$ (red line).

characteristics of the turbulence considered known. We determine the dispersion relation for such test modes. We consider the drift instability in slab geometry with constant magnetic field. The combined effect of the parallel motion of electrons (non-adiabatic response) and finite Larmor radius of the ions destabilizes the drift waves. The perturbations of the electron and ion distribution functions are obtained from the gyrokinetic equation as integrals along test particle trajectories of the source terms determined by the average density gradient. The gyrokinetic equations are not linearized around the unperturbed state as in linear theory but around a turbulent state with known spectrum.

The background turbulence produces two modifications of the equation for the linear modes. One consists in the stochastic ExB drift that appears in the trajectories and the other is the fluctuation of the diamagnetic velocity. Both effects are important for ions while the response of the electrons is approximately the same as in quiet plasma.

The average propagator of the modes is evaluated using the above results on trajectory statistics. In the first order it depends on the size $S(K)$ of the structures. The solution of the dispersion relation for a mode with frequency ω and wave number components k_1, k_2 is obtained as

$$\omega = \omega_{*e} \frac{\Gamma_0 \left(\frac{k_\perp^2 \rho_L^2}{2} \right) \exp \left(-\frac{1}{2} k_i^2 S^2 \right)}{2 - \Gamma_0 \left(\frac{k_\perp^2 \rho_L^2}{2} \right)} \tag{1}$$

$$\gamma = \sqrt{\pi} \frac{\omega \left(\omega_{*e} - \omega \right)}{2 - \Gamma_0} \frac{1}{|k_z| \, v_{Te}} - k_i^2 \mathcal{D}_i \cos(\omega \tau_c) + k_i k_j R_{ij} \omega / k_2 \tag{2}$$

where V_{*e} is the diamagnetic velocity, $\omega_{*e} = k_2 V_{*e}$ is the diamagnetic frequency, ρ_L is the ion Larmor radius and $\Gamma_0(b) = exp(-b)I_0(b)$. The tensor R_{ij} has the dimension of a length and is defined by

$$R_{ji}(\tau, t) \equiv \int_{\tau}^{t} d\theta' \int_{-\infty}^{\tau-\theta'} d\theta M_{ji}(|\theta|) \tag{3}$$

where M_{ij} is the Lagrangian correlation

$$M_{ji}(|\theta' - \theta|) \equiv \left\langle v_j \left(\mathbf{x}^i(\theta'), z, \theta'\right) \; \partial_2 v_i \left(\mathbf{x}^i(\theta), z, \theta\right)\right\rangle, \tag{4}$$

and v_j is the ExB drift velocity.

The trajectory trapping process has a complex influence on the mode. The quasi-coherent component of ion motion (the stochastically trapped ions) determines the S-dependent exponential factor in the frequency ω. Its effect is the displacement of the unstable k-range toward small values. The random component in the ion motion (the free ion trajectories) determines a diffusive damping term in the growth rate γ that produces the stabilization of the large wave numbers. It is similar with the term obtained in [20], but with the diffusion coefficient influenced by trapping. The fluctuations of the diamagnetic velocity determines the last term in the growth rate (2). The tensor R_{ij} contributes to the growth of the modes.

The test mode growth rate γ strongly depends on the characteristics of the background turbulence represented by the Kubo number as seen in Fig. 4 where ω and γ are plotted for a quiet background plasma (dashed lines) and for a turbulent one (continuous lines). At small Kubo numbers (Fig. 4a), the results of Dupree are obtained. The turbulence determines the stabilization of the large $k_\perp$ modes due to ion trajectory diffusion and it does not influence the frequency. At large Kubo numbers (Fig. 4b), a strong decay of the frequency appears due to the quasi-coherent component of ion motion [$S(K) \neq 0$ in Eq. (1)] and the maximum of the growth rate is displaced to smaller wave numbers.

The dynamics of the drift turbulence is determined starting from a large spectrum of modes with very small amplitudes (thermal bath). In this quasilinear regime only the diffusion of the ions influences the modes producing the damping of the modes with large wave numbers, $k_\perp \rho_L \gg 1$. The amplitude of the stochastic potential increases continuously while the correlation length remains comparable with ρ_L and the correlation time is $\tau_c = 1/\omega_{*e} \sim \rho_L/V_{*e}$. This amplitude increase eventually leads to $K > 1$ and produces trajectory trapping and coherent motion for a part of the ions. Small trajectory structures are formed and persist during the correlation time of the potential. This ordered motion of the ions acts similarly with the cyclotron gyration: it decreases the frequency of the modes and displaces the maximum of the spectrum toward smaller wave numbers. In this stage of the evolution the amplitude of the ExB velocity remain approximately constant, while the correlation length and the correlation time are slowly increasing. The spectrum is continuously displaced toward small wave numbers and narrowed due to the increase of the diffusion coefficient. Thus the energy taken by the instability from the electrons produces a motion of the ions with increasing coherent component. The size of the trajectory structures increases and is reflected in the turbulence that looses the random aspect: large ordered potential cells are produced. In the same time, as the Kubo number increases, the term determined by the fluctuation of the diamagnetic velocity is growing due to the fact that the turbulence becomes anisotropic. The evolution at this stage strongly depends on the tensor R_{ji}, on the amplitude of the potential and also on

(b)

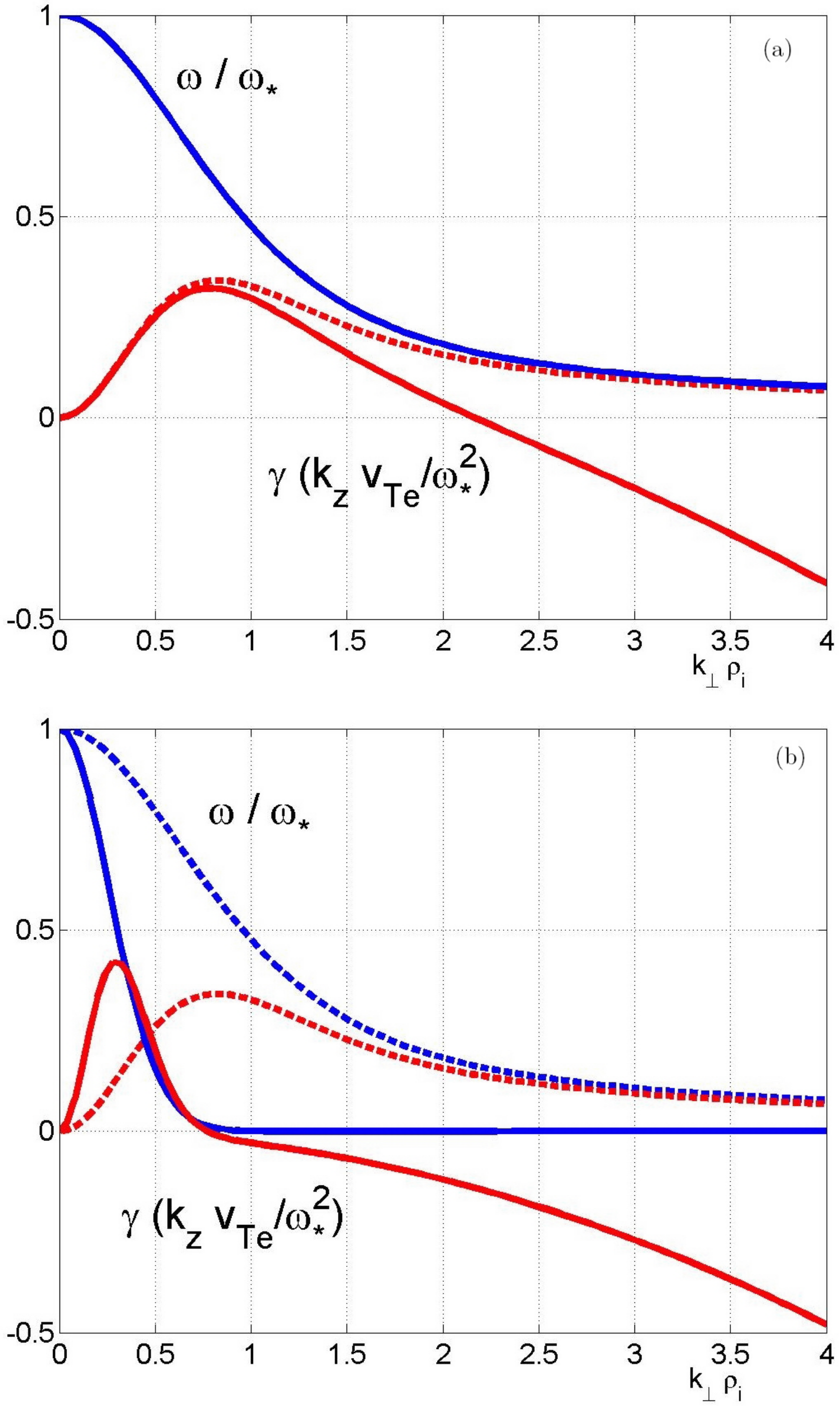

Figure 4: The wave number and the growth rate as function of the wave number for unperturbed plasma (dashed lines) and for turbulent plasma (continuous lines). The Kubo number of the background turbulence in $K = 0.1$ in (a) and $K = 10$ in (b).

the shape of the correlation (normal for such a strongly nonlinear state). We expect that the toroidal aspects have a strong influence on this tensor.

A different perspective on the inverse cascade is thus obtained. It does not appear as wave-wave interaction but as the consequence of ion ExB motion in the potential of the turbulence that strongly influences the test mode stability. Namely, the quasi-coherent motion of the trapped ions produces the destabilization of the modes with wave lengths of the order of the average size of the trajectory structures. These decreases the frequency of the modes and produces the increase of the size S of the trajectory structures leading to further decrease of the wave numbers of the unstable modes.

5 Conclusions

We have discussed the problem of stochastic advection of test particles by the ExB drift in turbulent plasmas. We have shown that the nonlinear effects are very strong in the case of static potentials. The trajectories are non-Gaussian, there is statistical memory and coherence, and the scaling laws are dependent on the Eulerian correlation of the stochastic potential.

These properties persist if the system is weakly perturbed by time variation of the potential or by other components of the motion (collisions, poloidal rotation, parallel motion). The memory effect determines anomalous diffusion regimes. A code was developed for the calculation of the diffusion coefficients for given Eulerian correlation of the turbulence, which takes into account multiple decorrelations.

These non-standard statistical properties of the trajectories are shown to be associated with order and structure formation in turbulent magnetized plasmas. Particle trajectories have a high degree of coherence when the perturbations are weak. The trajectory structures determine the evolution of the drift turbulence toward large scales (inverse cascade).

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