Physics AUC, vol. 17 (part I), 76-85 (2007)

Short wavelength ballooning mode in tokamaks

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Abstract

A skin size ($k_\perp \lesssim \omega_{pe}/c$) plasma mode characterized by a dispersion relation $\omega \simeq ck_\perp k_\parallel/k_{De}$ (k_{De} the electron Debye wavenumber), adiabatic ions, and $\omega \ll k_\parallel v_{Te}$, in a uniform plasma is destabilized in the tokamak geometry by a modest electron temperature gradient η_e and ballooning parameter α_e. When unstable, a large electron thermal diffusivity emerges because of the cross-filed wavelength much longer than that of the conventional electron temperature gradient (ETG) mode.

1 Introduction

The role played by turbulence having cross-field wavelength of the order of the electron skin depth c/ω_{pe} in anomalous transport has long been speculated in the past. For example, for tokamaks, Ohkawa [1] proposed the following electron thermal diffusivity,

$$\chi_e = \frac{v_{Te}}{qR}\left(\frac{c}{\omega_{pe}}\right)^2, \tag{1}$$

where v_{Te}/qR is the electron transit frequency with v_{Te} the electron thermal velocity and qR the connection length of the helical magnetic structure. In the formula, the skin depth plays the role of decorrelation length in the radial direction and the transit frequency plays the role of decorrelation rate. It is noted that the derivation of the diffusivity was heuristic, the presence of skin size instability was assumed but the origin of the turbulence (instability) was left unidentified. An attractive feature of the thermal diffusivity is that it gives a somewhat natural explanation for the empirical Alcator type scaling law for the energy confinement time [2],

$$\tau_E \propto n, \tag{2}$$

where n is the plasma density.

From the point of view of plasma wave theory, the Ohkawa diffusivity is strange because it is usually thought that the skin depth is pertinent only if the parallel phase velocity $\omega/k_\parallel$ is larger than the electron thermal velocity in which case plasma instabilities become predominantly electrostatic. In this case, the electron skin depth appears only as a small electromagnetic correction and should not play important roles. To circumvent this problem, it has been suggested [3] that the nonlinear Doppler shift $\mathbf{k}\cdot\mathbf{v}_E$ due to the

$E \times B$ drift $\mathbf{v}_E$ may exceed the electron transit frequency in strong turbulence, and skin size electromagnetic turbulence may exist *nonlinearly.*

In this paper, we wish to report on a linearly unstable electromagnetic mode characterized by adiabatic ions $(k_\perp \rho_i)^2 \gg 1$ (ρ_i the ion Larmor radius) and adiabatic electrons with a phase velocity parallel to the magnetic field smaller than the electron thermal speed $\omega < k_\parallel v_{Te}$. As will be shown, in these limits, a simple electromagnetic ballooning mode emerges which is symbolically described by the following dispersion relation,

$$(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e} \omega_{De} - \frac{(\omega - \omega_{*e})^2}{1+\tau} = \frac{c^2 k_\parallel k_\perp^2 k_\parallel}{k_{De}^2}, \tag{3}$$

where the notations are standard, namely, $\omega_{*e}(\omega_{De})$ is the electron diamagnetic (magnetic) drift frequency, η_e is the electron temperature gradient parameter, $\tau = T_e/T_i$, $k_{De} = \omega_{pe}/v_{Te}$ is the electron Debye wavenumber, and $k_\perp(k_\parallel)$ is the wavenumber perpendicular (parallel) to the ambient magnetic field. The mode is destabilized by a modest electron temperature gradient η_e and electron ballooning parameter defined by

$$\alpha_e = q^2 \frac{R}{L_n} (1 + \eta_e) \beta_e. \tag{4}$$

Here, q is the safety factor, β_e is the electron beta factor, $R/L_n = 1/\varepsilon_n$ is the density gradient aspect ratio, and $\eta_e = d \ln T_e / d \ln n_0$ is the electron temperature gradient. The mode is intrinsically electromagnetic (because it is a ballooning mode, although there is no resemblance to the ideal MHD ballooning mode), and is not a result of correction to electrostatic modes such as the familiar electron temperature gradient (ETG) mode. (Such a case has been analyzed by Kim and Horton [4].) The right hand side of Eq. (3) may be approximated by

$$\frac{c^2 k_\parallel k_\perp^2 k_\parallel}{k_{De}^2} \simeq \left(\frac{c k_\theta}{\omega_{pe}}\right)^2 v_{Te}^2 k_\parallel^2, \tag{5}$$

which indicates the electron transit mode and skin depth are intimately related. It is noted that the Debye screening factor $(k_\perp/k_{De})^2 \ll 1$ manifests itself in the dispersion relation even though charge neutrality holds.

2 Local analysis

We start with the electron drift kinetic equation under the assumption that the electron Larmor radius is ignorable $(k_\perp \rho_e)^2 \ll 1$,

$$f_e = \frac{e\phi}{T_e} f_{Me} - \frac{\omega - \hat{\omega}_{*e}(v^2)}{\omega - \hat{\omega}_{De}(\mathbf{v}) - k_\parallel v_\parallel} \left(\phi - \frac{v_\parallel}{c} A_\parallel\right) \frac{e}{T_e} f_{Me}, \tag{6}$$

where ϕ is the scalar potential, $A_\parallel$ is the vector potential, $f_{Me}(v^2)$ is Maxwellian distribution, and energy dependent electron diamagnetic drift frequency $\hat{\omega}_{*e}(v^2)$ and velocity dependent electron magnetic drift frequency $\hat{\omega}_{De}(\mathbf{v})$ are defined by

$$\hat{\omega}_{*e}(v^2) = \omega_{*e} \left[1 + \eta_e \left(\frac{v^2}{v_{Te}^2} - \frac{3}{2}\right)\right], \ \omega_{*e} = \frac{c T_e}{e B^2} (\nabla \ln n_0 \times \mathbf{B}) \cdot \mathbf{k}, \tag{7}$$

and

$$\hat{\omega}_{De}(\mathbf{v}) = \frac{mc}{eB^3}\left(\frac{1}{2}v_\perp^2 + v_\parallel^2\right)(\nabla B \times \mathbf{B})\cdot\mathbf{k}, \tag{8}$$

respectively. In the limit of adiabatic electrons $\omega < k_\parallel v_{Te}$, the electron density perturbation can be readily found to be

$$n_e = \left(\phi - \frac{\omega - \omega_{*e}}{ck_\parallel}A_\parallel\right)\frac{e}{T_e}n_0. \tag{9}$$

In the same limit, the electron parallel current perturbation can be calculated from

$$\begin{aligned} J_{\parallel e} &= \frac{n_0 e^2}{T_e}\int v_\parallel \frac{\omega - \hat{\omega}_{*e}(v^2)}{\omega - \hat{\omega}_{De}(\mathbf{v}) - k_\parallel v_\parallel}\left(\phi - \frac{v_\parallel}{c}A_\parallel\right) f_{Me} d^3v \\ &\simeq \frac{n_0 e^2}{k_\parallel T_e}\left[(\omega_{*e} - \omega)\phi + \frac{(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e}\omega_{De}}{ck_\parallel}A_\parallel\right]. \end{aligned} \tag{10}$$

Ions are assumed to be adiabatic as well in light of short wavelength nature $(k_\perp \rho_i)^2 \gg 1$,

$$n_i = -\frac{e\phi}{T_i}n_0.$$

Then charge neutrality $n_e = n_i$ and Ampere's law

$$k_\perp^2 A_\parallel = \frac{4\pi}{c}J_{\parallel e},$$

yield the following dispersion relation,

$$(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e}\omega_{De} - \frac{(\omega - \omega_{*e})^2}{1+\tau} = \frac{c^2 k_\parallel k_\perp^2 k_\parallel}{k_{De}^2}. \tag{11}$$

It is noted that the Debye screening factor $(k_\perp/k_{De})^2$ appears even though charge neutrality is assumed. In the RHS of Eq. (11), the small factor $(k_\perp/k_{De})^2 \ll 1$ is multiplied by the square of a large frequency $ck_\parallel$ (electromagnetic transit frequency) and balances the LHS which essentially describes the *electromagnetic* electron temperature gradient mode in the adiabatic limit.

In a uniform plasma $\omega_{*e} = \omega_{De} = 0$, the dispersion relation reduces to the known form [5],

$$\omega^2 = \frac{1+\tau}{\tau}\left(\frac{ck_\parallel k_\perp}{k_{De}}\right)^2 = \left(\frac{ck_\perp}{\omega_{pe}}\right)^2 k_\parallel^2 \frac{T_e + T_i}{m_e}. \tag{12}$$

The conditions of adiabatic ions $(k_\perp \rho_i)^2 \gg 1$ and adiabatic electrons $\omega < k_\parallel v_{Te}$ impose the range in the cross field wavenumber $k_\perp$ such that

$$\frac{1}{\rho_i} < k_\perp < \frac{\omega_{pe}}{c}, \tag{13}$$

where c/ω_{pe} is the collisionless electron skin depth. This is possible if the plasma β factor exceeds the electron/ion mass ratio, $\beta \gg m_e/m_i \simeq 3 \times 10^{-4}$, which is well satisfied in tokamaks. However, the dispersion relation in Eq. (11) pertinent to tokamaks is not subject to $ck_\perp < \omega_{pe}$.

Equation (11) should not be confused with the dispersion relation of the ETG mode derived by Kim and Horton [4] in the opposite, nonadiabatic limit $\omega > k_\parallel v_{Te}$,

$$\tau + \frac{\omega_{*e}}{\omega} + \left((k_\perp \rho_e)^2 - \frac{\omega_{De}}{\omega}\right)\left(1 - \frac{\omega_{*ep}}{\omega}\right) = \frac{\left(k_\parallel v_{Te}\right)^2}{\omega^2}\left(1 - \frac{\omega_{*ep}}{\omega}\right)\frac{k_\perp^2}{k_\perp^2 + \left(\omega_{pe}/c\right)^2}, \quad (14)$$

which is subject to $\omega \gg \omega_{De}$. Here, $\omega_{*ep} = (1+\eta_e)\,\omega_{*e}$. In this limit, electromagnetic effects appear as the inverse skin depth ω_{pe}/c in the right hand side, which is small because of the assumption $\omega > k_\parallel v_{Te}$. In contrast, the mode being discussed here is intrinsically electromagnetic and can be ballooning unstable in tokamaks.

Eq. (2) is the special case of the kinetic ballooning mode studied earlier [6],

$$(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e}\omega_{De} - \frac{(\omega - \omega_{*e})^2}{1 + \tau(1 - I_i)} = \frac{c^2 k_\parallel k_\perp^2 k_\parallel}{k_{De}^2}, \quad (15)$$

where

$$I_i = \int \frac{\omega + \hat{\omega}_{*i}(v^2)}{\omega + \hat{\omega}_{Di}(\mathbf{v})} J_0^2\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right) f_{Mi} d^3 v, \quad (16)$$

is the ion integral pertinent to the nonadiabatic part of the ion density perturbation,

$$n_i = -\frac{e\phi}{T_i}(1 - I_i)\, n_0. \quad (17)$$

Here $J_0(x)$ is the Bessel function, f_{Mi} is the unperturbed ion distribution assumed to be Maxwellian, and

$$\hat{\omega}_{*i}(v^2) = \omega_{*i}\left[1 + \eta_i\left(\frac{v^2}{v_{Ti}^2} - \frac{3}{2}\right)\right], \quad (18)$$

$$\hat{\omega}_{Di}(\mathbf{v}) = \frac{c m_i}{e B^3}\left(\frac{1}{2}v_\perp^2 + v_\parallel^2\right)(\nabla B \times \mathbf{B}) \cdot \mathbf{k}. \quad (19)$$

In the short wavelength limit $(k_\perp \rho_i)^2 \gg 1$, I_i vanishes, and we obtain the dispersion relation for the electron ballooning mode with adiabatic ions. The instability of interest here is of hydrodynamic nature (ballooning nature) even though both ions and electrons are adiabatic. The growth rate is large and, consequently, large electron thermal transport should emerge. (The instability does not contribute to particle transport because of adiabatic ions.)

The quadratic dispersion relation in Eq. (11) may be solved if the norm of the parallel gradient $k_\parallel$ is specified. As a rough estimate, we assume $k_\parallel \simeq 1/(2qR)$. Then the root is given by

$$\frac{\omega}{\omega_{*e}} = 1 - \frac{1+\tau}{2\tau}(1 - 2\varepsilon_n) + i\frac{\gamma}{\omega_{*e}}, \quad (20)$$

where γ/ω_{*e} is the normalized growth rate

$$\frac{\gamma}{\omega_{*e}} = \frac{1+\tau}{2\tau}\sqrt{\frac{4\tau}{1+\tau}\left(2\varepsilon_n\eta_e - \frac{\varepsilon_n(1+\eta_e)}{2\alpha_e}\right) - (1 - 2\varepsilon_n)^2}. \quad (21)$$

The condition for instability is given

$$\alpha_e\left[8\varepsilon_n\frac{\tau\eta_e}{1+\tau} - (1 - 2\varepsilon_n)^2\right] > 2\varepsilon_n\frac{\tau}{1+\tau}(1+\eta_e). \quad (22)$$

For typical tokamak discharge parameters, the instability criterion reduces to $\alpha_e \gtrsim 0.3$. The source of instability is in the interchange term $(1+\eta_e)\,\omega_{*e}\omega_{De}$ due to the combination of unfavorable magnetic curvature and electron pressure gradient. The mode described by Eq. (11) may be called an electron ballooning mode. When compared with the ideal MHD ballooning mode symbolically described by

$$\omega\left(\omega+\omega_{*i}\right)\left(k_\perp\rho_i\right)^2 = \left(k_\perp\rho_i\right)^2\left(k_\parallel V_A\right)^2 - \left(1+\eta_e\right)\omega_{De}\omega_{*i} - \left(1+\eta_i\right)\omega_{Di}\omega_{*i}, \qquad (23)$$

where V_A is the Alfven speed, the role of stabilizing Alfven frequency $k_\parallel V_A$ in MHD ballooning mode is played by the modified electron transit frequency $(ck_\perp/\omega_{pe})\,k_\parallel v_{Te}$. As is well known, the growth rate of the ideal MHD ballooning mode is essentially independent of the ion finite Larmor radius parameter $k_\perp\rho_i$ since $\omega_{De}\omega_{*i} \propto k_\perp^2$, while the growth rate of the electron ballooning mode sensitively depends on $k_\perp$.

The condition for the instability given in Eq. (22) is for hydrodynamic ballooning mode and may be relaxed if kinetic effects (electron resonance) are considered. As will be shown in the section to follow, fully kinetic analysis will reveal that the instability persists even in electrostatic limit (although the growth rate is small).

In the mode described by Eq. (12) for a uniform plasma, energy equipartition holds between the magnetic energy and thermal potential energy. They are out of phase and the sum of the two energy forms is constant, consistent with the general constraint on energy relationship in plasma waves [7]. The magnetic energy density associated with the wave is

$$\begin{aligned} U_m &= \frac{1}{8\pi}k_\perp^2 A_\parallel^2 = \frac{1}{8\pi}\frac{\left(1+\tau\right)^2 c^2 k_\parallel^2 k_\perp^2}{\omega^2}\phi^2 \\ &= \frac{1}{8\pi}\tau\left(1+\tau\right)k_{De}^2\phi^2 = \frac{1}{8\pi}\left(1+\tau\right)k_{Di}^2\phi^2, \end{aligned} \qquad (24)$$

while the potential energy density is

$$\begin{aligned} U_p &= \frac{1}{2}n_0 T_i\left(\frac{n_i}{n_0}\right)^2 + \frac{1}{2}n_0 T_e\left(\frac{n_e}{n_0}\right)^2 \\ &= \frac{1}{8\pi}k_{Di}^2\phi^2 + \frac{1}{2}\frac{n_0 e^2}{T_e}\left(\phi - \frac{\omega}{ck_\parallel}A_\parallel\right)^2 \\ &= \frac{1}{8\pi}\left(1+\tau\right)k_{Di}^2\phi^2, \end{aligned} \qquad (25)$$

in agreement with the magnetic energy density. Here the charge neutrality relationship $(1+\tau)\,ck_\parallel\phi = \omega A_\parallel$ has been substituted. It is noted that the dispersion relation is independent of electron and ion masses and thus no kinetic energy is involved in the wave.

3 Nonlocal analysis

In order to confirm destabilization of the mode by the ballooning effect in a more rigorous manner, a fully kinetic, electromagnetic integral equation code [8] has been employed to find the mode frequency and growth rate. We consider a high temperature, low β tokamak discharge with eccentric circular magnetic surfaces. Trapped electrons are ignored for simplicity. Also, the magnetosonic perturbation ($\mathbf{A}_\perp$) is ignored in light of the low β assumption and we employ the two-potential (ϕ and $A_\parallel$) approximation to describe electromagnetic modes. As in the preceding section, the basic field equations are the charge neutrality condition (subject to $k^2 \ll k_{De}^2$)

$$n_i(\phi, A_\parallel) = n_e(\phi,\, A_\parallel), \tag{26}$$

and the parallel Ampere's law,

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c} J_\parallel(\phi,\, A_\parallel), \tag{27}$$

where the density perturbations are given in terms of the perturbed velocity distribution functions f_i and f_e by

$$n_i = \int f_i d\mathbf{v}, \quad n_e = \int f_e d\mathbf{v}, \tag{28}$$

and the parallel current by

$$J_\parallel = e \int v_\parallel (f_i - f_e) d\mathbf{v}. \tag{29}$$

The perturbed distribution functions f_i and f_e can be found from the gyro-kinetic equation in the form

$$f_i = -\frac{e\phi}{T_i} f_{Mi} + g_i(v, \theta) J_0(\Lambda_i), \tag{30}$$

$$f_e = \frac{e\phi}{T_e} f_{Me} + g_e(v, \theta) J_0(\Lambda_e), \tag{31}$$

where $g_{i,e}$ are the nonadiabatic parts that satisfy

$$\left(i \frac{v_\parallel(\theta)}{qR} \frac{\partial}{\partial \theta} + \omega + \widehat{\omega}_{Di} \right) g_i = (\omega + \widehat{\omega}_{*i}) J_0(\Lambda_i) \left(\phi - \frac{v_\parallel}{c} A_\parallel \right) \frac{e}{T_i} f_{Mi}, \tag{32}$$

$$\left(i \frac{v_\parallel(\theta)}{qR} \frac{\partial}{\partial \theta} + \omega - \widehat{\omega}_{De} \right) g_e = -(\omega - \widehat{\omega}_{*e}) J_0(\Lambda_e) \left(\phi - \frac{v_\parallel}{c} A_\parallel \right) \frac{e}{T_e} f_{Me}. \tag{33}$$

Here, θ is the extended poloidal angle in the ballooning space, J_0 is the Bessel function with argument $\Lambda_{i,e} = k_\perp v_\perp / \omega_{ci,e}$,

$$k_\perp^2 = k_\theta^2 \left[1 + (s\theta - \alpha \sin\theta)^2 \right],$$

$$\widehat{\omega}_{Dj} = 2\varepsilon_n \omega_{*j} \left[\cos\theta + (s\theta - \alpha\sin\theta)\sin\theta \right] \left(\frac{1}{2} \hat{v}_\perp^2 + \hat{v}_\parallel^2 \right),$$

and qR is the connection length. For circulating particles, g_j ($j = i, e$) can be integrated as

$$v_\parallel > 0, \qquad g_j^+ = -i \frac{e_j f_{Mj}}{T_j} \int_{-\infty}^{\theta} d\theta' \frac{qR}{|v_\parallel|} e^{i\beta_j} (\omega - \widehat{\omega}_{*j}) J_0(\Lambda_j') \left(\phi(\theta') - \frac{|v_\parallel|}{c} A_\parallel(\theta') \right), \tag{34}$$

$$v_\parallel < 0, \qquad g_j^- = -i \frac{e_j f_{Mj}}{T_j} \int_{\theta}^{\infty} d\theta' \frac{qR}{|v_\parallel|} e^{-i\beta_j} (\omega - \widehat{\omega}_{*j}) J_0(\Lambda_j') \left(\phi(\theta') + \frac{|v_\parallel|}{c} A_\parallel(\theta') \right), \tag{35}$$

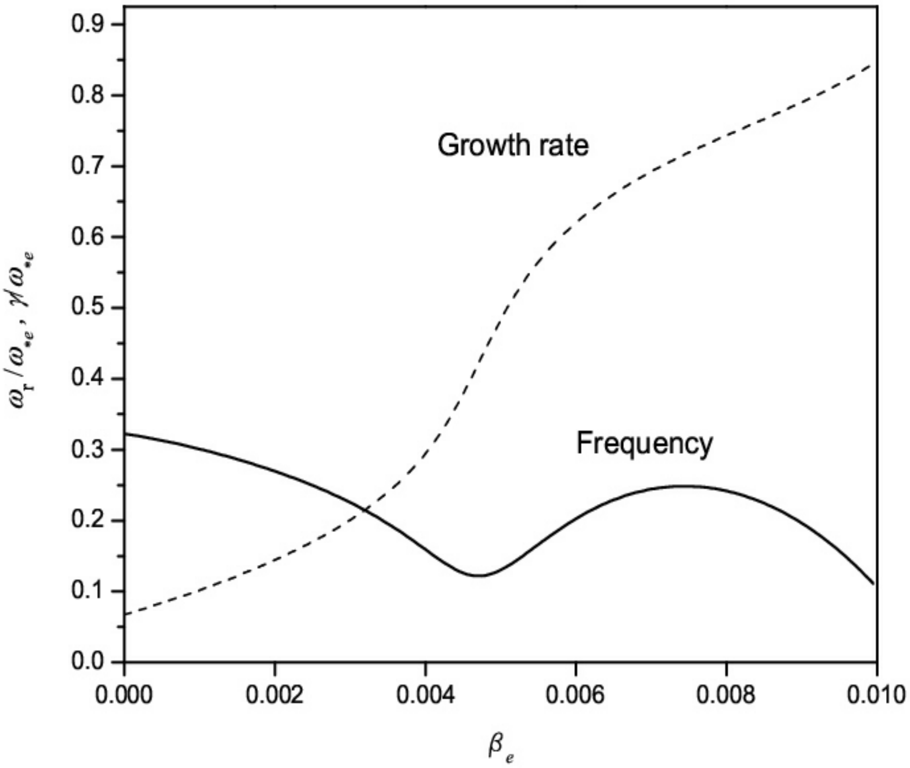

Figure 1: The normalized mode frequency ω_r/ω_{*e} and growth rate γ/ω_{*e} as functions of the electron beta β_e when $ck_\theta/\omega_{pe} = 0.3$, $L_n/R = 0.2$, $\eta_e = 2$, $q = 2$, $s = 1$.

where

$$\beta_j(\theta, \theta') = \int_{\theta'}^{\theta} \frac{qR}{|v_\parallel|}[\omega - \widehat{\omega}_{Dj}(\theta'')]d\theta''.$$

Substitution of perturbed distribution functions into charge neutrality and parallel Ampere's law yields

$$\nabla^2\phi = -4\pi \sum_{j=i,e} e_j \left(-\frac{e_j}{T_j}\phi + \int \left[g_j^+(\theta) + g_j^-(\theta)\right] J_0(\Lambda_j) d\mathbf{v} \right), \tag{36}$$

$$\nabla_\perp^2 A_\parallel(\theta) = -\frac{4\pi}{c} \sum_{j=i,e} e_j \int v_\parallel \left[g_j^+(\theta) - g_j^-(\theta)\right] J_0(\Lambda_j) d\mathbf{v}, \tag{37}$$

where $\int d\mathbf{v} = 2\pi \int_0^\infty v_\perp dv_\perp \int_0^\infty dv_\parallel$. This system of inhomogeneous integral equations can be solved by employing the method of Fredholm in which the integral equations are viewed as a system of linear algebraic equations [9]. In the numerical code, the velocity space integration is executed using Gauss-Hermite approximation.

Figure 1 shows the β_e dependence of the normalized eigenvalue ω/ω_{*e} (frequency ω_r/ω_{*e} and growth rate γ/ω_{*e}) when $ck_\theta/\omega_{pe} = 0.3$, $T_e = T_i$, $L_n/R = 0.3$, $\eta_e = 2$, $\beta_e = 0.5\%$, $s = 1$, $q = 2$, $m_i/m_e = 1836$ (hydrogen). The growth rate increases rapidly with the electron pressure (β_e) indicating that the instability is indeed driven by the ballooning effect. The growth rate found in the numerical analysis qualitatively agrees with the analytic expression presented in Eq. (21).

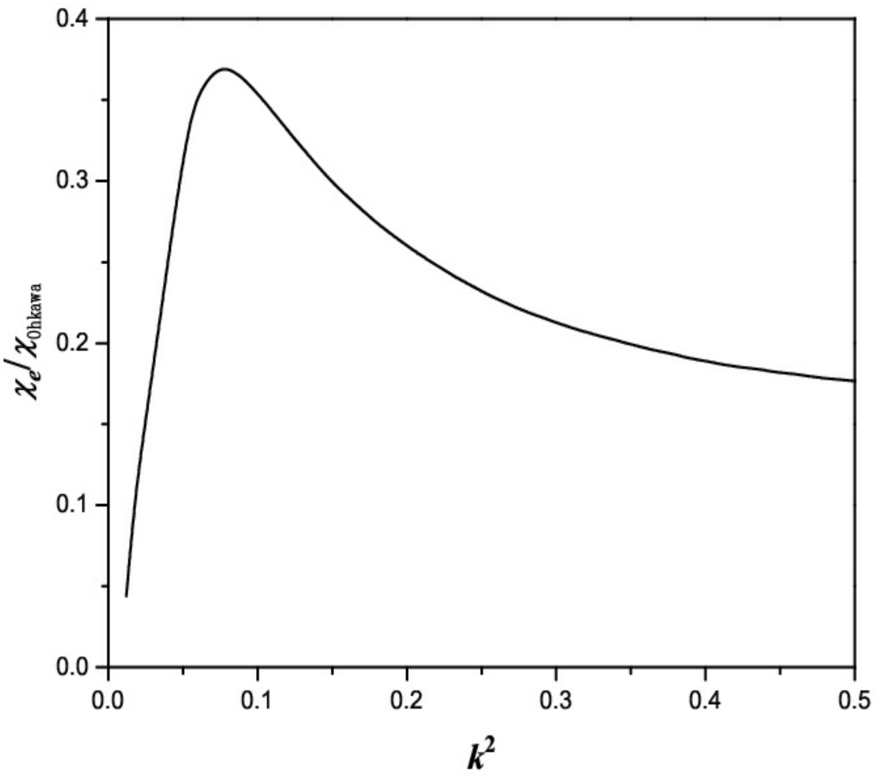

Figure 2: χ_e/χ_{Ohkawa} vs. $k^2 = (ck_\theta/\omega_{pe})^2$ when $L_n/R = 0.2$, $\eta_e = 2$, $s = 1$, $q = 2$, and $\beta_e = 0.004$.

The mixing length electron thermal diffusivity $\chi_e = \gamma/k_\perp^2$ normalized by the Ohkawa diffusivity

$$\chi_{Ohkawa} = \frac{v_{Te}}{qR}\left(\frac{c}{\omega_{pe}}\right)^2, \tag{38}$$

is shown in Figure 2 as a function of $k^2 = (ck_\theta/\omega_{pe})^2$ and for the case $\beta_e = 0.4\%$. The diffusivity increases with k^2 and attains a value which is a large fraction of the Ohkawa diffusivity. The maximum diffusivity occurs at $ck_\perp/\omega_{pe} \simeq 0.3$. This is demonstrated in Figure 3 in which the diffusivity is plotted as a function of β_e. The diffusivity in Figure 3 is normalized by the electron gyro-Bohm diffusivity given by

$$\chi_{egB} = \frac{v_{Te}}{L_n}\rho_e^2.$$

It is clearly seen that the diffusivity far exceeds the electron gyro-Bohm diffusivity which pertains to the ETG mode.

In light of the analytic dispersion relation found in this study, it may be concluded that a tokamak discharge can be strongly unstable in the wavelength regime $k_\perp \simeq \omega_{pe}/c$ which is the lower end of unstable $k_\perp$ spectrum. The growth rate is of the order of $\gamma \simeq \sqrt{\eta_e\omega_{*e}\omega_{De}}$. Therefore, for the electron thermal diffusivity, the following estimate emerges,

$$\chi_e \simeq \frac{\sqrt{\eta_e\omega_{*e}\omega_{De}}}{k_\perp^2} = \frac{qv_{Te}}{\sqrt{RL_T}}\left(\frac{c}{\omega_{pe}}\right)^2\sqrt{\beta_e},$$

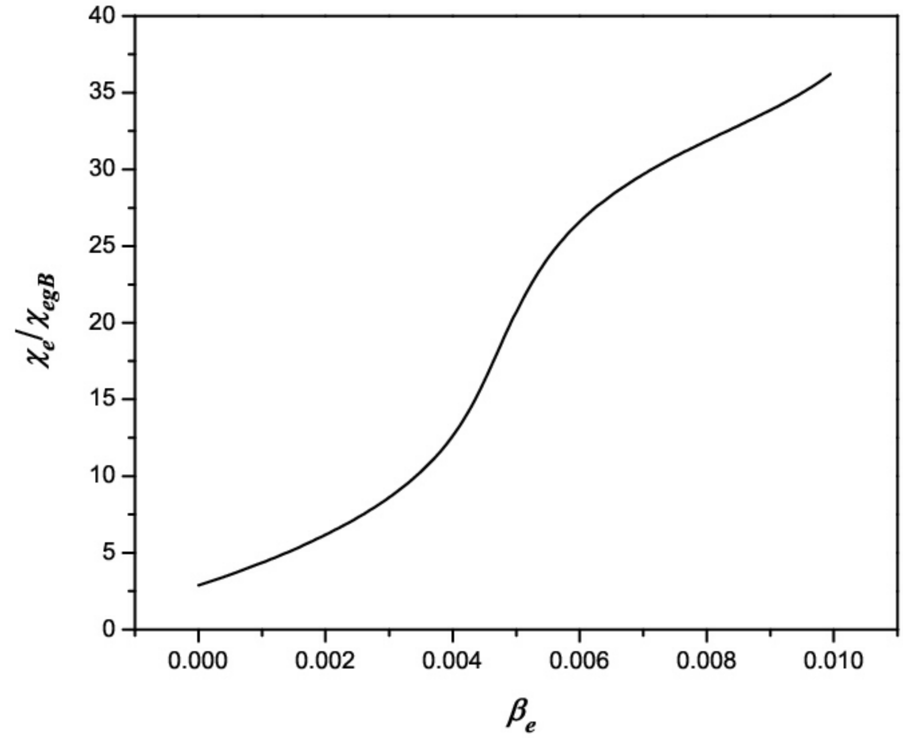

Figure 3: χ_e/χ_{egB} ($\chi_{egB} = v_{Te}\rho_e^2/L_n$ the electron gyro Bohm diffusivity) as a function of the electron β factor. $L_n/R = 0.3$, $\eta_e = 2$, $q = 2$, $s = 1$.

where L_T is the temperature gradient scale length. The proportionality of the diffusivity to the safety factor, $\chi_e \propto q$, stems from the condition of most active thermal transport, $\gamma \simeq k_\parallel v_{Te}$, which yields $k_\perp \propto 1/q$ [10].

4 Discussions and conclusions

The local dispersion relation

$$(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e}\omega_{De} - \frac{(\omega - \omega_{*e})^2}{1+\tau} = \frac{c^2 k_\parallel k_\perp^2 k_\parallel}{k_{De}^2},$$

derived in this study describes the short wavelength electron ballooning mode subject to the adiabatic electron response $\omega < k_\parallel v_{Te}$. The mode is intrinsically electromagnetic while the conventional ETG mode is subject to $\omega > k_\parallel v_{Te}$ for which electromagnetic effects appear only as a small correction. Because of the long wavelength nature of the instability ($c/\omega_{pe} \gg \rho_e$), large electron thermal transport emerges even in simple mixing length estimate. The following formula for the electron thermal diffusivity has been found,

$$\chi_e = \frac{q v_{Te}}{\sqrt{RL_T}} \left(\frac{c}{\omega_{pe}}\right)^2 \sqrt{\beta_e}.$$

In the previous investigations [Horton], it was proposed skin size plasma turbulence may exist if the nonlinear Doppler shift $\mathbf{k}\cdot\mathbf{v}_{E\times B}$ ($\mathbf{v}_{E\times B}$ being the $E \times B$ drift) exceeds the electron transit frequency $\mathbf{k} \cdot \mathbf{v}_{E\times B} > k_{\parallel} v_{Te}$. The main finding in the present investigation is that the skin depth manifests itself even in the adiabatic limit and governs the lower end of the k spectrum of the ETG mode. It is noted that in the limit of large ballooning parameter α_e, the dispersion relation reduces to

$$(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e} \omega_{De} - \frac{(\omega - \omega_{*e})^2}{1+\tau} = 0,$$

which resembles that of the ETG mode in the limit $\omega > k_{\parallel} v_{Te}$,

$$\tau - \frac{\omega_{*e} - \omega_{De}}{\omega - \omega_{De}} + \frac{\eta_e \omega_{*e} \omega_{De}}{(\omega - \omega_{De})^2} = 0.$$

In summary, a novel electromagnetic ballooning instability having cross-field wavelengths of the order of electron skin depth has been identified analytically and confirmed with a rigorous integral equation code which is fully kinetic and electromagnetic. In tokamaks, the mode is destabilized by a modest electron ballooning parameter α_e. A large electron thermal diffusivity emerges because of the long wavelength nature of the instability.

5 Acknowledgments

Helpful discussions with N. Joiner, A. Smolyakov and P. K. Shukla are acknowledged with gratitude.

This research has been supported by the Natural Sciences and Engineering Research Council of Canada and by Canada Research Chair Program.

References

[1] T. Ohkawa, Phys. Lett. **67A**, 35 (1978).

[2] P. R. Parker, et al., Nucl. Fsuion **25**, 1127 (1985).

[3] W. Horton, Phys. Reports **192**, 1 (1990).

[4] J.Y. Kim and W. Horton, Phys. Fluids B **4**, 3194 (1991).

[5] A. B. Mikhailovskii, Sov. Phys.-JTP **37**, 1365 (1967).

[6] A. Hirose, L. Zhang, and M. Elia, Phys. Rev. Lett. **72**, 3993 (1994).

[7] J. F. Denisse and J. L. Delcroix, in *Plasma Waves*, (Interscience Publsihers, New York, 1963), p. 52.

[8] M. Elia, Ph.D. thesis, University of Saskatchewan (2000).

[9] G. Rewoldt, et al., Phys. Fluids **25**, 480 (1982).

[10] A. Hirose et al., Nucl. Fusion **45**, 1628 (2005).