

Generalized Balescu Kinetic Equations

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Abstract

The famous Balescu-Lenard kinetic equation was generalized for weakly non-ideal plasma taking into account exchange correlation in the dielectric function. Starting with the quantum BBGKY-hierarchy for the distribution functions, we have obtained the quantum non-Markovian kinetic equation including the dynamical screening of the interaction potential, which exactly takes into account the exchange scattering in the plasma. The collisional integral is expressed in terms of the Green function of the linearized Hartree - Fock equation. This quantum kinetic equation satisfies the law of total energy conservation with account of the polarization and exchange interaction.

1 Preface

I had an enormous privilege to be close to this Famous Scientist and a person of integrity - the two qualities that are not often compatible. He was an honest and faithful student, and a modest and proud teacher. He was in peace with everybody. Radu Balescu as well as his teacher Ilya Prigogine directly and indirectly helped many peoples, although not all of them remember it.

I met Radu, or among friends, Balu, at the end of 1960s at the Moscow University. I was a PhD student of Klimontovich. One day, Klimontovich asked us to organize a party. So, Yuri L'vovich took advantage of this because many pretty girls came to Radu's party.

Next time Radu came to Moscow State University in 1971 as a guest professor. At that time editing house "MIR" had published the translation of his first book "Statistical Mechanics of Charged Particles" [1], which was translated by our famous specialists on plasma physics A.A. Rukhadze and V. P. Silin. By the way, that book as well as the other book, translated into Russian "Equilibrium and Nonequilibrium Statistical Mechanics" [2] are written with great pedagogical talent and do still serve students and scholars. He received a royalty from "MIR", which he had to spend within the USSR. As one of his daughters was starting studying a cello, Radu figured out to buy that instrument. He imagined naively that it would be an easy task to find it in the country of Rostropovich. However, it came out to be an illusion. Finally, through relationships ("po blatu") he managed to find a cello (made in the DDR) and a bow (made in Czechoslovakia), but without a bag. Thus it was carefully wrapped in old newspapers. After that Radu had

a big trouble with the customs, where he had to undo this beautiful package, so that the officer could check inside in order to verify the absence of dollars and of any prohibited document. Nevertheless, his daughter had never used this cello, because she gave up its study, but the history hold true.

Later, thanks to Radu I started a fruitful collaboration with Irina Veretennicoff and continued with Jean Wallenborn. Then I had a chance to have long meetings with him in Moscow, Tashkent, Brussels, and les Treille. In Treille Radu, Dilip Kondepudi and myself shared a cabana. Joint breakfasts on terrace are unforgettable. His dietary habits, tobacco and coffee worried us. But he repeated: I prefer the life short but nice. His dream came true, he passed away happy and he did not live a decay.

For the last time we met at the end of 2005. He was in a good form and full of joy. He got younger because of his granddaughter Luna. This time, I brought some Russian woody toys for her. They were both very glad (see photo).

Every scientist has a deep willingness to make a track in the Science, for example an equation or a theorem is holding their name. And not many of them get lucky to be successful in it. Professor Balescu was lucky, as by his name the famous Balescu-Lenard equation was called. Balescu successfully used Prigogine's diagram technique for plasma, and derived the closed kinetic equation taking dynamical screening due to plasma polarization into account [3]. Although this equation did not receive many applications, it is fundamental in plasma physics and is comparable with the Boltzmann equation for diluted gas. Later the same equation was derived by others more simple but not so rigorous methods.

2 Introduction

Before Balescu and Lenard [4] the plasma description was restricted to Vlasov equation [5] with self-consistent field or to Landau equation [6], which was derived from Boltzmann equation by expansion in powers of small momentum transfer. In these conditions, the presence of the Coulomb potential interaction leads to divergence of the collision integral for small wave-numbers. To avoid this divergence Landau introduced ad hoc the cutoff at the Debye wave number. Balescu, on the contrary, took into account collective interaction of particles in the collision integral and the divergence in his collision integral disappeared.

The Balescu-Lenard (BL) kinetic equation takes into account the polarization of the system only in the collision integral, while the thermodynamics corresponds to the ideal gas; the dissipative and non-dissipative phenomena are not treated on an equal footing. This discrepancy can be avoided if non-Markovian effects are considered. In the book by Kadanoff and Baym [9], general non-Markovian equations for two-time Green functions were derived. But in the two-time Green function method a question arose which sort of relationships occurs between these functions and one-time distribution function, in other words, of how the time argument in the distribution function should be chosen. In the non-Markovian case this question is especially important. Kadanoff and Baym make their choice on the basis of a certain hypothesis (the KB Ansatz). Klimontovich [10] proceeded from the BBGKY hierarchy for the one-time distribution functions where no such problem arises. He obtained explicit expression for non-Markovian Landau collision integrals of and Boltzman kinetic equations. Later the KB method and BBGKY hierarchy were used in deriving non-Markovian quantum equations for weakly coupled systems and

neutral particle systems with very short interaction range [11]. In order to describe non-Markovian processes in a polarizable plasma, Klimontovich [12] derived a set of equations for the particle distribution function and the electric field spectral function. However, the equation for the spectral function was not solved, and the collision integral was obtained only in the averaged potential approximation. The non-linear kinetic equation, which generalized the BL equation for a spatially-uniform weakly non-ideal multicomponent classical plasma has been obtained in [13] and for non-uniform plasma in [14].

The quantum kinetic equation for weakly coupled polarizable plasma has been derived by Balescu [7] and Guernsey [8]. In that equation the exchange interaction of particles was retained only in the distribution functions. But it is also necessary to take into account the exchange interaction in the scattering amplitude and in the dielectric function.

In the quantum case starting from the quantum BBGKY-hierarchy for the distribution function we have solved, in the so-called plasma approximation the equation for the quantum pair correlation function, the non-Markovian correction included. The solution to this equation can be expressed in terms of the resolvent of the linearized Hartree-Fock equation. As a result, we obtain a quantum non-Markovian kinetic equation, which takes into account the dynamical screening of the interaction potential and the exchange interaction in a non trivial way. In particular, this equation contains the dielectric function, which exactly describes the exchange scattering in a plasma. The quantum kinetic equation derived satisfies the law of total energy conservation with account of the polarization and exchange interaction.

3 The classical non-Markovian Balescu-Lenard kinetic equation

We consider a spatially-uniform weakly nonideal multicomponent plasma. Out of equilibrium, its evolution is exactly described by the BBGKY- hierarchy [15] for the reduced distribution function (df) or, equivalently, for the correlation functions. Since there are no mean field in an uniform system, the two first equation are:

$$\begin{aligned}
\frac{\partial}{\partial t} f_a(\mathbf{p}_1, t) = & \sum_b \int d\mathbf{r}_2 d\mathbf{p}_2 \left[\frac{\partial}{\partial \mathbf{r}_1} \Phi_{ab}(|\mathbf{r}_1 - \mathbf{r}_2|) \right] \cdot \frac{\partial}{\partial \mathbf{p}_1} g_{ab}(\mathbf{p}_1, \mathbf{p}_2, |\mathbf{r}_1 - \mathbf{r}_2|, t), \quad (1) \\
& \left[\frac{\partial}{\partial t} + \mathbf{v}_1 \cdot \frac{\partial}{\partial \mathbf{r}_1} + \mathbf{v}_2 \cdot \frac{\partial}{\partial \mathbf{r}_2} \right] g_{ab}(\mathbf{p}_1, \mathbf{p}_2, |\mathbf{r}_1 - \mathbf{r}_2|, t) \\
& - \sum_c \int d\mathbf{r}_3 d\mathbf{p}_3 \left\{ \frac{\partial}{\partial \mathbf{r}_1} \Phi_{ac}(|\mathbf{r}_1 - \mathbf{r}_3|) \cdot \frac{\partial}{\partial \mathbf{p}_1} f_a(\mathbf{p}_1, t) g_{bc}(\mathbf{p}_2, \mathbf{p}_3, |\mathbf{r}_2 - \mathbf{r}_3|, t) \right. \\
& + \frac{\partial}{\partial \mathbf{r}_2} \Phi_{bc}(|\mathbf{r}_2 - \mathbf{r}_3|) \cdot \frac{\partial}{\partial \mathbf{p}_2} f_b(\mathbf{p}_2, t) g_{ac}(\mathbf{p}_1, \mathbf{p}_3, |\mathbf{r}_1 - \mathbf{r}_3|, t) + \left[\frac{\partial}{\partial \mathbf{r}_1} \Phi_{ac}(|\mathbf{r}_1 - \mathbf{r}_3|) \cdot \frac{\partial}{\partial \mathbf{p}_1} \right. \\
& \left. + \frac{\partial}{\partial \mathbf{r}_2} \Phi_{bc}(|\mathbf{r}_2 - \mathbf{r}_3|) \cdot \frac{\partial}{\partial \mathbf{p}_2} \right] g_{abc}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, |\mathbf{r}_1 - \mathbf{r}_2|, |\mathbf{r}_2 - \mathbf{r}_3|, t) \left. \right\} \\
& - \frac{\partial}{\partial \mathbf{r}_1} \Phi_{ab}(|\mathbf{r}_1 - \mathbf{r}_2|) \cdot \left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_1} \right) g_{ab}(\mathbf{p}_1, \mathbf{p}_2, |\mathbf{r}_1 - \mathbf{r}_2|, t)
\end{aligned}$$

$$= \frac{\partial}{\partial \mathbf{r}_1} \Phi_{ab}(|\mathbf{r}_1 - \mathbf{r}_2|) \cdot \left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_1} \right) f_a(\mathbf{p}_1, t) f_b(\mathbf{p}_2, t), \quad (2)$$

where $f_a(\mathbf{p}_1, t)$ is the one-particle df for species a and $g_{ab}(\mathbf{p}_1, \mathbf{p}_2, |\mathbf{r}_1 - \mathbf{r}_2|, t)$ is the pair correlation function, $\mathbf{r}_1, \mathbf{p}_1, \mathbf{v}_1 = \frac{\mathbf{p}_1}{m_a}$ are respectively the position, the momentum and the velocity of particle 1, $\Phi_{ab}(|\mathbf{r}_1 - \mathbf{r}_2|)$ is the interaction potential between particle 1 of species a and particle 2 of species b. In order to close this set of equations, we make the usual so-called plasma approximation: triple correlation function as well as interaction between correlated particles are neglected. In this way, the last two terms of the l.h.s. of eq. (2) are dropped. In the Fourier space, the closed set of equations takes then the form:

$$\frac{\partial}{\partial t} f_a(\mathbf{p}_1, t) = J_a(\mathbf{p}_1, t) = -\frac{i}{8\pi^3} \sum_b \int d\mathbf{k} d\mathbf{p}_2 \Phi_{ab}(\mathbf{k}) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_1} g_{ab}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t), \quad (3)$$

$$\begin{aligned} & \left[\frac{\partial}{\partial t} + i\mathbf{k} \cdot (\mathbf{v}_1 - \mathbf{v}_2) \right] g_{ab}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) \\ & - i\mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_1} f_a(\mathbf{p}_1, t) \sum_c \Phi_{ac}(\mathbf{k}) \int d\mathbf{p}_3 g_{bc}(\mathbf{p}_2, \mathbf{p}_3, \mathbf{k}, t) \\ & + i\mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_2} f_b(\mathbf{p}_2, t) \sum_c \Phi_{bc}(\mathbf{k}) \int d\mathbf{p}_3 g_{ac}(\mathbf{p}_1, \mathbf{p}_3, \mathbf{k}, t) \\ & = i\Phi_{ab}(\mathbf{k}) \mathbf{k} \cdot \left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2} \right) f_a(\mathbf{p}_1, t) f_b(\mathbf{p}_2, t), \end{aligned} \quad (4)$$

where

$$g_{ab}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) = \int d\mathbf{r} \exp(-i\mathbf{k} \cdot \mathbf{r}) g_{ab}(\mathbf{p}_1, \mathbf{p}_2, |\mathbf{r}|, t), \quad (5)$$

and

$$\Phi_{ab}(\mathbf{k}) = \frac{4\pi e_a e_b}{k^2} \quad (6)$$

is the Fourier transform of the Coulomb potential. We also defined in eq. (3) the collision integral $J_a(\mathbf{p}_1, t)$. To solve eq. (4) it is generally assumed that the time dependence of the one-particle df can be neglected in regard with the collision time-scale. That results in the Markovian BL equation. To obtain a non-Markovian kinetic equation one has to consider explicitly the time dependence of all the functions. Nevertheless, it is necessary to distinguish between fast and slow time scales.

In order to avoid unnecessary complications in this study, we introduce compact notations. Let us define a scalar product as:

$$\underline{A} \cdot \underline{B} = \sum_{ab} \int d\mathbf{p}_1 d\mathbf{p}_2 A_{ab}(\mathbf{p}_1, \mathbf{p}_2) B_{ab}(\mathbf{p}_1, \mathbf{p}_2), \quad (7)$$

and rewrite eq.(4):

$$(\partial_t - \underline{L} \cdot + \underline{\Gamma}(t) \cdot) \underline{g}(t) = \underline{A}(t), \quad (8)$$

where

$$(\underline{L})_{aba'b'}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2) = -i\mathbf{k} \cdot \left(\frac{\mathbf{p}_1}{m_a} - \frac{\mathbf{p}_2}{m_b} \right) \delta_{aa'} \delta_{bb'} \delta(\mathbf{p}_1 - \mathbf{p}'_1) \delta(\mathbf{p}_2 - \mathbf{p}'_2) \quad (9)$$

$$\begin{aligned}
(\underline{\Gamma}(t))_{aba'b'}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2) &= -i\mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_1} f_a(\mathbf{p}_1, t) \Phi_{aa'}(\mathbf{k}) \delta_{bb'} \delta(\mathbf{p}_2 - \mathbf{p}'_2) \\
&\quad + i\mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_2} f_b(\mathbf{p}_2, t) \Phi_{bb'}(\mathbf{k}) \delta_{aa'} \delta(\mathbf{p}_1 - \mathbf{p}'_1)
\end{aligned} \tag{10}$$

$$\underline{A}_{ab}(\mathbf{p}_1, \mathbf{p}_2, t) = i\Phi_{ab}(\mathbf{k})\mathbf{k} \cdot \left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2} \right) f_a(\mathbf{p}_1, t) f_b(\mathbf{p}_2, t). \tag{11}$$

We shall formally solve eq. (8) by the method of Green's function's [1].

When t is large enough, the initial correlations have decayed and the solution of eq. (8) can be written as:

$$\underline{g}(t) = \int_{-\infty}^t dt' \underline{R}(t, t') \cdot \underline{A}(t'), \tag{12}$$

where the Green's function $\underline{R}(t, t')$ is the solution of the equation:

$$(\partial_t - \underline{L} \cdot + \underline{\Gamma}(t) \cdot) \underline{R}(t, t') = \delta(t - t') \underline{I}, \tag{13}$$

with the causality condition $\underline{R}(t, t') = 0$ if $t < t'$.

For stationary and spatially uniform systems the df f_a and the operator $\widehat{\Gamma}_a$ do not depend on time and space. In this case, the dependence on time and space of the Green function $\underline{R}(t, t')$ appears only through the difference $t - t'$. However, when the df $f_a(\mathbf{p}, \mu t)$ and $\widehat{\Gamma}_a(\mathbf{p}, \mu t)$ are slowly varying quantities in time, and when nonlocal effects are considered, the time dependence of $\underline{R}(t, t')$ is more subtle:

$$\underline{R}(t, t') = \underline{R}(\mathbf{p}, \mathbf{p}', t - t', \mu t'). \tag{14}$$

For the homogeneous case this non-trivial result was obtained for the first time in our previous work [13]. This result was extended to inhomogeneous systems [14]. Here we want to stress that the nonlocal effects appear due to the slow time dependences $\mu t'$. Introducing this result in eq. (12) yields:

$$\underline{g}(t) = \int_0^{\infty} d\tau \underline{R}(\tau, \mu(t - \tau)) \cdot \underline{A}(\mu(t - \tau)), \tag{15}$$

where we took into account that $\underline{A}(t)$ varies on the time scale of the one-particle df. The Markovian limit of eq. (15) is obtained when $t - \tau$ is replaced by t in the integrand, i.e. when the one-particle df varies slowly on the time scale τ of $\underline{R}(\tau, \mu t)$.

In this paper, we shall only treat the first nontrivial approximation which leads to the total energy conservation, i.e. we consider the Markovian limit of eq. (15) and its first non-Markovian correction. Let us point out that it is not a trivial approximation since it leads to the total energy conservation. For this, we expand the integrand of eq. (15) up to first order in τ :

$$\underline{g}(t) = \int_0^{\infty} d\tau [1 - \mu\tau \partial_{\mu t}] \underline{R}(\tau, \mu t) \cdot \underline{A}(\mu t). \tag{16}$$

In the following for simplicity we omit μ keeping in mind that derivative over time is taken with respect to the slowly varying variable μt .

It is useful to introduce the resolvent $\underline{R}(z, \mu t)$ as the Laplace transform of $\underline{R}(\tau, \mu t)$:

$$\underline{R}(z, \mu t) = \int_0^{\infty} d\tau \exp(iz\tau) \underline{R}(\tau, \mu t), \quad \text{Im}z > 0. \quad (17)$$

Eq. (16) takes then the form:

$$\underline{g}(t) = \lim_{z \rightarrow i\varepsilon} [1 + i\partial_t \frac{\partial}{\partial z}] \underline{R}(z, t) \cdot \underline{A}(t). \quad (18)$$

Specifically we obtain:

$$g_{ab}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) = g_{ab}^0(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) + g_{ab}^1(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t), \quad (19)$$

where $g_{ab}^0(\mathbf{p}_a, \mathbf{p}_b, t)$ is the well known result of Markovian theory [1]:

$$g_{ab}^0(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) = i\pi \delta_-(\mathbf{k} \cdot \mathbf{v}_1 - \mathbf{k} \cdot \mathbf{v}_2) G_{ab}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) \quad (20)$$

with

$$\begin{aligned} G_{ab}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) = & \Phi_{ab}(\mathbf{k}) \left\{ \frac{1}{\varepsilon(\mathbf{k} \cdot \mathbf{v}_2, \mathbf{k})} f_b(\mathbf{p}_2, t) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_1} f_a(\mathbf{p}_1, t) \right. \\ & - \frac{1}{\varepsilon^*(\mathbf{k} \cdot \mathbf{v}_1, \mathbf{k})} f_a(\mathbf{p}_1, t) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_2} f_b(\mathbf{p}_2, t) - \sum_c i\pi \int d\mathbf{p}_3 \frac{\Phi_{cc}(\mathbf{k})}{|\varepsilon(\mathbf{k} \cdot \mathbf{v}_3, \mathbf{k})|^2} [\delta_+(\mathbf{k} \cdot \mathbf{v}_3 - \mathbf{k} \cdot \mathbf{v}_1) \\ & \left. + \delta_+(\mathbf{k} \cdot \mathbf{v}_2 - \mathbf{k} \cdot \mathbf{v}_3)] f_c(\mathbf{p}_3, t) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_1} f_a(\mathbf{p}_1, t) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_2} f_b(\mathbf{p}_2, t) \right\}, \quad (21) \end{aligned}$$

and

$$\varepsilon(\mathbf{k} \cdot \mathbf{v}, \mathbf{k},) = 1 - i\pi \sum_a \Phi_{aa}(\mathbf{k}) \int d\mathbf{p} \delta_+(\mathbf{k} \cdot \mathbf{v} - \mathbf{k} \cdot \mathbf{v}_1) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_1} f_a(\mathbf{p}_1, t) \quad (22)$$

is the dynamical dielectric function. We also used the conventional notations: $i\pi\delta_-(\mathbf{x}) = \frac{1}{x-i0} = i\pi\delta(\mathbf{x}) + \frac{P}{x}$, $-i\pi\delta_+(\mathbf{x}) = \frac{1}{x+i0} = -i\pi\delta(\mathbf{x}) + \frac{P}{x}$, where P means the Cauchy principal part. It can be shown that expression (20) reduces at equilibrium to the Debye-Hückel pair correlation function. The non-Markovian correction to the pair correlation function is:

$$\begin{aligned} g_{ab}^1(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) = & i \frac{\partial}{\partial t} \{ i\pi \delta'_-(\mathbf{k} \cdot \mathbf{v}_1 - \mathbf{k} \cdot \mathbf{v}_2) G_{ab}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) \\ & + \sum_c \frac{\Phi_{bc}(\mathbf{k})}{\varepsilon^*(\mathbf{k} \cdot \mathbf{v}_1, \mathbf{k})} i\pi \delta_-(\mathbf{k} \cdot \mathbf{v}_1 - \mathbf{k} \cdot \mathbf{v}_2) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_2} f_b(\mathbf{p}_2, t) \int d\mathbf{p}_3 i\pi \delta'_-(\mathbf{k} \cdot \mathbf{v}_1 - \mathbf{k} \cdot \mathbf{v}_3) G_{ac}(\mathbf{p}_1, \mathbf{p}_3, \mathbf{k}, t) \\ & - \sum_c \frac{\Phi_{ac}(\mathbf{k})}{\varepsilon(\mathbf{k} \cdot \mathbf{v}_2, \mathbf{k})} i\pi \delta_-(\mathbf{k} \cdot \mathbf{v}_1 - \mathbf{k} \cdot \mathbf{v}_2) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_1} f_a(\mathbf{p}_1, t) \int d\mathbf{p}_3 i\pi \delta'_-(\mathbf{k} \cdot \mathbf{v}_3 - \mathbf{k} \cdot \mathbf{v}_2) G_{cb}(\mathbf{p}_3, \mathbf{p}_2, \mathbf{k}, t) \\ & + \sum_{cd} i\pi \frac{P}{(\mathbf{k} \cdot \mathbf{v}_1 - \mathbf{k} \cdot \mathbf{v}_2)} \Phi_{ac}(\mathbf{k}) \Phi_{bd}(\mathbf{k}) \int d\mathbf{p}_3 \left[\frac{\delta_-(\mathbf{k} \cdot \mathbf{v}_3 - \mathbf{k} \cdot \mathbf{v}_1)}{|\varepsilon(\mathbf{k} \cdot \mathbf{v}_1, \mathbf{k})|^2} \right. \\ & \left. + \frac{\delta_-(\mathbf{k} \cdot \mathbf{v}_3 - \mathbf{k} \cdot \mathbf{v}_2)}{|\varepsilon(\mathbf{k} \cdot \mathbf{v}_2, \mathbf{k})|^2} \right] \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_1} f_a(\mathbf{p}_1, t) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_2} f_b(\mathbf{p}_2, t) \int d\mathbf{p}_4 i\pi \delta'_-(\mathbf{k} \cdot \mathbf{v}_3 - \mathbf{k} \cdot \mathbf{v}_4) G_{cd}(\mathbf{p}_3, \mathbf{p}_4, \mathbf{k}, t) \}, \quad (23) \end{aligned}$$

where $\delta'_+(\mathbf{x})$ is the derivative of $\delta_+(\mathbf{x})$. Finally, the correction to the collision integral is obtained from eq.(1):

$$J_a^1(\mathbf{p}_1, t) = -\frac{i}{8\pi^3} \sum_b \int d\mathbf{k} d\mathbf{p}_2 \Phi_{ab}(\mathbf{k}) \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}_1} g_{ab}^1(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t). \quad (24)$$

The conservation laws are easily verified with the following symmetry property in mind:

$$g_{ab}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t) = g_{ba}(\mathbf{p}_2, \mathbf{p}_1, -\mathbf{k}, t). \quad (25)$$

The particle conservation is a trivial consequence of expression (1) while the momentum conservation follows directly from eqs. (1) and (25). We know that kinetic energy is conserved by $J_a^0(\mathbf{p}_1, t)$, the Balescu-Lenard collision integral. Therefore, the variation of the kinetic energy is due to the non-Markovian part of the collision integral $J_a^1(\mathbf{p}_1, t)$:

$$\partial_t \sum_a \left\langle \frac{\mathbf{p}_1^2}{2m_a} \right\rangle = -\frac{1}{8\pi^3} \sum_{ab} \int d\mathbf{k} d\mathbf{p}_1 d\mathbf{p}_2 \Phi_{ab}(\mathbf{k}) (\mathbf{k} \cdot \mathbf{v}_1 - \mathbf{k} \cdot \mathbf{v}_2) g_{ab}^1(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t), \quad (26)$$

where we used eqs. (1), (24) and (25). The three last terms of r.h.s. of eq. (23) give no contribution to r.h.s. of eq. (26) since they are proportional to $\int d\mathbf{p} \frac{\partial}{\partial \mathbf{p}} f_\alpha(\mathbf{p}, t) = 0$. The first term leads to:

$$\partial_t \sum_a \left\langle \frac{\mathbf{p}_1^2}{2m_a} \right\rangle = -\partial_t \frac{1}{8\pi^3} \frac{1}{2} \sum_{ab} \int d\mathbf{k} d\mathbf{p}_1 d\mathbf{p}_2 \Phi_{ab}(\mathbf{k}) g_{ab}^0(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t). \quad (27)$$

The r.h.s. of eq. (27) is precisely minus the variation of the potential energy at the order of approximation we consider. It is easy to show that this potential energy becomes at equilibrium the Debye-Hückel energy [1]. Eq. (27) achieves the consistence of the theory. Eq. (27) shows that, in the considered approximation, the total energy $E^K + E^P$ is conserved. By construction, E^P is of the first order in the plasma parameter; thus at equilibrium it is equal to the Debye-Hückel energy. It is easy to verify that $J_a^0(\mathbf{p}_1, t) + J_a^1(\mathbf{p}_1, t)$ vanishes at equilibrium. It is indeed a property of the BL collision integral $J_a^0(\mathbf{p}_1, t)$. On the other hand, the time derivative on front of the expression (23) for $g_{ab}^1(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}, t)$ guarantees that it vanishes when the one-particle df doesn't depend on time. As a consequence $J_a^1(\mathbf{p}_1, t)$ is zero for any stationary state.

4 The quantum non-Markovian kinetic equation

The quantum hierarchy for a multicomponent plasma in the operator techniques takes the form

$$\frac{\partial}{\partial t} f_a(1) = [H_a(1), f_a(1)] + \sum_b Sp_{(2)} [U_{ab}(12), f_{ab}(12)], \quad (28)$$

$$\frac{\partial}{\partial t} f_{ab}(12) = [H_{ab}(12), f_{ab}(12)] + \sum_c Sp_{(3)} [U_{ac}(13) + U_{bc}(23), f_{abc}(123)], \quad (29)$$

where $f_a(1)$, $f_{ab}(12)$ and $f_{abc}(123)$ are one-, two- and three- particle density matrices, $[A, B]$ is the Poisson bracket.

$$H_a(1) = \frac{p^2(1)}{2m_a} \text{ is the kinetic energy,} \quad (30)$$

$$H_{ab}(12) = \frac{p^2(1)}{2m_a} + \frac{p^2(2)}{2m_b} + U_{ab}(12) \quad (31)$$

is the two-particle Hamiltonian, and $U_{ab}(12)$ is the two-particle interaction potential. Let us introduce the new operators [16]:

$$f_{ab}(12) = \gamma_{ab}(12)f'_{ab}(12), \quad (32)$$

$$f_{abc}(123) = \gamma_{abc}(123)f'_{abc}(123), \quad (33)$$

where the symmetrization operators are

$$\gamma_{ab}(12) = 1 + \delta_{ab}\eta_a P(12), \quad (34)$$

$$\gamma_{abc}(123) = \gamma_{ab}(12) \{1 + \delta_{ac}\eta_a P(13) + \delta_{bc}\eta_b P(23)\}, \quad (35)$$

$\eta_a = 1(\text{Bose}), -1(\text{Fermi})$; $P(12)$ is the permutation operator. Therefore,

$$\frac{\partial}{\partial t} f_a(1) = [H_a(1), f_a(1)] + \sum_b Sp_{(2)} [U_{ab}(12), \gamma_{ab}(12)f'_{ab}(12)], \quad (36)$$

$$\begin{aligned} & \frac{\partial}{\partial t} f'_{ab}(12) = [H_{ab}(12), f'_{ab}(12)] \\ & + \sum_c Sp_{(3)} [U_{ac}(13) + U_{bc}(23), (1 + \delta_{ac}\eta_a P(13) + \delta_{bc}\eta_b P(23))f'_{abc}(123)]. \end{aligned} \quad (37)$$

The symmetrization operators (34), (35) are convenient in that they give the possibility to partially transmit the permutation operator $P(12)$ from the density matrix to the interaction potentials. The density matrices $f_{ab}(12)$ etc. possess the quantum symmetry properties: $P(12)f_{ab}(12) = f_{ab}(12)P(12)$ etc., whereas the density matrices $f'_{ab}(12)$ etc. possess only the classical symmetry properties: $P(12)f'_{ab}(12)P(12) = f'_{ab}(12)$ etc. For the classically symmetric density matrices the usual conditions for disentanglement of equations hold, which are the same as those in the classical statistics. Specifically, in the plasma approximation, when the triple correlation function is neglected:

$$f'_{ab}(12) = f_a(1)f_b(2) + g'_{ab}(12), \quad (38)$$

$$f'_{abc}(123) = f_a(1)f_b(2)f_c(3) + g'_{ab}(12)f_c(3) + g'_{ac}(13)f_b(2) + g'_{bc}(23)f_a(1), \quad (39)$$

where $g'_{ab}(12)$ is the pair correlation function. By substituting eqs. (32), (33), (38) and (39) into eqs. (28) and (29) one obtains a closed set of equations for the one-particle distribution function and two-particle correlation function.

$$\frac{\partial}{\partial t} f_a(1) = [H'_a(1), f_a(1)] + \sum_b Sp_{(2)} [U'_{ab}(12), \gamma_{ab}(12)g'_{ab}(12)], \quad (40)$$

$$\begin{aligned} \frac{\partial}{\partial t} g'_{ab}(12) &= [H'_a(1) + H'_b(1), g'_{ab}(12)] + A'_{ab}(12) \\ &+ \sum_c Sp_{(3)} \{ [U'_{bc}(23), f_b(2)g'_{ac}(13)] + [U'_{ac}(13), f_a(1)g'_{bc}(23)] \}, \end{aligned} \quad (41)$$

where

$$H'_a(1) = \frac{p^2(1)}{2m_a} + U_a(1), \quad (42)$$

$$U_a(1) = \sum_b Sp_{(2)} [U'_{ab}(12), f_b(2)] \equiv U_a^H(1) + U_a^F(1), \quad (43)$$

$$U'_{ab}(12) = \gamma_{ab}(12)U_{ab}(12), \quad (44)$$

$$\begin{aligned} i\hbar A'_{ab}(12) &= [1 + \eta_a f_a(1)][1 + \eta_b f_b(2)]U_{ab}(12)f_a(1)f_b(2) \\ &- f_a(1)f_b(2)U_{ab}(12)[1 + \eta_a f_a(1)][1 + \eta_b f_b(2)], \end{aligned} \quad (45)$$

$$U_a^H(1) = \sum_b Sp_{(2)} [U_{ab}(12), f_b(2)] \quad (46)$$

is the Hartree field, i.e. mean self-consistent field [9] and

$$U_a^F(1) = \sum_b Sp_{(2)} \delta_{ab} \eta_a P(12) [U_{ab}(12), f_b(2)] \quad (47)$$

is the Fock field, mean field, taking into account only exchange interaction (Pauli's principle) [9].

In the plasma approximation in eq. (40) the term $[U'_{ab}(12), g'_{ab}(12)]$, which describes direct interaction of pair particles (1,2) is not taken into account.

Let us consider the homogeneous case. In the Wigner representation the kinetic equation (40) takes the form:

$$\frac{\partial}{\partial t} f_a(\mathbf{p}) = J_a(\mathbf{p}) = 2\hbar^2 \sum_b \int d\mathbf{p}' d\mathbf{k} [U_{ab}(\mathbf{k}) + \delta_{ab} \eta_a U_{ab}(\mathbf{p}' - \mathbf{p})] g'_{ab}(\mathbf{p}, \mathbf{p}', \mathbf{k}). \quad (48)$$

Here the spin variables are omitted for simplicity.

The solution of the equation for the pair correlation function $g'_{ab}(\mathbf{p}, \mathbf{p}', \mathbf{k})$ in the non-Markovian approximation in the Wigner form can be expressed in the spatially homogeneous case in terms of the resolvent of the equation (41) and its source (45).

$$\begin{aligned} &g'_{ab}(p, p', \mathbf{k}, t) \\ &= (1 - \frac{\partial}{\partial t} \frac{\partial}{\partial \Delta}) \sum_{a'b'} \int d\mathbf{q} d\mathbf{q}' R_{ab, a'b'}(\mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, z, \mu t) A'_{a'b'}(\mathbf{q}, \mathbf{q}', \mathbf{k}, \mu t) \Big|_{z=0}, \end{aligned} \quad (49)$$

$$z = \omega + i\Delta,$$

$$\begin{aligned} i\hbar A'_{ab}(\mathbf{p}, \mathbf{p}', \mathbf{k}, \mu t) &= U_{ab}(\mathbf{k}) \{f_a(\mathbf{p})f_b(\mathbf{p}') [1 + \eta_a f_a(\mathbf{p} + \frac{\hbar\mathbf{k}}{2})] [1 + \eta_b f_b(\mathbf{p}' - \frac{\hbar\mathbf{k}}{2})] \\ &\quad - f_a(\mathbf{p} + \frac{\hbar\mathbf{k}}{2})f_b(\mathbf{p}' - \frac{\hbar\mathbf{k}}{2}) [1 + \eta_a f_a(\mathbf{p})] [1 + \eta_b f_b(\mathbf{p}')] \} \end{aligned} \quad (50)$$

with the resolvent $R_{ab,a'b'}(\mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, z, \mu t)$ in eq. (49) being a product of two resolvents

$$R_{ab,a'b'}(\mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, t - t', \mu t') = R_{aa'}(\mathbf{p}, \mathbf{q}, \mathbf{k}, t - t', \mu t') R_{bb'}(\mathbf{p}', \mathbf{q}', \mathbf{k}, t - t', \mu t'), \quad (51)$$

which satisfy the linearized Hartree-Fock equation

$$\begin{aligned} [\hbar z + \Delta_k E_a(\mathbf{p})] R_{aa'}(\mathbf{p}, \mathbf{q}, \mathbf{k}, z, t) &= \delta_{aa'} \delta(\mathbf{p} - \mathbf{q}) \\ &+ e_a \Delta_k f_a(\mathbf{p}) \sum_c e_c \int d\mathbf{p}' [\Phi(\mathbf{k}) + \delta_{ac} \eta_a \Phi(\frac{\mathbf{p} - \mathbf{p}'}{\hbar})] R_{ca'}(\mathbf{p}', \mathbf{q}, \mathbf{k}, z, t), \end{aligned} \quad (52)$$

where

$$\begin{aligned} U_{ab}(\mathbf{k}) &= e_a e_b \Phi(\mathbf{k}) \\ \Delta_k E_a(\mathbf{p}) &= E_a(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}) - E_a(\mathbf{p} - \frac{\hbar\mathbf{k}}{2}); \quad \Delta_k f_a(\mathbf{p}) = f_a(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}) - f_a(\mathbf{p} - \frac{\hbar\mathbf{k}}{2}), \end{aligned} \quad (53)$$

$$E_a(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_a} + \eta_a \int d\mathbf{p}' U_{aa}(\frac{\mathbf{p} - \mathbf{p}'}{\hbar}) f_a(\mathbf{p}'). \quad (54)$$

The solution of eq. (52) takes the form

$$R_{aa'}(\mathbf{p}, \mathbf{p}', \mathbf{k}, z, t) = \frac{\Gamma_a(\mathbf{p}, \mathbf{p}') \delta_{aa'}}{\hbar z - \Delta_k E_{a'}(\mathbf{p}')} + \frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(\omega, \mathbf{k})} \Psi_a^{(1)}(\mathbf{p}) \Psi_{a'}^{(2)}(\mathbf{p}'), \quad (55)$$

where we introduced the notations

$$\Psi_a^{(1)}(\mathbf{p}) = e_a \int d\mathbf{p}'' \frac{\Gamma_a(\mathbf{p}, \mathbf{p}'') \Delta_k f_a(\mathbf{p}'')}{\hbar z - \Delta_k E_a(\mathbf{p}'')}, \quad (56)$$

$$\Psi_{a'}^{(2)}(\mathbf{p}') = e_{a'} \int d\mathbf{p}'' \frac{\Gamma_{a'}(\mathbf{p}'', \mathbf{p}')}{\hbar z - \Delta_k E_{a'}(\mathbf{p}')}, \quad (57)$$

and

$$\varepsilon^{HF}(\omega, \mathbf{k}) = 1 - \Phi(\mathbf{k}) \sum_a e_a^2 \int d\mathbf{p} d\mathbf{p}' \frac{\Gamma_a(\mathbf{p}, \mathbf{p}') \Delta_k f_a(\mathbf{p}')}{\hbar z - \Delta_k E_a(\mathbf{p}')} \quad (58)$$

is the dielectric function with exchange interaction.

The exchange scattering amplitude $\Gamma_a(\mathbf{p}, \mathbf{p}')$ for eqs. (55-58) satisfies an integral equation, which contains only the exchange interaction potential:

$$\Gamma_a(\mathbf{p}, \mathbf{p}') = \delta(\mathbf{p} - \mathbf{p}') + e_a^2 \eta_a \frac{\Delta_k f_a(\mathbf{p})}{\hbar z - \Delta_k E_a(\mathbf{p})} \int d\mathbf{p}'' \Phi(\frac{\mathbf{p} - \mathbf{p}''}{\hbar}) \Gamma_a(\mathbf{p}'', \mathbf{p}'). \quad (59)$$

$\Gamma_a(\mathbf{p}, \mathbf{p}')$ depends on \mathbf{k} and z as on parameters and is similar to the vertex-function, well-known in many-particle perturbation theory.

The formulae (55-59) yield the general expression for the pair correlation function in the non-Markovian approximation with complete description of the polarization and the exchange interaction of the particles:

$$g'_{ab}(\mathbf{p}, \mathbf{p}', \mathbf{k}, t) = -\frac{i}{\hbar} \left(1 - \frac{\partial}{\partial t} \frac{\partial}{\partial \Delta}\right) \Phi(\mathbf{k}) \sum_c e_c \int dz d\mathbf{q} f_c(\mathbf{q} + \frac{\hbar\mathbf{k}}{2}) [1 + \eta_a f_c(\mathbf{q} - \frac{\hbar\mathbf{k}}{2})]$$

$$\left\{ \frac{\Psi_a^{(1)}(\mathbf{p})}{\varepsilon^{HF}(z, \mathbf{k})} \left[\frac{\Gamma_b(\mathbf{p}', \mathbf{q}) \delta_{bc}}{\hbar z - \Delta_k E_c(\mathbf{q})} + \frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z, \mathbf{k})} \Psi_b^{(1)}(\mathbf{p}') \Psi_c^{(2)}(\mathbf{q}) \right]^* \right.$$

$$\left. - \frac{1}{\varepsilon^{*HF}(z, \mathbf{k})} \Psi_b^{*(1)}(\mathbf{p}') \left[\frac{\Gamma_a(\mathbf{p}, \mathbf{q}) \delta_{ac}}{\hbar z - \Delta_k E_c(\mathbf{q})} + \frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z, \mathbf{k})} \Psi_a^{(1)}(\mathbf{p}) \Psi_c^{(2)}(\mathbf{q}) \right] \right\}. \quad (60)$$

The expressions for the collision integral and the internal energy take the forms:

$$J_a(\mathbf{p}) = -2\hbar \left(1 - \frac{\partial}{\partial t} \frac{\partial}{\partial \Delta}\right) \text{Im } i \sum_{bc} e_a e_b e_c \int \Phi(\mathbf{k}) d\mathbf{p}' d\mathbf{k} dz d\mathbf{q} [\Phi(\mathbf{k})$$

$$+ \delta_{ab} \eta_a \Phi(\frac{\mathbf{p}' - \mathbf{p}}{\hbar})] f_c(\mathbf{q} + \frac{\hbar\mathbf{k}}{2}) [1 + \eta_a f_c(\mathbf{q} - \frac{\hbar\mathbf{k}}{2})]$$

$$\left\{ \frac{\Psi_a^{(1)}(\mathbf{p} + \frac{\hbar\mathbf{k}}{2})}{\varepsilon^{HF}(z, \mathbf{k})} \left[\frac{\Gamma_b(\mathbf{p}' + \frac{\hbar\mathbf{k}}{2}, \mathbf{q}) \delta_{bc}}{\hbar z - \Delta_k E_c(\mathbf{q})} + \frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z, \mathbf{k})} \Psi_b^{(1)}(\mathbf{p}' + \frac{\hbar\mathbf{k}}{2}) \Psi_c^{(2)}(\mathbf{q}) \right]^* \right.$$

$$\left. - \frac{\Psi_b^{*(1)}(\mathbf{p}' + \frac{\hbar\mathbf{k}}{2})}{\varepsilon^{*HF}(z, \mathbf{k})} \left[\frac{\Gamma_a(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}, \mathbf{q}) \delta_{ac}}{\hbar z - \Delta_k E_c(\mathbf{q})} + \frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z, \mathbf{k})} \Psi_a^{(1)}(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}) \Psi_c^{(2)}(\mathbf{q}) \right] \right\}, \quad (61)$$

$$U = 2 \text{Re } i \sum_{abc} \int \frac{e_a e_b e_c}{\hbar} d\mathbf{p} d\mathbf{p}' d\mathbf{k} dz d\mathbf{q} \Phi(\mathbf{k}) [\Phi(\mathbf{k})$$

$$+ \delta_{ab} \eta_a \Phi(\frac{\mathbf{p}' - \mathbf{p}}{\hbar})] f_c(\mathbf{q} + \frac{\hbar\mathbf{k}}{2}) [1 + \eta_a f_c(\mathbf{q} - \frac{\hbar\mathbf{k}}{2})]$$

$$\frac{1}{\varepsilon^{*HF}(z, \mathbf{k})} \Psi_b^{*(1)}(\mathbf{p}') \left[\frac{\Gamma_a(\mathbf{p}, \mathbf{q}) \delta_{ac}}{\hbar z - \Delta_k E_c(\mathbf{q})} + \frac{\Phi(\mathbf{k})}{\varepsilon^{HF}(z, \mathbf{k})} \Psi_a^{(1)}(\mathbf{p}) \Psi_c^{(2)}(\mathbf{q}) \right]. \quad (62)$$

The complete description of the exchange interaction of particles is reduced to the solution of the linear integral equation (59).

5 The law of total energy conservation

Lets demonstrate that the kinetic equation derived (61) satisfies the law of total energy conservation taking into account potential energy U (62).

$$\frac{\partial}{\partial t} (E^{HF} + U) = 0, \quad (63)$$

where E^{HF} is the sum of kinetic energy system and energy of self consistent field:

$$E^{HF} = \sum_a \int d\mathbf{p} f_a(\mathbf{p}) \frac{\mathbf{p}^2}{2m_a} + \frac{1}{2} \sum_a \int d\mathbf{p} d\mathbf{p}' U_{aa} \left(\frac{\mathbf{p} - \mathbf{p}'}{\hbar} \right) f_a(\mathbf{p}) f_a(\mathbf{p}'). \quad (64)$$

The second term in eq. (64) describes the energy of the exchange interaction of the particles. Self-consistent Hartree field vanished due to space homogeneity of system.

It is not difficult to see that

$$\frac{\partial}{\partial t} E^{HF} = \sum_a \int d\mathbf{p} E_a(\mathbf{p}) \frac{\partial}{\partial t} f_a(\mathbf{p}), \quad (65)$$

where $E_a(\mathbf{p})$ is defined by eq. (54). Multiply equation (48) on $E_a(\mathbf{p})$, integrate over the impulse and sum up over the plasma components. Taking into account eq. (49) we receive the following:

$$\begin{aligned} \frac{\partial}{\partial t} E^{HF} &= 2\hbar^2 \sum_{ab} \int d\mathbf{p} d\mathbf{p}' d\mathbf{k} E_a(\mathbf{p}) [U_{ab}(\mathbf{k}) + \delta_{ab} \eta_a U_{ab}(\mathbf{p}' - \mathbf{p})] \\ &\times \left(1 - \frac{\partial}{\partial t} \frac{\partial}{\partial \Delta} \right) \sum_{a'b'} \int d\mathbf{q} d\mathbf{q}' R_{ab,a'b'}(\mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, z, \mu t) A'_{a'b'}(\mathbf{q}, \mathbf{q}', \mathbf{k}, \mu t) \Big|_{z=0}. \end{aligned} \quad (66)$$

From eq. (52) follows an integral equation for the resolvent (51)

$$\begin{aligned} R_{ab,a'b'}(\mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, z, \mu t) &= \frac{1}{\hbar z + \Delta_k E_a(\mathbf{p}) + \Delta_k E_b(\mathbf{p}')} \{ \delta_{aa'} \delta_{bb'} \delta(\mathbf{p} - \mathbf{q}) \delta(\mathbf{p}' - \mathbf{q}') \\ &+ e_a \Delta_k f_a(\mathbf{p}) \sum_c e_c \int d\mathbf{p}_1 [\Phi(\mathbf{k}) + \delta_{ac} \eta_a \Phi(\frac{\mathbf{p} - \mathbf{p}_1}{\hbar})] R_{cb,a'b'}(\mathbf{p}_1, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, z, \mu t) \\ &+ e_b \Delta_k f_b(\mathbf{p}') \sum_c e_c \int d\mathbf{p}_2 [\Phi(\mathbf{k}) + \delta_{bc} \eta_b \Phi(\frac{\mathbf{p}' - \mathbf{p}_2}{\hbar})] R_{ac,a'b'}(\mathbf{p}, \mathbf{p}_2, \mathbf{q}, \mathbf{q}', \mathbf{k}, z, \mu t) \}. \end{aligned} \quad (67)$$

The derivative $\frac{\partial}{\partial \Delta}$ of expression (67) gives two terms. In the case $z = 0$, it follows

$$\begin{aligned} \frac{\partial}{\partial \Delta} R_{ab,a'b'}(\mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, \mu t) &= -i\hbar \left(\frac{1}{\Delta_k E_a(\mathbf{p}) + \Delta_k E_b(\mathbf{p}')} \right)^2 \\ &\times \{ \delta_{aa'} \delta_{bb'} \delta(\mathbf{p} - \mathbf{q}) \delta(\mathbf{p}' - \mathbf{q}') + e_a \Delta_k f_a(\mathbf{p}) \\ &\sum_c e_c \int d\mathbf{p}_1 [\Phi(\mathbf{k}) + \delta_{ac} \eta_a \Phi(\frac{\mathbf{p} - \mathbf{p}_1}{\hbar})] R_{cb,a'b'}(\mathbf{p}_1, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, \mu t) \\ &+ e_b \Delta_k f_b(\mathbf{p}') \sum_c e_c \int d\mathbf{p}_2 [\Phi(\mathbf{k}) + \delta_{bc} \eta_b \Phi(\frac{\mathbf{p}' - \mathbf{p}_2}{\hbar})] R_{ac,a'b'}(\mathbf{p}, \mathbf{p}_2, \mathbf{q}, \mathbf{q}', \mathbf{k}, \mu t) \} \\ &+ \frac{1}{\Delta_k E_a(\mathbf{p}) + \Delta_k E_b(\mathbf{p}')} \{ e_a \Delta_k f_a(\mathbf{p}) \end{aligned}$$

$$\begin{aligned} & \sum_c e_c \int d\mathbf{p}_1 [\Phi(\mathbf{k}) + \delta_{ac}\eta_a \Phi(\frac{\mathbf{p}-\mathbf{p}_1}{\hbar})] \frac{\partial}{\partial \Delta} R_{cb,a'b'}(\mathbf{p}_1, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, z, \mu t) \\ & + e_b \Delta_k f_b(\mathbf{p}') \sum_c e_c \int d\mathbf{p}_2 [\Phi(\mathbf{k}) + \delta_{bc}\eta_b \Phi(\frac{\mathbf{p}-\mathbf{p}_2}{\hbar})] \frac{\partial}{\partial \Delta} R_{ac,a'b'}(\mathbf{p}, \mathbf{p}_2, \mathbf{q}, \mathbf{q}', \mathbf{k}, \mu t) \}. \end{aligned} \quad (68)$$

Substituting eq. (68) into eq. (67) and symmetrizing the integral it is not difficult to be convinced that a different from zero contribution to eq. (67) gives only the first term in eq. (68) and that the right hand side of eq. (67) represents itself derivation of the potential energy, which is determined by eq. (62). As a result we obtain the law of total energy conservation (63).

6 Kinetic equation with averaged exchange interaction

The collision integral (61) and the internal energy (62) are expressed by the amplitude of the scattering interaction $\Gamma_a(\mathbf{p}, \mathbf{p}')$ which satisfies the linear integral equation (59). The solution of this equation in case of Coulomb interaction of the particles is difficult and requires an appropriate approximation. The simplest approximation is the replacement of the expression under the integral of (59) by the averaged over the impulse value

$$\Phi(\mathbf{k})F(z, \mathbf{k}) = \int d\mathbf{p}'' \Phi(\frac{\mathbf{p}-\mathbf{p}''}{\hbar}) \Gamma_a(\mathbf{p}'', \mathbf{p}'). \quad (69)$$

Then, the dielectric function taking into account exchange interaction particles, takes the form:

$$\varepsilon^{HF}(z, \mathbf{k}) = 1 - P(z, \mathbf{k})[1 + P(z, \mathbf{k})F(z, \mathbf{k})]^{-1}, \quad (70)$$

where

$$P(z, \mathbf{k}) = \Phi(\mathbf{k}) \sum_a e_a^2 \int d\mathbf{p} \frac{\Delta_k f_a(\mathbf{p})}{\hbar z - \Delta_k E_a(\mathbf{p})} \quad (71)$$

is the polarisation.

In the special case of the Hubbard approximation [17], one has

$$F(z, \mathbf{k}) = F(\mathbf{k}) = \frac{1}{2} \frac{k^2}{k^2 + k_f^2}. \quad (72)$$

Using the expression for the pair correlation function, we find the Markovian collision integral

$$\begin{aligned} J_a(\mathbf{p}) = & 4\pi^2 e^4 \int \Phi^2(\mathbf{k})(1 - F(z, \mathbf{k})) d\mathbf{k} d\mathbf{q} \frac{\delta(\Delta_k E(\mathbf{q}) - \Delta_k E(\mathbf{p}))}{\left| \tilde{\varepsilon}(\Delta_k E(\mathbf{q})/\hbar, \mathbf{k}) \right|^2} \\ & \left\{ f(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}) f(\mathbf{q} - \frac{\hbar\mathbf{k}}{2}) [1 - f(\mathbf{p} - \frac{\hbar\mathbf{k}}{2})] [1 - f(\mathbf{q} + \frac{\hbar\mathbf{k}}{2})] \right. \\ & \left. - f(\mathbf{p} - \frac{\hbar\mathbf{k}}{2}) f(\mathbf{q} + \frac{\hbar\mathbf{k}}{2}) [1 - f(\mathbf{p} + \frac{\hbar\mathbf{k}}{2})] [1 - f(\mathbf{q} - \frac{\hbar\mathbf{k}}{2})] \right\}, \end{aligned} \quad (73)$$

and the internal energy of particles

$$U = e^4 \int d\mathbf{p} d\mathbf{p}' d\mathbf{k} \frac{\Phi^2(\mathbf{k})}{\Delta_k E(\mathbf{q}) - \Delta_k E(\mathbf{p})} \frac{1}{\left| \tilde{\varepsilon}(\Delta_k E(\mathbf{q})/\hbar, \mathbf{k}) \right|^2}$$

$$\begin{aligned} & \{f(\mathbf{p} + \frac{\hbar\mathbf{k}}{2})f(\mathbf{p}' - \frac{\hbar\mathbf{k}}{2})[1 - f(\mathbf{p} - \frac{\hbar\mathbf{k}}{2})][1 - f(\mathbf{p}' + \frac{\hbar\mathbf{k}}{2})] \\ & - f(\mathbf{p} - \frac{\hbar\mathbf{k}}{2})f(\mathbf{p}' + \frac{\hbar\mathbf{k}}{2})[1 - f(\mathbf{p} + \frac{\hbar\mathbf{k}}{2})][1 - f(\mathbf{p}' - \frac{\hbar\mathbf{k}}{2})]\}, \end{aligned} \quad (74)$$

where

$$\tilde{\varepsilon}(z, \mathbf{k}) = 1 - (1 - F(z, \mathbf{k}))P(z, \mathbf{k}). \quad (75)$$

From expressions for the internal energy of particles (74) and for the collision integral (73) follows that $|\tilde{\varepsilon}(z, \mathbf{k})|^2$ play the role of the screening of the interaction potential $\Phi(\mathbf{k})$. It is interesting to note that eqs. (73) and (74) are differed from the corresponding Balescu's expressions by taking into account the exchange interaction in this screening. Moreover the colision integral (73) contains the additional renormalization of the interaction $(1 - F(z, \mathbf{k}))$. However $\tilde{\varepsilon}(z, \mathbf{k})$ dose not serve as linear rersponce function, in contrast to the Hartree-Fock dielectric function $\varepsilon^{HF}(z, \mathbf{k})$ in eq. (70).

In the equilibrium state, expression (74) satisfies the fluctuation-dissipation theorem since

$$\text{Im} \frac{P(z, \mathbf{k})}{|\tilde{\varepsilon}(z, \mathbf{k})|^2} = \text{Im} \frac{P(z, \mathbf{k})}{\varepsilon^{HF}(z, \mathbf{k})}. \quad (76)$$

7 Conclusion

Using the operator technique within BBGKY hierarchy we obtained a closed set of equations for the one- and two- particle density matrix, referring to the plasma approximation which considers also the exchange interaction. The equation for the pair correlation function is solved with the help of the resolvent of the Hartree-Fock equation. The expression obtained for the pair correlation function takes into account the non-Markovian effect and the exchange interaction. The latter is described by the scattering amplitude which is subject to the integral equation formulated above. Expression for the time-nonlocal collision integral and the internal energy are obtained with the exchange interaction and polarisation taken into account. The quantum kinetic equation derived satisfies the law of total energy conservation in the case of polarization and exchange interaction. The Hubbard approximation was generalized to the non-equilibrium case.

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References

- [1] Balescu R. 1963 *Statistical Mechanics of Charged Particles*, New York : Wiley - Interscience
- [2] Balescu R. 1975 *Equilibrium and Nonequilibrium Statistical Mechanics*, New York: Wiley - Interscience

- [3] Balescu R. 1960 *Phys. of Fluids* **3** 52
- [4] Lenard A. 1960 *Ann. Phys. (NY)* **10** 390
- [5] Vlasov A.A. 1938 *Sov. Phys. - JETP* **8** 291
- [6] Landau L.D. 1936 *Phys. Zs. Sow. Union* **10** 154
- [7] Balescu R. 1961 *Phys. of Fluids* **4** 94
- [8] Guernsey R. L. 1962 *Phys. Rev. Lett.* **127** 1446
- [9] Kadanoff L.P. and Baym G. 1964 *Quantum statistical mechanics*, New York: Benjamin
- [10] Klimontovich Yu.L. 1971 *Sov. Phys. - JETP* **30** 1352; Klimontovich Yu.L. 1975 *Kinetic theory of nonideal gases and nonideal plasma* New York: Academic Press
- [11] Klimontovich Yu.L. and Ebeling W. 1973 *Sov. Phys. - JETP* **36** 476; Morawetz K. and Roepke G. 1995 *Phys. Rev. E* **51** 4246; Bonitz M. Scott D.C. Binder R. Kremp D. Kraeft W.D. Kohler H.S. 1995 *in Physics of Strongly Coupled Plasmas*, p.185 (W.D. Kraeft and M. Schlanges eds.) Singapore: World Scientific; Bornath Th. Kremp D. Schlanges M. 1996 *Phys. Rev. E* **54** 3274; Kremp D. Bonitz M. Kraeft W.D. Schlanges M. 1997 *Ann.of Phys. (NY)* **258** 320
- [12] Klimontovich Yu. L. 1972 *Sov. Phys. - JETP* **35** 920
- [13] Belyi V.V. Kukharenko Yu.A. Wallenborn J. 1996 *Phys. Rev. Lett.* **76** 3554; Belyi V.V. Kukharenko Yu.A. Wallenborn 1998 *J. Plasma Phys.* **59** 657
- [14] Belyi V.V. Kukharenko Yu.A. Wallenborn J. 2002 *Contrib. Plasma Phys.* **42** 3
- [15] Bogoliubov N. 1962 in *Studies in Statistical Mechanics*, edited by J. de Boer and G.E. Uhlenbeck, Amsterdam: North-Holland
- [16] Bogoliubov N. 1947 *J. Phys. (Moscow)* **11** 23
- [17] Hubbard J. 1958 *Proc. Roy. Soc. A* **243** 336