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Relativistic kinematics: transformation of spatial coordinates and time during the transition from an inertial system of reference to a non-inertial one

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Abstract

The report highlights some issues of relativistic kinematics (or Einstein's special theory of relativity – STR). Formulas approximating the transformations of spatial coordinates and time during the transition from an inertial systemof reference (ISR) to a non-inertial system moving relative to ISR with constant acceleration are given. In the light of the proposed additions to classical relativistic kinematics, the laws of the evolution of the Universe at the last stage of its expansion – after the formation of galaxies – are considered. It is shown that there is no need to accept the hypothesis of the existence of dark energy to explain these regularities, the age and size of the Metagalaxy are also clarified.

Keywords: theory of relativity, acceleration of the systems of reference, expansion of Metagalaxy.

1. Introduction

Relativistic kinematics is one of the most important components of modern physics. In fact, it is a physical theory of philosophical categories – space and time (Fok, 1955). It strictly proves the inseparable (and far from obvious) connection of these categories, the impossibility of overcoming the "light barrier" *c*, the equivalence of mass and energy, the monotonous increase in the mass of objects with increasing speed of their movement.

The conclusions of relativistic kinematics (more precisely, the kinematics of objects moving at very high speeds) go far beyond the scope of "everyday" experience, and therefore they seem paradoxical. One of them – the statement about the slowing down of the passage of time in a moving inertial system of reference – generally looks implausible. However, this conclusion, like all other provisions of STR, is the result of unmistakable mathematical transformations of the original equations; it is experimentally confirmed, for example, by the results of measurement by a motionless observer of the lifetime of unstable elementary particles accelerated in the CERN ring accelerator to a speed approaching *c* (Bailey, 1974).

In STR, two inertial systems ofreference are studied, the first of which, having space-time characteristics $\{x, y, z, t\}$, is motionless, and the second with characteristics $\{\xi, \eta, \zeta, \tau\}$ at $t > 0$ is moving away from the first at a constant speed $\vec{v}_0$, which can tend to *c*. The systems coincide at $t = \tau = 0$, then the system ofreference II moves so that the equally directed axes x and ξ continue to remain on the same straight line, and the other two pairs of axes maintain parallelism.

In these circumstances, at $v_0 \to c$ the simple rule of addition of velocities $\frac{dx}{dt} = \frac{d\xi}{d\tau} + v_0$ of Galileo-Newton's kinematics, corresponding to "everyday" experience, is not fulfilled (Fok, 1955). In the general case

$$\frac{dx}{dt} = \left(\frac{d\xi}{d\tau} + v_0\right) \Bigg/ \left(1 + \frac{d\xi}{d\tau}\frac{v_0}{c^2}\right) \qquad (1)$$

that is, an experimenter who, being in system I, measures the speed of an object moving at a speed $\frac{d\xi}{d\tau}$ in system II, receives its value $\frac{dx}{dt} < \frac{d\xi}{d\tau} + v_0$. Even if $v_0 = c$ and $\frac{d\xi}{d\tau} = c$, the speed $\frac{dx}{dt}$ does not exceed the "light barrier", $\frac{dx}{dt} = c = \frac{d\xi}{d\tau}$. So, equality (1) is a mathematical record of one of the postulates of the special theory of relativity formulated by A. Einstein: the speed of electromagnetic waves is not affected by the movement of emitters and receivers relative to each other, *c* is a world constant.

The inertial systems of reference considered in the special theory of relativity are "mirror-like" indistinguishable: the second of them also moves away from the first with a constant velocity $(-\vec{v}_0)$, in both of them the laws of Newton's dynamics are fulfilled. However, this indistinguishability of the ISR disappears if one of them, for example, the second one begins to move with acceleration. Inertia forces are detected in such a system – it becomes non-inertial. Transformations of spatial coordinates and time during the transition from a motionless inertial system I to a non-inertial system II are not described by STR formulas, in this part relativistic kinematics requires clarifications. The purpose of this work is to find such clarifications.

2. Materials and Methods

In order to achieve the above goal, it was necessary to solve the following tasks: *firstly*, to determine the direction of the necessary mathematical transformations (their basis); *secondl*y, to specify the methods of transformations and write down calculation formulas; *thirdly*, to carry out calculations using these formulas in relation to objects studied in relativistic kinematics.

The special theory of relativity is the basis of the calculations I have made. STR according to the criteria proposed by A. Einstein – the simplicity of the postulates, the significant difference between the categories connected, the breadth of application – deserves the highest praise and represents a solid foundation for the development of theoretical physics, improving ideas about the world around us (Fok, 1955).

The calculations do not go beyond of the "school" mathematical analysis and are not too complicated (Vygodsky, 2019).

The results of the calculations are used to describe the motion of galaxies, that is, massive material objects moving in outer space, as shown by astronomical observations, at very high speeds (Weinberg, 2008).

3. Results

Let's assume that at the initial moment of time in the center of the reference system I $\{x, y, z, t\}$ there is a real object (a material point – MP). The center of system II $\{\xi, \eta, \zeta, \tau\}$, which coincides with the center of the first system, is rigidly connected with it. At this moment, the MP begins to move along the *x*-axis with acceleration $\vec{a}$. Together with it, the second system also moves rapidly, without rotating.

The MP in system II remains motionless during the entire observation time, so that $\xi = \eta = \varsigma = 0$, $\frac{d\xi}{dt} = 0$. If the reference systems were moving apart at a constant speed $\vec{v}_0$, then. as follows from equality (1), an observer located in the center of a motionless system would have recorded the coincidence of the velocities of MP and system II, $\frac{dx}{dt} = v_0$. It is natural to assume that in a more difficult situation $\frac{dx}{dt} = v(t)$. At the same time, however, one should not forget about the existence of an insurmountable "light barrier", so that the rapidity of increase in the speed of MP and system II should decrease to zero over time. By the way, it should be notedthat for the interpretation of this phenomenon it is not necessary to invent some retarding forces – it is explained within the framework of relativistic kinematics by the fundamental properties of space-time.

In this message, the exact type of dependence *v*(*t*) is not determined; it is proposed to approximate it with a function that (at $t \to 0$)tends to *at*, and whose slope to the abscissa axis (at $t \to \infty$)monotonically decreases to zero,

$$v(t) = \frac{dx}{dt} = c\left[1 - \exp\left(-\frac{at}{c}\right)\right] \tag{2}$$

Integrating equality (2) in the range from 0 to *t*, we obtain the time dependence of the *x* coordinate:

$$x = \int_0^t v(t)dt = ct + \frac{c^2}{a}\exp\left(-\frac{at}{c}\right) - \frac{c^2}{a} \tag{3}$$

It follows from formula (3) that

- when $t \to 0$ the path length of the object gaining speed is proportional to the square of the observation time (as in the classical kinematics of a material point, $x = at^2/2$);

- when $t \to \infty$, the motionless observer will fix the approximate proportionality of the values *x* and *t* ($x \approx ct - c^2/a$).

If the object and the non-inertial reference system II accelerated away from the stationary system I, reached a speed $\vec{v}_0$, and then at *t* = 0 began to approach the stationary observer with acceleration ($-\vec{a}$) and move away from the "light barrier", then up to a stop at $t = v/a$ the modulus of their speed decreased:

$$v(t) = \frac{dx}{dt} = v_0 - at \tag{4}$$

As you know, the results of measuring time intervals Δt and $\Delta\tau$ do not coincide if the experimenters are in two inertial systems of reference that are moving apart at a speed $v_0 = const$ (Fok, 1955).Then the “effect of twins”is observed:

$$\Delta\tau < \Delta t; \quad \Delta\tau = \Delta t\sqrt{1-\left(v_0^2 / c^2\right)} \qquad (5)$$

In the situation under discussion, system II is non-inertial, $v = v(t)$. Then the ratio between the values Δt and $\Delta\tau$ must be calculated using the integral formulas given below, which will be obtained after replacement v_0 in equality (5) on the functions $v = v(t)$ (2) and (4):

$$\Delta\tau = \int_0^{\Delta t}\left[1-\left(1-e^{-at/c}\right)^2\right]^{1/2} dt \qquad (6)$$

$$\Delta\tau = \int_0^{\Delta t}\left[1-\left(\frac{v_0 - at}{c}\right)^2\right]^{1/2} dt \qquad (7)$$

4. Discussion

The special theory of relativity, including the proposed additions, becomes relevant when the objects under study move at speeds approaching the "light barrier". Such "expresses" include accelerated elementary particles (Bailey, 1974) and distant galaxies (Weinberg, 2008).

Taking into account the additions to the STR, it is possible to clarify the existing ideas about the micro- and macrocosm, in particular, to find an answer to the question of the universality of the "twins paradox" observed in high-speed reference systems: yes, as follows from equations (5), (6) and (7), in any mobile reference system, the passage of time slows down, $\Delta\tau < \Delta t$ regardless of the direction of the vectors $\vec{v}$ and $\vec{a}$.

Let us now determine the kinematic characteristics of high-speed galaxies.

It has been experimentally established that the total number of Metagalactic elements (observed galaxies, quasars and other macro objects) is approximately 10^{11}-10^{12}. With rare exceptions, these elements are moving away from the Milky Way, and Hubble's law is fulfilled (though not strictly), declaring the proportionality of the velocities of the scattering galaxies and the distances R from them to terrestrial observers (Hubble, 1929):

$$v = HR \qquad (8)$$

where $H \approx 22 \cdot 10^{-19}\ \frac{1}{c}$ is the Hubble constant. Extrapolation of these experimental data into the distant past leads to the hypothesis of the Big Bang (BB), from which the age of Metagalaxy should be counted.

Currently, the implementation of the Big Bang is beyond doubt, within the framework of modern cosmology, this event is considered as an axiom of the generally accepted standard model of the birth and transformation of "all things" (Weinberg, 2008).

It is assumed that the current moment separates approximately $\frac{1}{H} = 4{,}5 \cdot 10^{17}$ seconds (~14 billion years) from the Big Bang. After about $5 \cdot 10^{16}$ seconds BB, in the era of reionization of matter, a set of elements of Metagalaxy was formed (Kuznetsov, 2006), which then evolved in accordance with the laws of STR. Thus, during most of the existence of Metagalaxy (about 90%), the movements of its large-scale elements should be described by the equations of relativistic kinematics proposed in this message.

It should be noted that Hubble's law is of great importance for the formation of modern cosmology as a science, since the generally accepted ideas about the emergence of superdense and superheated substance in the Big Bang began to take shape precisely after the fact of the scattering of metagalactic elements was established. Nevertheless, the regularity corresponding to formula (8) can in no case be considered accurate and strictly justified for the following reasons:

1) There are a small number of galaxies that are approaching the Earth observer, and not moving away from him. These include, for example, the nearest neighbor of the Milky Way – the Andromeda nebula.

2) There is no reason to believe that the velocities of all galaxies and quasars equidistant from the Milky Way are the same or close. This assumption is clearly refuted by the results of measuring the quantities v and R, which form a point cloud in the "space" $v - R$, the width of which reaches 10^6 m/s (Hubble, 1929).

3) The validity of Hubble's law was postulated for space objects distant from the Milky Way "only" at $R \sim 10^{23}$ m, while the farthest of the discovered galaxies is located at a distance of about a thousand times greater from the terrestrial observer. The comparison of the above numbers raises serious doubts that the effect of this law applies to the entire Metagalaxy at all times.

4) It is impossible to agree with the opinion about the validity of equation (8) for all star clusters observed at a certain point in time, since information about their velocities is received by the observer with a time delay, the value of which varies significantly with R (at $R \sim 10^{23}$ m corresponds to ten million years, and at $R \sim 10^{26}$m the time delay is approximately ten billion years).

5) Experimental data presented in the "space" $v - R$ by a set of widely scattered points relate to various space objects characterized by mismatched initial velocities and different equations of motion$R = f(t)$.

6) If the path x traversed by the object under study and a velocity of this object $\frac{dx}{dt}$ are related by equation (8), then both of these values increase over time equally infinitely – exponentially. However, the speed of any object in accordance with the fundamental provisions of the special theory of relativity cannot exceed the speed of propagation of electromagnetic radiation in a vacuum. So, equation (8) is just an imperfect approximation of the real dependence $v = f(R)$.

Proceeding from the above, we consider the movement of the elements of Metagalaxy, taking into account formulas (2) and (3), which approximate time dependencies $v(t)$ and $x(t)$ when changing the argument in an infinitely wide interval, $0 < t < \infty$.

These equalities are reduced to the form

$$at = -c \ln(1 - v/c) \qquad (9)$$

$$aR = cat - cv \qquad (10)$$

Obviously, if the velocity of a non-inertial reference system v is found, which is associated with a specific receding space object detected at a measured distance R from the Milky Way, then it is not difficult to calculate unknown values of the acceleration modulus a and the travel time t using equations (9) and (10).

In cosmology, the radial velocity is measured, not the total velocity of the objects under study. Let us take into account the physical meaning of this value and the incomplete adequacy of Hubble's law. Then we will come to the following conclusions: the scattering velocity of galaxies and quasars v, located approximately at the same distance R from the observer, must be found by spectroscopic method (by redshift) and evaluated not by the average value of the measured parameter v, but by the maximum v_{max}; at the same time, the distance R must be determined independently – for example, by photometry (by measuring the periodically changing brightness of cepheid stars).

Experiments have shown that when $R \approx 3 \cdot 10^{22}\, m$ the speed of removal of Metagalactic elements away the Milky Way is about $0,8 \cdot 10^6\, m/s$, and with an increase in the specified distance by half, galactic star clusters scatter at a speed $\sim 1,2 \cdot 10^6\, m/s$ (Hubble E., 1929).

Calculations performed on the basis of the above data give the following results:

- the acceleration of fleeing galaxies is negligible, it is equal to $(1,1-1,2) \cdot 10^{-11}\, m/s^2$; it means that these objects are affected by negligibly small forces, the nature of which is not discussed in kinematics; in fact, fleeing galaxies move uniformly;

- some of the elements of Metagalaxy overcame a distance $R \approx 3 \cdot 10^{22}\, m$ at $\sim 7,5 \cdot 10^{16}\, s$, others moved $6 \cdot 10^{22}\, m$ away from the Milky Way with $\sim 10^{17}\, s$.

Suppose that the galaxies discovered in recent years, the most distant from the terrestrial observer and having a velocity of about 0.98 c, are sent after the Big Bang to the boundaries of the observable Universe with the same acceleration of $1,2 \cdot 10^{-11}\, m/s^2$ as slower moving space objects, whose velocity at $t = 0$ was much lower. Then it follows from equations (9) and (10) that such high-speed objects reached the boundaries of Metagalaxy in ~$9,8 \cdot 10^{19} s$, having moved away from the Milky Way by ~$22 \cdot 10^{22} m$ during this time.

In cosmology, it is customary to estimate the age of the Metagalaxy and its size based on Hubble's law – by the values of "Hubble time" $1/H \approx 4,5 \cdot 10^{17}\, c$ and "Hubble distance" $c/H \approx 13,5 \cdot 10^{25}\, m$ (Weinberg, 2008). However, a critical analysis of the mentioned law (see above) proves that these estimates cannot be considered reliable. The characteristics of Metagalaxy given in this report are more realistic; it follows from them that the age and size of the observable Universe are underestimated by Hubble calculations by 220 and 160 times, respectively.

An alternative statement to the concept presented here is that the galaxies farthest from the Milky Way are moving with a large positive acceleration, and not almost uniformly. It is based on the discrepancy between the results of measuring distances to these objects R_b by the brightness of "standard candles" (type 1a supernovae located in them (Pearlmutter, 2013; Riess, 1998) and R_{rs} by the redshift of lines in their emission spectra (that is, by Hubble's law, recognized as universal). It turned out that the brightness of supernovae is weaker than expected and $R_b > R_{rs}$.

It was proposed to consider this discrepancy between photometry and spectrometry data within the framework of Einstein's general theory of relativity (GTR), the fundamental equations of which contain a cosmological constant, traditionally denoted by the Greek letter Λ. The Λ-term in the GTR equations makes sense of antigravity, the source of which is considered to be dark energy, evenly filling Universe, accounting for about 70% of its total energy-mass and providing accelerated scattering of galaxies under the influence of negative pressure (Chernin, 2005).

Although the hypothesis of the existence of dark energy has many very authoritative supporters, there are numerous reasons for its criticism:

1) A. Einstein introduced the cosmological constant into the GTR equations, taking for truth the hypothesis of a stationary universe, long ago refuted by astronomical observations.

2) Back in 1922-1924, A. A. Friedman solved the GTR equations for the real universe, that is, non-stationary, which did not contain the "antigravity" Λ-term (Weinberg, 2018).

3) Any models of a non-stationary Universe based on the recognition of the universality of Hubble's law cannot but cause doubts.

4) The introduction of the antigravity force into theoretical constructions contradicts the principle of universal simplicity – one of the main provisions of the methodology of scientific research formulated by W. Occam back in the XIV century: entities should not be multiplied unnecessarily. Dark energy is obviously a redundant entity, since the "unexpectedly" weak brilliance of distant galaxies is easily explained by the results of the calculations of the distances R to these objects from the Milky Way given in this message, performed according to the equations of relativistic kinematics, in which there is no concept of dark energy.

5) If the accelerations of distant galactic star clusters are $\sim 10^{-11}\ m/s^2$, then there is no need to talk about the effect of a large negative pressure on them, exceeding the gravitational attraction. "Galaxies don't run away because some mysterious force is pulling them apart. Similarly, a stone does not fly up because it is repelled by the Earth. Galaxies are moving away from each other because they were scattered by the Big Bang in the past" (Weinberg, 2018).

6) Without the introduction of redundant entities (provisions that complement the fundamental concepts of the Universe), one can do not only when studying the laws of the evolution of Metagalaxy after the reionization of matter, but also when describing its "maturation" in much earlier times – at the dust stage and in the "dark ages". In any case, it was not necessary to accept the dark energy hypothesis in order to construct the theory of primary nucleosynthesis (Alpher, 1948).

5. Conclusion

The main results of the work performed are as follows:

1) Mathematical expressions are proposed that approximate the time dependence of the velocity of an object that is moving away from a stationary observer with constant acceleration, and the equation of motion of this object. Thus, logically justified additions have been made to the classical relativistic kinematics, which indicates the achievement of the declared research aim.

2) These additions are practically significant. They made it possible to prove the universality of the relativistic "twins paradox" and calculate the acceleration of the observed expansion of the Metagalaxy. It turned out that this acceleration is vanishingly small; therefore, the hypothesis of the existence of dark energy cannot be considered convincing.

The conducted research opens up the prospect of using modified relativistic kinematics to clarify ideas about the laws of motion of any objects, including elementary particles, with velocities approaching the "light barrier".

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