Anisotropic dark energy embedding viscosity in scalar tensor theories

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Abstract

In the present work, we investigate the role of embedded viscosity in anisotropic dark energy in Saez-Ballester and Brans-Dicke theories of gravitation with Bianchi type II, VIII and IX metric. Hyperbolic volumetric expansion is considered to solve the field equations. It is observed that the universe is expanding and accelerating. The universe tends to be isotropic at later stage.

Keywords: Brans-Dicke, Saez-Ballester, anisotropic dark energy, Viscosity, Bianchi type II, VIII, IX metric

1. Introduction

Scalar tensor theories (STT) are generalizations of general relativity, in order to incorporate Mach's principle. Scalar tensor theories of gravitation's are useful to discuss the problem of singularity. In STT, scalar field ϕ is a dynamical variable. STT are divided into two categories. In the first category, ϕG^{-1} and in the second category, ϕ is dimensionless scalar field. The Brans-Dicke theory (BDT)[1] is of first category and Saez-Ballester theory (SBT)[2] is of second category. Nordtvedt [3], Lyra [4], Sen and Dunn [5] are some of the scalar tensor theories. They are also called alternative theory. Singha and Debnath [6] studied chaplygin gas in Brans-Dicke Theory.

The high red-shift supernovae, the cosmic microwave background anisotropy, and galaxy clustering [7-9] indicate that the present universe is expanding and accelerating. It is assumed that some unknown exotic fluid known as dark energy is responsible for the accelerated expansion. The dark energy has strong negative pressure with equation of state parameter $\omega = \frac{P}{\rho}$, where P is pressure and ρ is energy density of dark energy. The cosmological constant with $\omega = -1$ is candidate of dark energy. However, the cosmological constant has a fine tuning problem. For this region, other candidates of dark energy were proposed. When $\omega < -1$, the candidate for dark energy is called Phantom [10]. A cosmological model with anisotropic equations of state parameters was investigated by Koivisto and Mota [11]. Singh and Beesham [12] studied anisotropic dark energy in the general theory of relativity.

Bulk viscosity leads to galaxy formation [13]. Padmanabhan and Chitre [14] have obtained inflationary like solutions in the presence of bulk viscosity. Inflationary solutions are also discussed in the literature [15, 16]. Singh et al.[17] have studied higher-dimensional model with variable bulk viscosity in Lyra geometry. One can develop a singularity free model by considering imperfect cosmic fluids [18]. Singularity-free cosmological models with viscosity and zero mass scalar fields are obtained by Roy and Maiti [19].

Motivating with the above discussion, we consider anisotropic dark energy with viscosity in the context of BDT and SBT. Section 2, devoted to basic equation. Section 3, represents the solution of BDT field equations. In section 4, we derive solution SBT field equations. In section 5, we conclude our results.

2. Basic equation

Bianchi type I-IX space-times are important for the study of early stages of evolution of the universe. The present state of the universe is well described by Friedman-Robertson-Walker (FRW) universe. Early stages of the universe may not be equal to the present. Therefore, Bianchi type II,VIII and IX are of special interest because it correspond to familiar solutions like Friedman-Robertson-Walker metric with positive curvature, the de Sitter universe, the Taub-Nut solutions etc. We consider Bianchi type II, VIII and IX metric in the following form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + \left(B^2I^2 + A^2h^2\right)dz^2 + 2A^2h(y)dxdz \quad (1)$$

where A and B are functions of cosmic time t. It represents

Bianchi type II: if $I(y) = 1$ and $h(y) = y$.

Bianchi type VIII: if $I(y) = \cosh y$ and $h(y) = \sinh y$

Bianchi type IX: if $I(y) = \sin y$ and $h(y) = \cos y$

The examination of isotropy of the universe has a long history. When metric and fluid are allowed to exhibit anisotropy, what happens at early and late time? Whether both metric and fluid tends to be isotropy? Such questions require the study of anisotropic models. Late tile isotropization of the universe is related to dark energy. The general form of the anisotropy parameter with Bianchi type III was studied by Akarsu and Kilinc [20]. In this paper, we consider the energy momentum tensor as

$$T_j^i = dia\left(\omega\rho - 3\zeta H, (\omega+\gamma)\rho - 3\zeta H, (\omega+\eta)\rho - 3\zeta H, \rho\right) \quad (2)$$

where ρ is the energy density of fluid, ω is equation of state parameter, γ and η are deviation parameters. γ and η are deviations from ω on both y and z axis respectively. We assume that ω, γ and η are not necessarily constants and can be functions of t.

3. Brans-Dicke Theory

BDT is simple and reduces to general relativity in some limit; therefore it si natural choice as generalization of general relativity. The present acceleration is derived in BDT in the matter-dominated era. However, the transition is not achieved in BDT [21]. Singh and Kale [22] have studied role of viscosity in BDT. The field equations of Brans-Dicke theory are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi\phi^{-1}T_{ij} - w\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}\left(\phi_{ij} - g_{ij}\phi_{;k}^{;k}\right) \quad (3)$$

where R is the scalar curvature. The semicolon denotes covariant differentiation. The comma denotes partial differentiation. The scalar field ϕ satisfy the equation.

$$\phi = \frac{8\pi T}{(3+2w)\phi} \quad (4)$$

where w is a dimensionless coupling constant. When $w \to 0$, BDT resembles to super gravity [23].

For the metric (1) and energy momentum tensor (2), Brans-Dicke field equations written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\delta}{B^2} - \frac{3}{4}\frac{A^2}{B^4} + \frac{w}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{\ddot{\phi}}{\phi} = -8\pi\phi^{-1}\left(\omega\rho - 3\zeta H\right) \quad (5)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{1}{4}\frac{A^2}{B^4}+\frac{w}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)+\frac{\ddot{\phi}}{\phi}=-8\pi\phi^{-1}\left((\omega+\gamma)\rho-3\zeta H\right) \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{1}{4}\frac{A^2}{B^4}+\frac{w}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+2\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}\right)+\frac{\ddot{\phi}}{\phi}=-8\pi\phi^{-1}\left((\omega+\eta)\rho-3\zeta H\right) \tag{7}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\delta}{B^2}-\frac{1}{4}\frac{A^2}{B^4}+\frac{w}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2+\frac{\dot{\phi}}{\phi}\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=8\pi\phi^{-1}\rho \tag{8}$$

From equations (6) and (7), we obtain

$$\gamma=\eta+\frac{\dot{\phi}}{8\pi\rho}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right) \tag{9}$$

Using equations (5),(6) and (9), we get

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}-\frac{\delta}{B^2}+\frac{1}{4}\frac{A^2}{B^4}=-8\pi\phi^{-1}\rho\gamma \tag{10}$$

We assume the volume of the universe is given by

$$V=AB^2 \tag{11}$$

Using equation (11), we arrive at the following equation

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{\lambda}{V}+\frac{1}{V}\int V\left(\frac{\delta}{B^2}-\frac{A^2}{B^4}-8\pi\phi^{-1}\rho\gamma\right)dt \tag{12}$$

The integral term in equation (12) vanish for the following value of deviation parameter

$$\gamma=\frac{\phi}{8\pi\rho}\left(\frac{\delta}{B^2}-\frac{A^2}{B^4}\right) \tag{13}$$

We obtain solutions under three assumptions viz. hyperbolic volumetric expansion, relation between coefficient of bulk viscosity and energy density and relation between scalar field and volume of the universe. The following hyperbolic volumetric expansion assumed to solve equation (12):

$$V=\cosh^2 t \tag{14}$$

Using equations (13), (14) in (12), the metric potentials A and B are obtained as

$$A=(\cosh t)^{2/3}\exp\left(\frac{2\lambda}{3}\tanh t\right) \tag{15}$$

$$B=(\cosh t)^{2/3}\exp\left(\frac{-\lambda}{3}\tanh t\right) \tag{16}$$

The relation between scalar field ϕ and volume V is assumed as

$$\phi=bV^m=b\cosh^m t \tag{17}$$

where m is constant. The deviation parameter is found to be

$$\gamma=\left(\delta(\cosh t)^{-4/3}\exp\left(\frac{2\lambda}{3}\tanh t\right)-(\cosh t)^{-4/3}\exp\left(\frac{8\lambda}{3}\tanh t\right)\right)\frac{\phi}{8\pi\rho} \tag{18}$$

We assume that coefficient of viscosity is proportional to the energy density

$$\zeta=k\rho^\alpha \tag{19}$$

This type of assumption is also known as radiation fluid when $\alpha=1$. Arab [24] has found de Sitter type inflationary solutions. Roy and Maiti [25] developed a singularity free FRW viscous fluid cosmological model. Bali and Yadav [26] have investigated Bulk viscous model in Bianchi type IX universe. Weinberg [27] has assumed bulk viscosity as a power function of energy density. Murphy [27] has studied bulk viscous models. Using equations

(5),(8),(15),(16) and (19), we obtain the expressions of energy density (ρ), and equation of state parameter (ω) as

$$8\pi\phi^{-1}\rho = 3\tanh^2 t - 3\lambda^2 \cosh^{-4} t + (\delta - 0.25)\cosh^{\frac{-4}{3}} t \exp\left(\frac{8\lambda}{3}\tanh t\right) + 2m\tanh t\left[(2 - wm)\tanh t + \lambda\cosh^{-2} t\right] \tag{20}$$

$$-8\pi\phi^{-1}\omega\rho = -24\pi\phi^{-1}\rho^{\alpha}\tanh t + 1.33 + 3\lambda^2\cosh^{-4} t + (\delta - 0.75)\cosh^{\frac{-4}{3}} t \exp\left(\frac{8\lambda}{3}\tanh t\right) + 2m\tanh t\left[(1.33 + wm)\tanh t + \lambda\cosh^{-2} t\right] \tag{21}$$

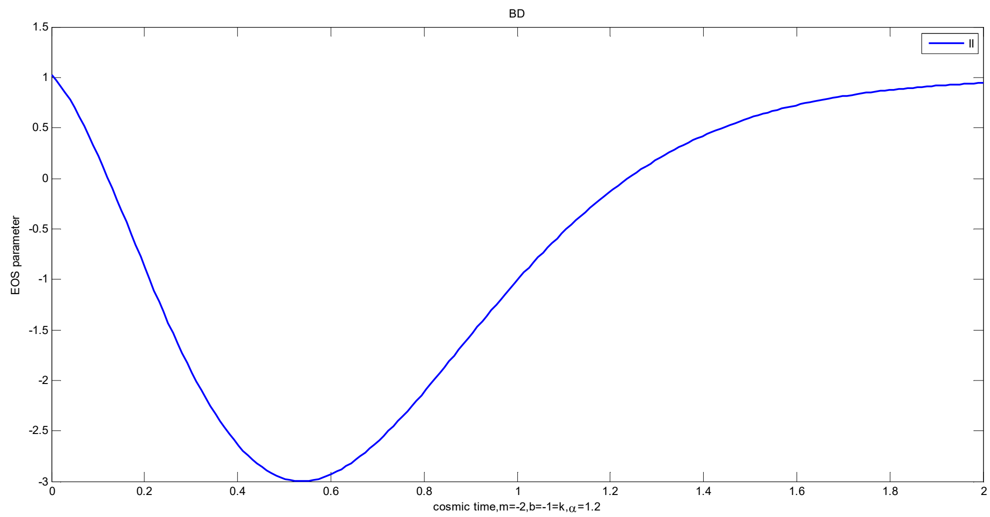

Fig.1 Plot of equation of state parameter versus time.

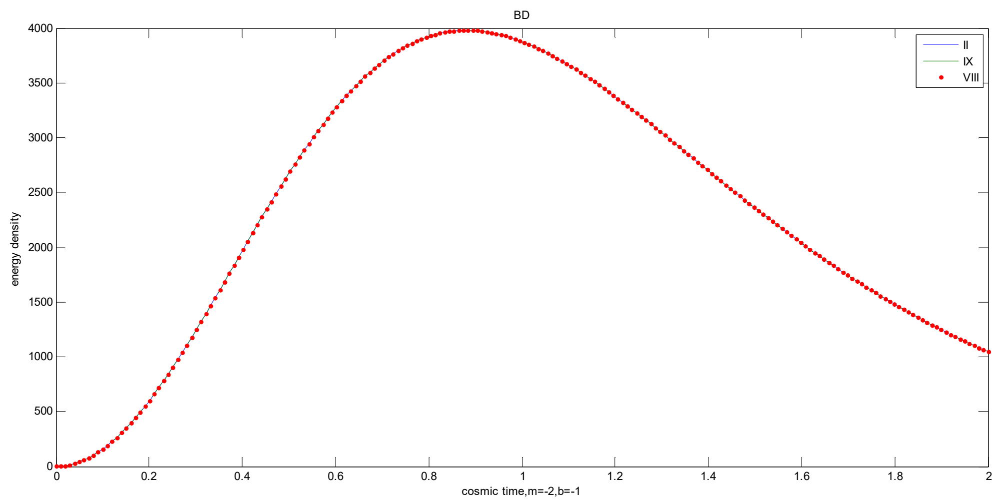

Fig2. Plot of energy density versus time

Figure (1) describes plot of equation of state parameter with cosmic time in the range $0 \le t \le 2$.We observe that at $t = 0, \omega = 1$ i.e. universe is matter-dominated and as t increases, ω crosses phantom divide line and as $t \to 2$, $\omega \to 1$. Trajectory of ω is same for all Bianchi type II, VIII and IX. It is note that for constants $k = b = 1$, $\alpha = 1.2$ and for all other values of these constant, the equation of state (EoS) parameter, $\omega \ge 1$. Here, the coupling constant is taken as $w = 10{,}000$. Also, when we take small values of w, the EoS parameter becomes greater than one. It implies that $\alpha = 1.2$ is condition of dark energy.

Figure (2) describes evolution of energy density with time ($\backslash 0 \le t \le 2$) for Bianchi type II, VIII and IX universe. It is seen that at $t = 0, \rho = 0$ and as t increases, ρ increase to its maximum and then decrease to constant. When the amount of energy density is zero it is called as a zero-energy universe or a universe from nothingness [29]. Thus, the universe starts to evaluate from a state of nothing to its maximum.

4. Saez-Ballester theory

SBT is useful to give answers to the problem of missing matter. In SBT, the metric is coupled with scalar field ϕ. The dimensionless scalar field coupling with metric describes weak fields in SBT.Reddy [30] studied cosmic strings in SBT. Singh [31] presented Bianchi type V cosmology in SBT. The Saez-Ballester field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R - \beta \phi^n \left(\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -T_{ij} \tag{22}$$

The scalar field ϕ satisfy the equation

$$2\phi^n \phi^i_{;i} + n\phi^{n-1} \phi_{,k} \phi^{,k} = 0 \tag{23}$$

where n and β are constants. Bianchi type II, VIII and IX space time for anisotropic dark energy in Saez-Ballester theory of gravitation is investigated by Rao et al.[32]. Adhav [33] has explored Bianchi type II for anisotropic dark energy in the general theory of relativity. For the metric (1) and energy momentum tensor (2), Saez-Ballester field equations written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\delta}{B^2} - \frac{3}{4}\frac{A^2}{B^4} - \frac{\beta}{2}\phi^n \left(\dot{\phi}\right)^2 = -(\omega\rho - 3\zeta H) \tag{24}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{A^2}{B^4} - \frac{\beta}{2}\phi^n \left(\dot{\phi}\right)^2 = -((\omega + \gamma)\rho - 3\zeta H) \tag{25}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{A^2}{B^4} - \frac{\beta}{2}\phi^n \left(\dot{\phi}\right)^2 = -((\omega + \eta)\rho - 3\zeta H) \tag{26}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{\delta}{B^2} - \frac{1}{4}\frac{A^2}{B^4} + \frac{\beta}{2}\phi^n \left(\dot{\phi}\right)^2 = \rho \tag{27}$$

From equations (25) and (26), we get

$$\gamma = \eta \tag{28}$$

In order to avoid the repetition and proceeding as in the case of Brans-Dicke theory, we get the same equation (10) and the value of the deviation parameter γ as

$$\gamma = \frac{1}{\rho}\left(\frac{\delta}{B^2} - \frac{A^2}{B^4} \right) \tag{29}$$

The scalar field ϕ is found to be

$$\phi = \left(\frac{n+2}{2} \right)^{2/n+2} (\tanh t)^{2/n+2} \tag{30}$$

For the solution (15) and (16), the expansion scalar (θ), mean anisotropic parameter (Δ), Shear scalar (σ) and deceleration parameter (q) are obtained as

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} = 2\tanh t \tag{31}$$

$$\Delta = \frac{2}{9\left(\sinh^2 t \cosh^2 t\right)} \tag{32}$$

$$\sigma^2 = \frac{\lambda^2 \cosh^{-4} t}{3} \tag{33}$$

$$q = -\frac{1}{\tanh^2 t} \tag{34}$$

Using equations (5),(8),(24) and (27), we obtain the expressions of energy density (ρ), deviation parameter (γ) and equation of state parameter (ω) as

$$\rho = 3\tanh^2 t - 3\lambda^2 \cosh^{-4} t + (\delta - 0.25)\cosh^{\frac{-4}{3}} t \exp\left(\frac{8\lambda}{3}\tanh t\right) + \frac{\beta}{2}\sec h^4 t \tag{35}$$

$$\gamma = \frac{1}{\rho}\left(\delta \exp\left(\frac{2\lambda}{3}\tanh t\right) - \exp\left(\frac{8\lambda}{3}\tanh t\right)\right)(\cosh t)^{-4/3} \exp\left(\frac{8\lambda}{3}\tanh t\right) \tag{36}$$

$$\omega\rho = -9\rho^{\alpha}\tanh t - 1.33 - 3\lambda^2 \cosh^{-4} t + (0.75 - \delta)\cosh^{\frac{-4}{3}} t \exp\left(\frac{8\lambda}{3}\tanh t\right) + \frac{\beta}{2}\sec h^4 t \tag{37}$$

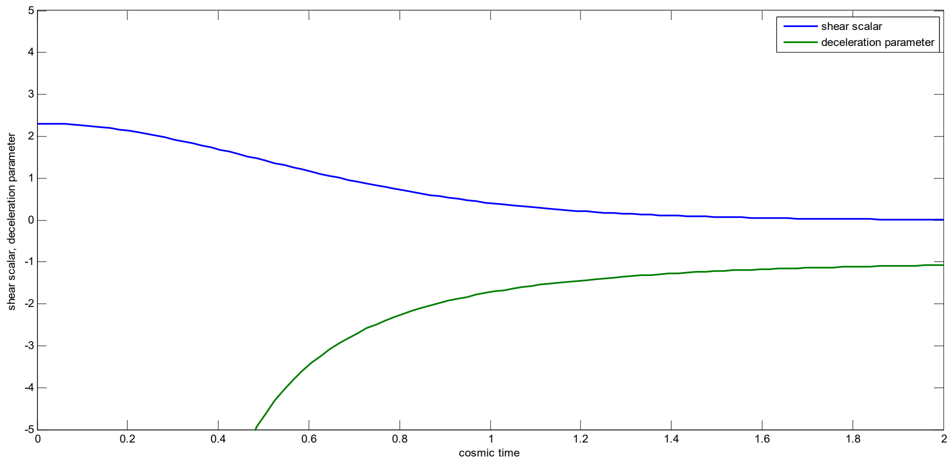

Fig 3. Plot of shear scalar and deceleration parameter versus time

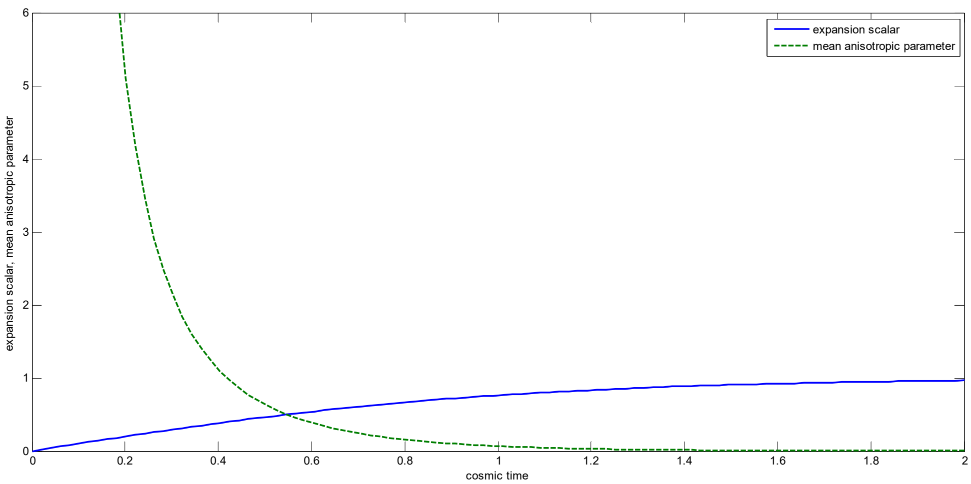

Fig4. Plot of expansion scalar and mean anisotropic parameter versus time

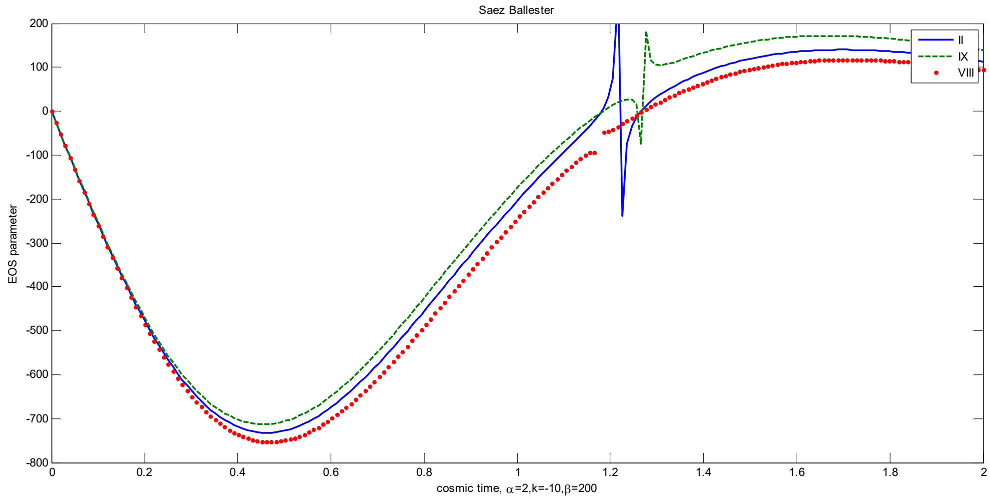

Fig5. Plot of equation of state parameter versus time

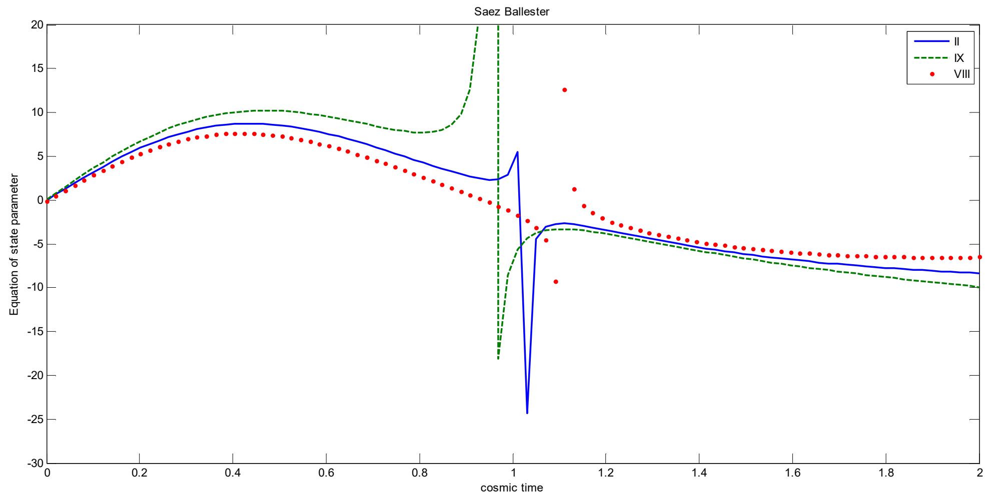

Fig6. Plot of equation of state parameter versus time

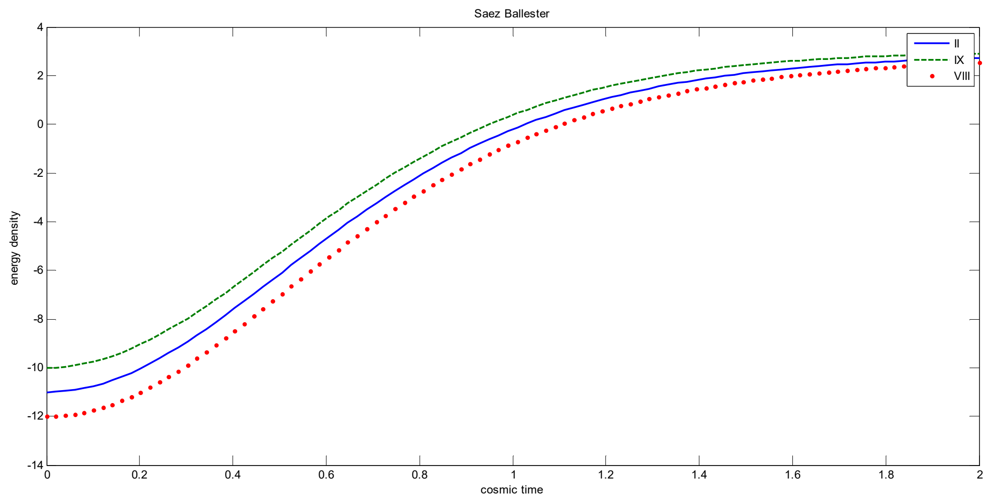

Fig7. Plot of energy density versus time

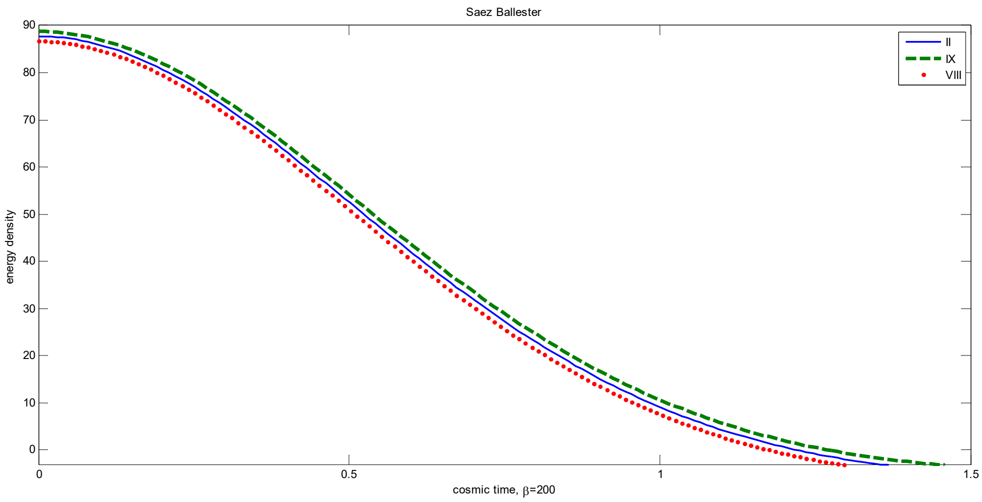

Fig8. Plot of energy density versus time

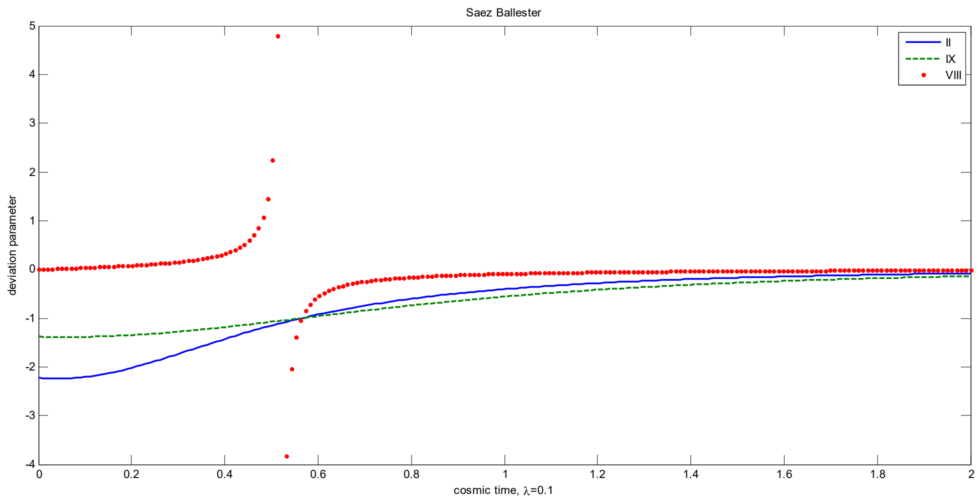

Fig9. Plot of deviation parameter versus time

The sign of the deceleration parameter is negative. The positive sign of the deceleration parameter indicates a decelerating universe, whereas the negative sign stands for an accelerating universe. From figure (3), it is clear that the sign of the deceleration parameter is negative; i.e. the universe is accelerating. From figure (4), it is observed that the mean anisotropic parameter tends to zero; i.e. the universe is isotropic at late time. Initially it was anisotropic. It is consistent with observational results. The expansion scalar is constant, i.e. the universe is expanding.

Figure (5) and (6), describes evolution of equation of state parameter versus cosmic time for $\beta = 200$, and $\beta = 2$ respectively. We found that when β is large, the EoS parameter is negative near $t = 0$, and tends to positive value for large value of t. When β is small, the EoS parameter is positive near $t = 0$, and tends to negative value for large value of t.

An important point is that the EoS parameter behaves alike for large value of w in BDT and β in SBT. For small values of w in BDT EoS parameter is positive large whereas for small values of β in SBT, EoS parameter tends to negative at later time. Also note that value of α in BDT is 1.2 and in SBT is 2. Adhav [33] have found that the EoS parameter tends to -1 in the case of general relativity. In scale covariant theory, the trajectory of EoS parameter lies in the range $-5 < \omega < 4$ [34] whereas it is $-3 < \omega < 1$ in BDT and $-800 < \omega < 200$ in SBT. This variation of trajectory in different theories shows the effect of scalar field and viscosity.

Figure (7) and (8) describes evolution of energy density with time $0 \le t \le 2$, for $\beta = 2$ and $\beta = 200$, respectively. It indicates that when β large energy density is is decreasing function of time. It is large near $t = 0$, and tends to zero at large times. When β is small, it is negative near $t = 0$, and tends to constant positive value at large time.
Figure (9), it is clear that the deviation parameter is negative near $t = 0$, and tends to zero as time increases; i.e. the anisotopic fluid tends to be isotropic at initial later time. It is interesting to note that both metric and fluid (figures 4 and 9) were initially anisotropic and tend to be isotropic at later times.

5. Conclusion

In this study we have considered anisotropic dark energy embedding viscosity in the context of Brans-Dicke and Saez-Ballester theory of gravitation for Bianchi type II,VIII and IX metric. We found that the universe is accelerating and expanding, which is consistent with observational results [7-9]. The universe was anisotropic in the past and it is isotropic in the present and in the future. The viscosity and scalar field contributed significantly to the deviation of the EoS parameter. In BDT [35] and SBT [32] in the absence of viscosity, the behaviour of the energy density is a decreasing function of time. Initially it is constant and tends to zero at infinite times. In the presence of viscosity in BDT start from Vacuum and tends to constant. In SBT, for small values of β, energy density initiated from negative and tends to constant, whereas for large value of β, it starts to decrease from constant and tends to zero at later time. The isotropy of the model is not affected by viscosity. In self-creation theory, models do not approach isotropy [36]. The novelty of the work is that, due to viscosity there are constraints on constants. The coupling constant value should be large. The constant m is negative. Further, for small values of β we get negative energy density. The range of EoS parameter is too small as compare to the observational limit $-1.46 < \omega < -0.785$ [7-9].

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