Self-Organization in Highly Dissipative Arrays of Coupled Josephson Junctions

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Abstract

Our theoretical study shows the appearance of the second hysteresis zone, formed by traveling wave and outermost branches in current-voltage characteristics (CVCs) in highly dissipative systems of coupled Josephson junctions. The manifestation and width of the second hysteresis zone depend on coupling and dissipation parameters, and number of junctions in the system. We demonstrate existence in the studied systems of self-organizing phenomena, which consist of traveling waves (TW) generation, decrease of energy loses for resistive dissipation and THz electromagnetic radiation emission. The detailed analysis of the charge oscillations on the superconducting layers shows formation of complex wave package, which ensures the TW branch stability. Due to high visibility of this phenomenon in I-V characteristics, it can be easily detected experimentally and can be used in quantum computing, traveling wave parametric amplifiers, and radiation THz emitters.

1 Introduction

The Josephson junction (JJ) is a superconducting-based heterostructure, which behaviour is determined by macroscopic quantum effects observed in these systems since 1980s [1, 29, 27, 5, 23]. Moreover, layered high-T_C superoconductors can be studied as the intrinsic Josephson junction (IJJs) arrays [28], and represent a great example of self-organization [15] and nonlinear wave interaction [16]. From the applicative point of view JJs have promising applications in superconducting electronics, plasmon based devices and quantum metrology [26, 30, 12, 25]. Self-organization due to quantum correleations is discussed in the works [33, 2, 4, 31, 32].

Simulations of the stack of IJJs represent an efficient way to predict its properties [6]. Current theoretical models allow simulation of the behavior of a stack of IJJs using coupling [7] (linked to the structure of IJJs) and dissipation [14] (connected to the temperature of the device) parameters. Their variation strongly influence the system behavior, that can be easily observed experimentally by the CVC measurements [24].

Recent results put in evidence affluent complex of phenomena which occur in JJs arrays because of finite coupling and dissipation parameters. In [21, 19, 20, 9] was discussed branching in the CVCs due to breakpoint. After that in [22, 17, 10] was proven the existence of traveling wave branch, with possibility of its experimental detection [17]. An interesting topic with important applications in sensing, imaging, and spectroscopy represents the THz wave emission by the long JJ arrays [13, 8].

However the main studies were focused in the McCumber hysteresis region in CVCs. In this Letter, within the framework of CCJJ + DC model [11] we reveal and describe formation of stable wave packages in Josephson plasma for highly dissipative systems for currents greater than critical current. This phenomenon manifests in CVC in form of the second hysteresis zone. In this paper, we study in the framework of CCJJ+DC [11] model highly dissipative systems of coupled IJJs. We have discovered the second hysteresis zone, which appears for currents greater than critical currents. The phase dynamics of such systems are investigated below, along with the plasma waves that appear. Some sapects of the discovered phenomenona are presented in the work [3]/

2 Model and Method

A stack of Josephson junctions is described by a system of non-linear differential equations, which do not have analytical solutions. Different models are used to describe the behavior of Josephson junctions, among which is capacitively coupled Josephson junction (CCJJ) model. Because of charging of superconducting layers in the system appears capacitive coupling between IJJs [21]. The CCJJ+ DC model was derived in [11] on the microscopic level, due to the importance of diffusion current and its influence upon the dynamics of the system of IJJs. The mechanism of switching from one branch to another in the CVC is caused by resonance between the Josephson and plasma oscillations, which causes the Breakpoint [19, 20] (BP) region. To investigate the system of capacitively coupled IJJs is used CCJJ+DC [11] model, described by the following equations:

$$\frac{d}{dt}V_l = I - \sin\phi_l - \beta\frac{d}{dt}\phi_l \tag{1}$$

$$\frac{d}{dt}\phi_l = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l) \tag{2}$$

where V_l is voltage between layers l and $l+1$, normalized to the value $V_0 = \hbar\omega_p/2e$, where ω_p is plasma frequency and $\omega_p^2 = 2eI_c/\hbar C$ with C being the capacitance of IJJs, $\phi_l(t) = \theta_{l+1}(t) - \theta_l(t) - \frac{2e}{\hbar}\int_l^{l+1} dz A_z(z,t)$ is gauge-invariant phase difference between layers l and $l+1$, A_z is vector potential in the barrier, α and β are coupling, and respectively dissipation parameters of the system, current I is measured in the units of the critical current I_c, and time t is normalized to ω_p^{-1}. We solve numerically the system of equations 1 using the fourth-order Runge-Kutta method and obtain the phase-dynamics of our system. The charge density Q_l on the superconducting layers (we will address it as charge)is obtained using the Maxwell equations.

$$Q_l = Q_0\alpha(V_{l+1} - V_l) \tag{3}$$

where $Q_0 = \varepsilon\varepsilon_0 V_0/r_D^2$, r_D is Debay screening length, V_l and V_{l+1} is voltage on the consecutive insulating layers, and ε is electric permittivity. In this work we use periodic boundary conditions for our system. The simulation procedure is described laboriously in [20, 18]. The power of electromagnetic radiation was calculated as:

$$P = V_{ac}^2/R_z \tag{4}$$

where $V_{ac} = d_I E_{ac}$ with E_{ac} being the alternating electric field, and $R_Z = (d_I/W)Z$, where W is the thickness of the junction and Z is the impedance of emission. For calculations was used method proposed in [13].

3 Manifestation of Second Hysteresis Zone on CVCs

In his section we propose to put in evidence the influence of coupling and dissipation parameters on CVCs of arrays of IJJs. Figure 1 shows CVCs in an array of nine Josephson junctions with fixed coupling $\alpha = 1$ and different values of dissipation parameter. This figure demonstrates an appearance of the second hysteresis (SH) for arrays with large dissipation. For $\beta = 0.709$, corresponding to the critical dissipation parameter value manifests a small SH (Insert of figure 1). For greater values of β the second hysteresis becomes larger and shifts to the area of greater currents.

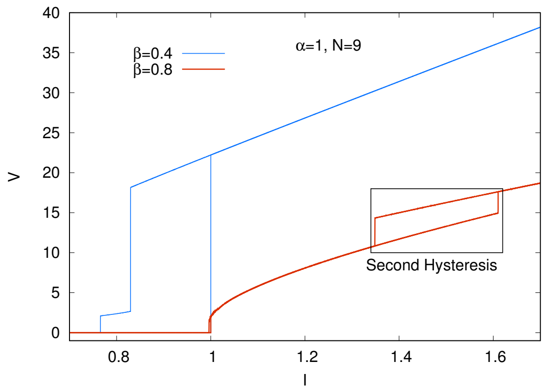

Figure 1: The CVCs for $\beta = 0.4$ and $\beta = 0.8$ in a system of $N = 9$ IJJs and with coupling of $\alpha = 1$. With black rectangle is put in evidence second hysteresis zone.

The appearance of the SH in CVC is a manifestation of a complex phenomenon that takes place in the strongly coupled and highly dissipative system of JJs. Due to internal capacitance of Josephson junctions, the superconducting layers are charged when through the stack flows the bias current. If the system follows the outermost branch, the amplitude of the charge oscillations is comparable with noise and cannot be detected. The charge oscillations on the TW branch self-organize and propagate through the stack as a longitudinal plasma wave (Figure 2). The main difference between the traveling plasma wave corresponding to the SHZ and TW observed in McCumber region [17] is its nonlinearity (insert 2).

Let us investigate the influence on CVC of coupling between the junctions in the system. In figure 3 is presented existence of tree types of behavior in highly dissipative arrays of Josephson junctions, corresponding to different coupling parameters. For weak coupling (dashed line 3), the array of JJs manifests well known behaviour with large McCumber hysteresis on CVC and existence of only outermost branch for currents greater than critical one [19]. The form of CVC changes with the increase of coupling parameter, McCumber hysteresis becoming smaller. For the critical coupling parameter $\alpha = 0.52$ (insert 3) and $\beta = 0.8$ system has transient behaviour with manifestation of chaos, appearance of SHZ and well pronounced branching in McCumber hysteresis. The same behavior was observer as well for critical dissipation parameter ($\alpha = 1$ and $\beta = 0.709$. Critical dissipation and coupling parameters, along with the physical properties of tran-

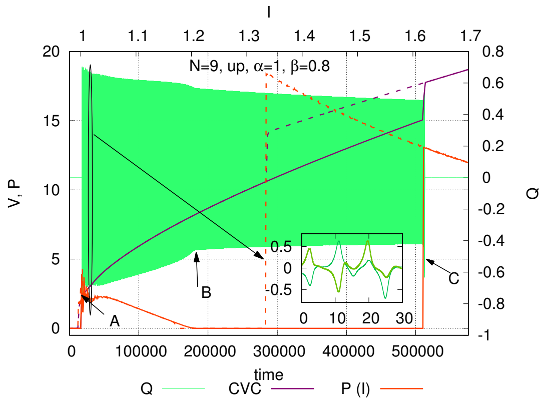

Figure 2: Temporal charge dependence on the first layer for the increase of current, and CVC with radiation emission, as functions of current. With dashed lines are shown CVC and radiation emission for the decrease of current. Inset shows the form of charge oscillations in the first and second layers.

sient chaotic systems will be discussed in a forthcoming paper. The SHZ becomes more pronounced with the increase of coupling parameter ($\alpha > 0.52$), the system becomes more stable and McCumber hysteresis abruptly reduces (blue line 3). Below we find the origins of physical phenomena behind the formation of SHZ and demonstrate that the second hysteresis is formed by the traveling wave (TW) branch and the outermost branch, and find origins of TW branch stability.

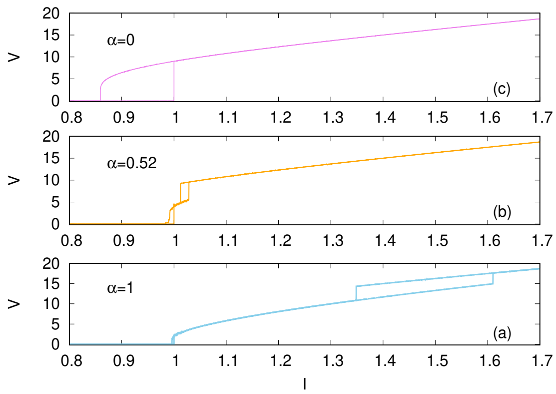

Figure 3: CVCs for a system of nine IJJs with $\beta = 0.8$ with different coupling between the junctions: $\alpha = 0$, $\alpha = 0.52$ and $\alpha = 1$.

4 Conclusions

In summary, we predict the appearance of second hysteresis region, formed by traveling wave and outermost branches, in I-V characteristic in strongly coupled and highly dissipative systems of Josephson junctions and layered high-T_c superconductors. The stability of TW branch is ensured by the complex structure of wave package. Due to large the distance between outermost and TW brances in CVC, we hope that discussed phenomenon could prompt an experimental validation.

The plasmon generation on TW branch can be applied into TW parametric amplifiers, for current locking into point contacts and micro bridges. Furthermore, new type of JJ arrays behaviour can be used for multi-state elements in solid state quantum computers.

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