Mass predictions for a vector field in interaction with various collections of real scalar fields: A brief review

Constantin Bizdadea, Eugen-Mihăiţă Cioroianu, Solange-Odile Saliu
Department of Physics, University of Craiova,
13 Al. I. Cuza Str., Craiova, 200585, Romania

Abstract

Starting from the observation that gauge fields could acquire mass when they mediate interactions between matter fields, lately, new mass generation schemes [1, 2] have been proposed. Recently, one of them [1] has been applied [3, 4, 5] to generate mass for a vector field in the context of its interaction with various collections of real scalar fields. This paper aims to collect the predictions made in our previous analyses [3, 4, 5].

1 Introduction

The famous unification theory that puts together weak and electromagnetic interactions involves two massive vector bosons $W^{\pm}$. Originally, this was explained by a mass-generation mechanism for gauge fields [6, 7, 8, 9]. This procedure assumes a potential term, which depends only on the matter fields and displays at least one local minimum configuration. With this setting, within the initial mass-generation mechanism [6, 7, 8, 9], the gauge fields acquire mass via some appropriate affine matter field redefinitions (Higgs mechanism) such that, in this scheme, the local minimum configuration of the potential corresponds to the trivial configuration for the new matter fields. At the quantum level, these redefinitions imply trivial vacuum expectation values for the new field operators. Lately, two mass-generation mechanisms for gauge vector fields have been proposed. None of these accounts in any way for the Higgs mechanism. The former [1] regards the mass as an effect of interaction such that the mass for an Abelian 1-form appear in the context of its interactions with an arbitrary finite set of real scalar fields. The latter [2] considers a $SU(2)$-Yang-Mills gauge theory coupled with a set of three scalar fields and the mass-production for gauge fields is gauge-invariant (without breaking the original gauge symmetry) achieved by means of some suitable constraints imposed on the scalar fields.

This paper aims to synthetically review our previous results [3, 4, 5] concerning the mass predictions for a vector field that is consistently coupled with various collections of real scalar fields. These have been obtained via implementing the multi-step program from [1] adapted to various sets of real scalar fields: (1) one starts with the Lagrangian description of the free theory; (2) one infers a general class of gauge theories that does not contain free parameters with negative mass dimensions and whose free limit is that from step (1); (3) one performs some conveniently chosen redefinitions of the free parameters that label interacting theories from (2) such that the mass terms become manifest in the new free limit. The employed mechanism reveals the next outputs: (A) the vector field acquires mass; (B) the scalar fields display non-trivial gauge transformations; (C) the gauge algebra of each interacting theory is Abelian; (D) the propagator of the massive vector field emerging from each gauge-fixed action possesses the same asymptotic behaviour [in the UV regime] like that from the massless case.

The paper is organized into five sections. In Section 2 we collect the main results from [1] concerning the consistent interactions that can be added to a free theory consisting of an Abelian 1-form and a finite collection of real scalar fields. Section 3 exhibits some predictions for the mass spectrum corresponding to an interacting theory that involves a vector field and four real scalar fields. In Section 4 we analyze the mass spectrum in the case where the matter field spectrum consists of five real scalar fields. Section 5 ends the paper with the main conclusions.

2 Real scalar fields interactions mediated by a gauge field: Main results

In this section we collect the main aspects concerning the consistent couplings between a gauge field and a collection of real scalar fields [1]. These results have been obtained by means of the deformation of the solution to the classical master equation [11, 12] with the help of local BRST cohomology [13, 14, 15].

Concretely, one starts from a free theory that consists of one Abelian vector field A_μ and a collection of massless real scalar fields $\{\varphi^A\}_{A=\overline{1,N_0}}$, whose Lagrangian dynamics is generated by

$$S_0^{\mathrm{L}}[A,\varphi] = \int d^4x \Big[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}k_{AB}\big(\partial_\mu\varphi^A\big)\partial^\mu\varphi^B \Big]. \tag{1}$$

The fields evolve on a 4-dimensional Minkowski spacetime of mostly minus signature, $\mathbb{R}^{1|3}$. Also, we consider that the space parameterized by the scalar fields (known as the target space in Poisson Sigma Models) is Euclidian, with the metric tensor components k_{AB}. The previous functional is found to be manifestly invariant under the gauge transformations

$$\delta_\epsilon A^\mu = \partial^\mu\epsilon, \qquad \delta_\epsilon\varphi^A = 0, \quad A = \overline{1,N_0}, \tag{2}$$

that are Abelian and irreducible (independent). At this stage, it is clear that the number of physical degrees of freedom of the starting theory is equal to $N_0 + 2$.

Using the technique of constructing consistent interactions [11, 12], it has been found [1] that the interacting theory, which complies with the standard hypotheses from field theory (analyticity in the coupling constant–g, Lorentz covariance, spacetime locality, Poincaré invariance, and at most two spacetime derivatives in the interaction vertices) is described by the Lagrangian action

$$\begin{aligned}\bar{S}_0^{\mathrm{L}}[A,\varphi] = \int d^4x \Big[& -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2k_{AB}n^An^BA_\mu A^\mu - g\mathcal{V}\left(\varphi^A\right) \\ & + \tfrac{1}{2}k_{AB}(1+g\omega\left(\varphi^A\right)\big(D_\mu\varphi^A - 2gn^AA_\mu\big)D^\mu\varphi^B \\ & + \tfrac{1}{2}gF_{\mu\nu}\big(\vartheta\left(\varphi^A\right)F^{\mu\nu} + \varepsilon^{\mu\nu\rho\lambda}\varkappa\left(\varphi^A\right)F_{\rho\lambda}\big) \\ & + \tfrac{1}{2}g^3k_{AB}\omega\left(\varphi^A\right)n^An^BA_\mu A^\mu\Big],\end{aligned} \tag{3}$$

which is invariant under the Abelian generating set of gauge transformations

$$\bar{\delta}_\epsilon A^\mu = \partial^\mu\epsilon, \qquad \bar{\delta}_\epsilon\varphi^A = g\big(T^A{}_B\varphi^B + n^A\big)\epsilon, \quad A = \overline{1,N_0}. \tag{4}$$

Previously, $\mathcal{V}$, ω, ϑ and $\varkappa$ are some smooth functions that depend only on the undifferentiated scalar fields and are subject to some consistency equations [1], $T^{A}{}_{B}$ are some real numbers that enjoy the property

$$k_{AM}T^{M}{}_{B}+k_{BM}T^{M}{}_{A}=0,\quad A,B=\overline{1,N_0}, \tag{5}$$

and $\{n^A : A=\overline{1,N_0}\}$ is a collection of arbitrary real numbers. We also used the notation

$$D_\mu\varphi^A=\partial_\mu\varphi^A-gT^{A}{}_{B}\varphi^B A_\mu \tag{6}$$

for the covariant derivatives of the scalar fields.

Functional (3) contains the term $\frac{1}{2}g^2k_{AB}n^An^BA_\mu A^\mu$, which, up to some redefinitions of the constants that parameterize (3), is a mass term for the vector field as long as

$$k_{AB}n^An^B>0. \tag{7}$$

Finally, the term $\mathcal{V}$ is responsible for the masses of the involved scalar fields.

3 Mass predictions in the context of a 4-dimensional target space

In this section we consider the case $N_0=4$. Motivated by our aim—to endow the involved gauge field with mass, we perform some simplifications of the general results (3)–(4). First, by making an appropriate linear reparametrization of the target space, we can consider that

$$k_{AB}=\delta_{AB}. \tag{8}$$

Second, since the terms proportional to ω, ϑ, and $\varkappa$ do not contribute to the mass spectrum, we can take

$$\omega=\vartheta=\varkappa=0. \tag{9}$$

Third, using the canonical expression available for any skew-symmetric matrix, it results that there exists an orthogonal matrix $\hat{O}\in O(4)$ such that

$$\hat{O}\hat{T}\hat{O}^\top=\begin{pmatrix}0&\beta_1&0&0\\-\beta_1&0&0&0\\0&0&0&\beta_2\\0&0&-\beta_2&0\end{pmatrix}, \tag{10}$$

so, modulo an orthogonal reparametrization of the target space, we are free to take

$$\left(T^{A}{}_{B}\right)\equiv\hat{T}\equiv\begin{pmatrix}0&\beta_1&0&0\\-\beta_1&0&0&0\\0&0&0&\beta_2\\0&0&-\beta_2&0\end{pmatrix},\quad \beta_1,\beta_2\in\mathbb{R}^*. \tag{11}$$

Fourth, motivated by the fact that for every non-vanishing vector $\mathbf{n}\in\mathbb{R}^4$ there is a rotation matrix $\hat{R}\in O(4)$ such that

$$\mathbf{n}=\hat{R}\mathbf{v},\qquad \mathbf{v}\equiv\sqrt{\mathbf{n}\cdot\mathbf{n}}\,(0\ 1\ 0\ 0)^\top, \tag{12}$$

we can choose from the beginning the shifts n^A as

$$\left(n^A\right) \equiv \mathbf{n} \equiv v\,(0\ \ -1\ 0\ 0)^\top \qquad v \neq 0. \tag{13}$$

In the light of choices (8), (9), (13), and (11), the generating set of gauge transformations (4) reduces to

$$\bar{\delta}_\epsilon A_\mu = \partial_\mu \epsilon, \bar{\delta}_\epsilon \varphi_1 = g\beta_1\varphi_2\epsilon, \bar{\delta}_\epsilon \varphi_2 = -g\left(\beta_1\varphi_1 + v\right)\epsilon, \bar{\delta}_\epsilon \varphi_3 = g\beta_2\varphi_4\epsilon, \bar{\delta}_\epsilon \varphi_4 = -g\beta_2\varphi_3\epsilon. \tag{14}$$

With these results at hand, the consistency condition verified by the scalar fiel potential $\mathcal{V}\left(\varphi^A\right)$ reads

$$\bar{\delta}_\epsilon \mathcal{V} \equiv g\left[\beta_1\left(\frac{\partial\mathcal{V}}{\partial\varphi_1}\varphi_2 - \frac{\partial\mathcal{V}}{\partial\varphi_2}\varphi_1\right) + \beta_2\left(\frac{\partial\mathcal{V}}{\partial\varphi_3}\varphi_4 - \frac{\partial\mathcal{V}}{\partial\varphi_4}\varphi_3\right)\right] - gv\frac{\partial\mathcal{V}}{\partial\varphi_2} = 0. \tag{15}$$

To display a renormalizable (at least power-counting [16]) interacting gauge theory, we restrict ourselves to those solutions of (15) that are polynomials of at most order four in the real scalar fields, i.e.

$$\mathcal{V} = \frac{c_1}{2}\left[\left(\beta_1\varphi_1 + v\right)^2 + {\beta_1}^2\varphi_2^2\right] + \frac{\tilde{c}_1}{4}\left[\left(\beta_1\varphi_1 + v\right)^2 + {\beta_1}^2\varphi_2^2\right]^2 + \frac{c_2}{2}\left(\varphi_3^2 + \varphi_4^2\right) + \frac{\tilde{c}_2}{4}\left(\varphi_3^2 + \varphi_4^2\right)^2. \tag{16}$$

Finally, inserting expression (16) into the local functional from (4), we obtain the interacting Lagrangian action

$$\begin{aligned}
\bar{S}_0^{\mathrm{L}}[A,\varphi] = \int \mathrm{d}^4x \Bigg\{ & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2v^2A_\mu A^\mu + \frac{1}{2}\sum_{A=1}^{4}\partial_\mu\varphi_A\partial^\mu\varphi_A - gvA^\mu\partial_\mu\varphi_1 \\
& + gA^\mu\left[\beta_1\left(\varphi_1\partial_\mu\varphi_2 - \varphi_2\partial_\mu\varphi_1\right) + \beta_2\left(\varphi_3\partial_\mu\varphi_4 - \varphi_4\partial_\mu\varphi_3\right)\right] \\
& + \frac{1}{2}g^2A_\mu A^\mu\left[\beta_1^2\left(\varphi_1^2 + \varphi_2^2\right) + \beta_2^2\left(\varphi_3^2 + \varphi_4^2\right) + 2v\beta_1\varphi_1\right] \\
& - \frac{1}{2}g\beta_1^2\left(c_1 + 3v^2\tilde{c}_1\right)\varphi_1^2 - \frac{1}{2}g\beta_1^2\left(c_1 + v^2\tilde{c}_1\right)\varphi_2^2 - \frac{1}{2}gc_2\left(\varphi_3^2 + \varphi_4^2\right) \\
& - gv\beta_1\left(c_1 + v^2\tilde{c}_1\right)\varphi_1 - \frac{1}{2}g\tilde{c}_2\left(\varphi_3^2 + \varphi_4^2\right)^2 \\
& - \frac{1}{4}g\tilde{c}_1\beta_1^2\left(\varphi_1^2 + \varphi_2^2\right)\left(\varphi_1^2 + \varphi_2^2 + 4v\beta_1\varphi_1\right)\Bigg\}.
\end{aligned} \tag{17}$$

Functional (17) exhibits the following desirable features: i) the appearance of mass-like terms for the vector field as well as for some of the scalar fields (i.e terms proportional to $A_\mu A^\mu$ and $\varphi_A\varphi_B$ respectively), but these appear in the second order in the coupling constant g and ii) a strong discrepancy between the polynomial degree and the order of perturbation theory. Motivated by these aspects, we have to perform some redefinitions of the free parameters that label the Lagrangian structure of the interacting theories such that 1. the mass-like terms become true mass terms and 2. the polynomial degree of each term from (17) agrees with the perturbation order. The free parameters in (17) consist of $\Lambda \equiv (g, v, \beta_1, \beta_2, c_1, c_2, \tilde{c}_1, \tilde{c}_2)$, while their redefinitions $\Lambda \leftrightarrow \Lambda' \equiv (g', m_A, \beta_1', \beta_2', c_1', c_2', \tilde{c}_1', \tilde{c}_2')$ read

$$g' = g, \quad m_A = gv, \quad \beta_1' = \beta_1, \quad \beta_2' = \beta_2 \tag{18}$$

$$c_1' = gc_1, \quad c_2' = gc_2, \quad \tilde{c}_1' = \tilde{c}_1/g, \quad \tilde{c}_2' = \tilde{c}_2/g. \tag{19}$$

Inserting the previous redefinitions into (17), we exhibit the mass $m_A = gv$ for the vector field as well as the scalar field 'masses'

$$\mu_{\varphi_1}^2 = \beta_1'^2 \left(c_1' + 3m_A^2 \tilde{c}_1'\right), \quad \mu_{\varphi_2}^2 = \beta_1'^2 \left(c_1' + m_A^2 \tilde{c}_1'\right), \quad \mu_{\varphi_3}^2 = c_2' = \mu_{\varphi_4}^2. \tag{20}$$

Regarding the nature of the field spectrum components, we conclude that all the physical fields A^μ, φ_2, φ_3 and φ_4 are massive as well as the nonphysical degree of freedom φ_1.

4 Mass predictions in the context of a 5-dimensional target space

In this section we review the results [4, 5] envisaging the mass-predictions for a vector field consistently coupled with five real scalar fields. First, we simplify the general results (3) and (4) by making choices (8) and (9). Second, the skew-symmetric matrix $\left(T^A{}_B\right) \equiv \hat{T}$ (in the light of (9), conditions (5) reduce to the skew-symmetry of $\hat{T}$) is considered to be in the canonical form

$$\hat{T} \equiv (T_{AB}) = \begin{pmatrix} 0 & \beta_1 & 0 & 0 & 0 \\ -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 \\ 0 & 0 & -\beta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{21}$$

with some arbitrary non-vanishing real parameters β_1 and β_2

$$(\beta_1 \beta_2)^2 > 0. \tag{22}$$

This can always be done via an orthogonal transformation in the space of scalar fields. In fact, expression (21) is nothing but the decomposition of the five-dimensional representation

$$u(1) \simeq so(2) \to \mathrm{End}\left(\mathbb{R}^5\right), \quad \tau \to \hat{T} \tag{23}$$

into its irreducible components

$$2 \oplus 2 \oplus 1. \tag{24}$$

Third, using the same argument like in the 4-dimensional target space, we are free to fix the shift vector as

$$\left(n^A\right) \equiv \mathbf{n} \equiv (n, 0, 0, 0, 0) \in \mathbb{R}^5, \quad n^2 > 0. \tag{25}$$

Taking into account all these simplifications, the generating set of gauge transformations (4) reduces to

$$\bar{\delta}_\epsilon A^\mu = \partial^\mu \epsilon, \quad \bar{\delta}_\epsilon \varphi_1 = g\left(\beta_1 \varphi_2 + n\right)\epsilon, \quad \bar{\delta}_\epsilon \varphi_2 = -g\beta_1 \varphi_1 \epsilon, \tag{26}$$

$$\bar{\delta}_\epsilon \varphi_3 = g\beta_2 \varphi_4 \epsilon, \quad \bar{\delta}_\epsilon \varphi_4 = -g\beta_2 \varphi_3 \epsilon, \quad \bar{\delta}_\epsilon \varphi_5 = 0, \tag{27}$$

which displays the consistency equation for the scalar field potential

$$\bar{\delta}_\epsilon \mathcal{V} \equiv \beta_1 \left(\frac{\partial \mathcal{V}}{\partial \varphi_1}\varphi_2 - \frac{\partial \mathcal{V}}{\partial \varphi_2}\varphi_1\right) + \beta_2 \left(\frac{\partial \mathcal{V}}{\varphi_3}\varphi_4 - \frac{\partial \mathcal{V}}{\varphi_4}\varphi_3\right) = 0. \tag{28}$$

The most general polynomial function of at most order four that verifies (28) reads

$$\mathcal{V}(\varphi) = \rho_2 I_2 + \frac{1}{2}\sum_{\alpha,\beta=1}^{3} \rho_{\alpha\beta} I_\alpha I_\beta + (I_3)^2 \sum_{\alpha=1}^{3} \bar{\rho}_\alpha I_\alpha + \bar{\rho}_5 (I_3)^4 , \tag{29}$$

where

$$I_1 \equiv \frac{1}{2}\beta_1^2 \left(\varphi_1^2 + \varphi_2^2\right) + n\beta_1\varphi_2, \quad I_2 \equiv \frac{1}{2}\beta_2^2 \left(\varphi_3^2 + \varphi_4^2\right), \quad I_3 \equiv \varphi_5, \tag{30}$$

with ρ some arbitrary real constants. However, for reasons that will become transparent later, we take

$$\rho_2 > 0, \quad \rho_{\alpha\beta} = \tilde{\rho}_\alpha \delta_{\alpha\beta}, \quad \tilde{\rho}_\alpha > 0, \quad \alpha = \overline{1,3}. \tag{31}$$

With all these preparations at hand, according to (3), the Lagrangian action of the interacting theory reads

$$\begin{aligned}\bar{S}_0^{(\mathrm{L}} [A,\varphi] = \int d^4x \Bigg[& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\sum_{A=1}^{5}\partial_\mu\varphi_A\partial^\mu\varphi_A + g\left(\beta_1\left(\varphi_1\partial_\mu\varphi_2 - \varphi_2\partial_\mu\varphi_1\right)\right.\\ & \left.+\beta_2\left(\varphi_3\partial_\mu\varphi_4 - \varphi_4\partial_\mu\varphi_3\right)\right)A^\mu - gnA^\mu\partial_\mu\varphi_1 - g\mathcal{V}(\varphi) + \frac{1}{2}g^2n^2A_\mu A^\mu\\ & +ng^2\beta_1\varphi_2 A_\mu A^\mu + \frac{1}{2}g^2\left(\beta_1^2\left(\varphi_1^2+\varphi_2^2\right) + \beta_2^2\left(\varphi_3^2+\varphi_4^2\right)\right)A_\mu A^\mu\Bigg], \end{aligned} \tag{32}$$

where the potential of the scalar fields is given in (29). By construction, (32) is invariant under the Abelian generating set of gauge transformations (26)–(27).

In the remaining part of this section we focus on the mass predictions that can be extracted from (32). It is transparent that the Lagrangian action of the interacting theory (32) contains mass-like terms for the vector field as well as for some of the scalar fields (i.e terms proportional to $A_\mu A^\mu$ and $\varphi_A\varphi_B$ respectively), but these appear in the second-order in the coupling constant g. Moreover, there is an obvious, strong discrepancy between the polynomial degree and the perturbation order of the terms that appear in the same functional. In order to solve these unsuitable aspects, we proceed like in the previous situation, i.e., we perform some appropriate redefinitions of the free parameters that label the Lagrangian structure of the interacting theory such that 1. the mass-like terms become true mass terms and 2. the polynomial degree of each term from (32) agrees with the perturbation order. If we collectively denote by Λ the free parameters that label (32), $\Lambda = (g,\beta_1,\beta_2,n,\rho_2,\tilde{\rho}_\alpha,\bar{\rho}_\alpha,\bar{\rho}_5)$, then we implement the new free parameters Λ', $\Lambda' = (g',\beta_1',\beta_2',n',\rho_2',\tilde{\rho}_\alpha',\bar{\rho}_\alpha',\bar{\rho}_5')$, via

$$g' = g, \quad \beta_1' = \beta_1, \quad \beta_2' = \beta_2, \quad n' \equiv M_A = gn, \tag{33}$$

$$\rho_2' \equiv M_{34}^2 = g\rho_2, \quad \tilde{\rho}_1' = \frac{1}{g}\tilde{\rho}_1, \quad \tilde{\rho}_2' = \frac{1}{g}\tilde{\rho}_2, \quad \tilde{\rho}_3' \equiv M_5^2 = g\tilde{\rho}_3, \tag{34}$$

$$\bar{\rho}_1' = \frac{1}{g}\bar{\rho}_1, \quad \bar{\rho}_2' = \frac{1}{g}\bar{\rho}_2, \quad \bar{\rho}_3' = \bar{\rho}_3, \quad \bar{\rho}_5' = \frac{1}{g}\bar{\rho}_5. \tag{35}$$

Inserting (33)–(35) into (32) we get

$$\bar{S}_0^{\prime\mathrm{L}}[A,\varphi] = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\sum_{A=1}^{5}\partial_\mu\varphi_A\partial^\mu\varphi_A - M_A A^\mu\partial_\mu\varphi_1 + \frac{1}{2}M_A^2 A_\mu A^\mu \right.$$
$$- \frac{1}{2}\left(M_{34}^2\beta_2^{\prime 2}\left(\varphi_3^2+\varphi_4^2\right) + \tilde{\rho}_1'\beta_1^{\prime 2}M_A^2\varphi_2^2 + M_5^2\varphi_5^2\right) - g'\bar{\rho}_1'\beta'_1 M_A\varphi_2\varphi_5^2$$
$$- g'\tilde{\rho}_1'\beta_1^{\prime 2}M_A\varphi_2\left(\varphi_1^2+\varphi_2^2\right) - g'\bar{\rho}_3'\varphi_5^3$$
$$+ g'\left(\beta_1'\left(\varphi_1\partial_\mu\varphi_2 - \varphi_2\partial_\mu\varphi_1\right) + \beta_2'\left(\varphi_3\partial_\mu\varphi_4 - \varphi_4\partial_\mu\varphi_3\right)\right)A^\mu$$
$$+ g'\beta_1' M_A\varphi_2 A_\mu A^\mu - g'^2\bar{\rho}_5'\varphi_5^4 - \frac{1}{2}g'^2\bar{\rho}_1'\beta_1^{\prime 2}\varphi_5^2\left(\varphi_1^2+\varphi_2^2\right)$$
$$- \frac{1}{2}g'^2\bar{\rho}_2'\beta_2^{\prime 2}\varphi_5^2\left(\varphi_3^2+\varphi_4^2\right) - \frac{1}{4}g'^2\tilde{\rho}_1'\beta_1^{\prime 4}\left(\varphi_1^2+\varphi_2^2\right)^2 - \frac{1}{8}g'^2\tilde{\rho}_2'\beta_2^{\prime 2}\left(\varphi_3^2+\varphi_4^2\right)^2$$
$$\left. + \frac{1}{2}g'^2\left(\beta_1^{\prime 2}\left(\varphi_1^2+\varphi_2^2\right) + \beta_2^{\prime 2}\left(\varphi_3^2+\varphi_4^2\right)\right)A_\mu A^\mu \right]. \tag{36}$$

Developing a similar procedure with respect to gauge transformations (26)– (27), it results that the Lagrangian action (36) is invariant under the generating set of gauge transformations

$$\bar{\delta}_\epsilon' A^\mu = \partial^\mu\epsilon, \quad \bar{\delta}_\epsilon'\varphi_1 = \left(g'\beta_1'\varphi_2 + M_A\right)\epsilon, \quad \bar{\delta}_\epsilon'\varphi_2 = -g'\beta_1'\varphi_1\epsilon, \tag{37}$$
$$\bar{\delta}_\epsilon'\varphi_3 = g'\beta_2'\varphi_4\epsilon, \quad \bar{\delta}_\epsilon'\varphi_4 = -g'\beta_2'\varphi_3\epsilon, \quad \bar{\delta}_\epsilon'\varphi_5 = 0. \tag{38}$$

Finally, from (36) we identify the mass spectrum

$$\mu_A^2 = M_A^2, \quad \mu_{\varphi_1}^2 = 0, \quad \mu_{\varphi_2}^2 = \tilde{\rho}_1'\beta_1^{\prime 2}M_A^2, \quad \mu_{\varphi_3}^2 = \mu_{\varphi_4}^2 = M_{34}^2\beta_1^{\prime 2}, \quad \mu_{\varphi_5}^2 = M_5^2. \tag{39}$$

Regarding the nature of the field spectrum components, we conclude that all the physical fields A^μ, φ_2, φ_3, φ_4 and φ_5 are massive, while the nonphysical degree of freedom φ_1 is massless.

5 Conclusions

In this paper we reviewed our results [3, 4, 5] concerning the mass predictions for a vector field that is consistently coupled to various collections of real scalar fields. Firstly, starting from the general prescriptions [1], we constructed an interacting theory comprising a vector field and four real scalar fields. Then, by means of some appropriate redefinitions of the free parameters of the interacting theory, we were able to conclude that all the physical and nonphysical fields are massive. Secondly, by performing the same analysis, but for a vector field in interaction with a collection of five scalar fields, we proved that all the physical modes are massive, while the nonphysical one is massless.

References

[1] C. Bizdadea, S.O. Saliu, *A novel mass generation scheme for an Abelian vector field* (preprint, arxiv: 1603.02543).

[2] K.I. Kondo, Phys. Lett. B **762** (2016) 219–224

[3] C. Bizdadea, E.M. Cioroianu, S.O. Saliu, AIP Conf. Proc. **1796** (1) (2017) 020003

[4] C. Bizdadea, E.M. Cioroianu, S.O. Saliu, AIP Conf. Proc. **1916** (1) (2017) 020003

[5] C. Bizdadea, E.M. Cioroianu, S.O. Saliu, AIP Conf. Proc. **2071** (1) (2019) 020006

[6] F. Englert, R. Brout, Phys. Rev. Lett. **13** (9) (1964) 321–323

[7] P.W. Higgs, Phys. Lett. **12** (2) (1964) 132–133

[8] P.W. Higgs, Phys. Rev. Lett. **13** (16) (1964) 508–508

[9] G. S. Guralnik, C.R. Hagen, T.W.B. Kibble, Phys. Rev. Lett. **13** (20) (1964) 585–587

[10] C. Bizdadea, E.M. Cioroianu, S.O. Saliu, AIP Conf. Proc. **2218** (1) (2020) 050006

[11] G. Barnich, M. Henneaux, Phys. Lett. B **311** (1993) 123–129

[12] M. Henneaux, Contemp. Math. **219** (1998) 93–110

[13] G. Barnich, F. Brandt, M. Henneaux, Commun. Math. Phys. **174** (1995) 57–91

[14] G. Barnich, F. Brandt, M. Henneaux, Commun. Math. Phys. **174** (1995) 93–116

[15] G. Barnich, F. Brandt, M. Henneaux, Phys. Rep. **338** (2000) 439–569

[16] M. Peskin, D. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley Publishing Com- pany, Reading, 1995