Physics AUC, vol. 30 (part II), 140-149 (2020) PHYSICS AUC

On Non-supersymmetric Heterotic Pati-Salam Models

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Abstract

We study a class of 4d heterotic string compactifications with Pati–Salam gauge symmetry exhibiting spontaneous supersymmetry breaking via the Scherk–Schwarz mechanism. Through the combined use of the free-fermionic and orbifold constructions, we identify vacua with interesting phenomenological properties, such as the presence of chiral matter and the existence of Pati–Salam and Standard Model breaking Higgs bosons, while avoiding the appearance of physical tachyons and suppressing the one-loop contribution to the cosmological constant.

Talk presented by K. Violaris-Gountonis at the SEENET-MTP 2020 Workshop: "Quantum Fields and Non-Linear Phenomena", September 27 - 29 2020, University of Craiova, Romania.

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1 Introduction

String theory provides a consistent framework for the unified description of all interactions including gravity. In particular, heterotic string compactifications with $\mathcal{N}=1$ supersymmetry are known to incorporate the salient features of minimal supersymmetric extensions of the Standard Model. However, despite considerable progress in the reconstruction of the tree-level string effective action, making further contact with low energy data ultimately necessitates the incorporation of quantum corrections. Indeed, the contribution of the infinite tower of massive string excitations to loop corrections is non-negligible and indicates that supersymmetry breaking must be realized within a fully-fledged worldsheet CFT approach.

A way of breaking supersymmetry while retaining the perturbative worldsheet description is the stringy version [1–4] of the Scherk–Schwarz mechanism [5,6]. This can be thought of as a generalisation of Kaluza-Klein compactifications, in which one exploits a symmetry operator Q of the theory to introduce non-trivial monodromies to fields and vertex operators of the theory around non-trivial cycles of the compactification manifold. This essentially results in shifting the Kaluza–Klein masses of charged fields, and introducing a mass gap inversely proportional to the compactification radius R. Identifying Q with the spacetime fermion number assigns different masses to fields within the same supermultiplet and results in the spontaneous breaking of supersymmetry at scales $m_{3/2} \sim 1/R$.

In the absence of supersymmetry, questions of perturbative instabilities become especially relevant. On the one hand, the cosmological constant no longer vanishes automatically, indicating the presence of a dilaton tadpole at one loop. On the other hand, the massless spectrum is typically plagued by tachyonic instabilities in regions of the perturbative moduli space of the order of the string scale. In addition, a set of phenomenological constraints is required to ensure that the massless string spectrum be compatible with low energy observation. Recently, there has been a revived interest in non-supersymmetric string constructions aspiring to address these questions and make contact with low energy phenomenology [7–22].

In this report, we construct a class of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds with Pati–Salam gauge symmetry, exhibiting a spontaneous $\mathcal{N}=1 \to 0$ breaking of supersymmetry and satisfying a number of semi-realistic phenomenological features, including the presence of chiral matter. Section 2 provides the basis of our construction at a special point of moduli space, where all internal coordinates are consistently fermionised. In section 3, we proceed to reinterpret them as orbifold compactifications at generic points of the perturbative moduli space and investigate the shape of the one-loop effective potential as a function of the compactification moduli. Section 4 contains our conclusions.

2 Models and phenomenological criteria

The starting point of our construction is based on the $SO(10)$ models studied in [16]. These are defined in the free fermionic formulation of heterotic strings [23–25] using a set of nine basis vectors $B_9 = \{\beta_1, \beta_2, \ldots, \beta_9\}$,

$$\begin{aligned}
\beta_1 &= \mathbb{1} = \{\psi^\mu,\ \chi^{1,\ldots,6}, y^{1,\ldots,6}, \omega^{1,\ldots,6} | \overline{y}^{1,\ldots,6}, \bar{\omega}^{1,\ldots,6}, \overline{\eta}^{1,2,3}, \overline{\psi}^{1,\ldots,5}, \overline{\phi}^{1,\ldots,8}\} \\
\beta_2 &= S = \{\psi^\mu, \chi^{1,\ldots,6}\} \\
\beta_3 &= T_1 = \{y^{12}, \omega^{12} | \overline{y}^{12}, \overline{\omega}^{12}\} \\
\beta_4 &= T_2 = \{y^{34}, \omega^{34} | \overline{y}^{34}, \overline{\omega}^{34}\} \\
\beta_5 &= T_3 = \{y^{56}, \omega^{56} | \overline{y}^{56}, \overline{\omega}^{56}\} \\
\beta_6 &= b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \overline{y}^{34}, \overline{y}^{56}, \overline{\psi}^{1,\ldots,5}, \overline{\eta}^1\} \\
\beta_7 &= b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \overline{y}^{12}, \overline{y}^{56}, \overline{\psi}^{1,\ldots,5}, \overline{\eta}^2\} \\
\beta_8 &= z_1 = \{\overline{\phi}^{1,\ldots,4}\} \\
\beta_9 &= z_2 = \{\overline{\phi}^{5,\ldots,8}\}\,,
\end{aligned} \tag{1}$$

accompanied by a set of phases $c\left[\begin{smallmatrix}\beta_i \\ \beta_j\end{smallmatrix}\right] = \pm 1$, $i \leq j \in \{1, \ldots, 9\}$ associated with GGSO projections. The simplest way to break the $SO(10)$ gauge symmetry is to introduce an additional basis vector

$$\beta_{10} = \alpha = \{\overline{\psi}^{4,5}, \overline{\phi}^{1,2}\}, \tag{2}$$

and the associated GGSO phases $c\left[\begin{smallmatrix}\beta_i \\ \beta_{10}\end{smallmatrix}\right]$, $i = 1, \ldots, 9$. In this description a string model is defined in terms of $10(10-1)/2 + 1 = 46$ independent phases. Consequently, the augmented set of basis vectors $B_{10} = \{\beta_1, \ldots, \beta_{10}\}$ gives rise to $2^{46} \sim 10^{14}$ a priori distinct models. It turns out that apart from special choices of the GGSO phases the gauge symmetry of the models under consideration is

$$G = \{SU(4) \times SU(2)_L \times SU(2)_R\}_{\text{observable}} \times U(1)^3 \times SU(2)^4 \times SO(8)\,. \tag{3}$$

As expected, G comprises the Pati–Salam (PS) gauge group $SU(4) \times SU(2)_L \times SU(2)_R$ which we consider as the "observable" gauge symmetry of our models [26, 27].

In this framework, we can easily break space-time supersymmetry by projecting out the gravitino state from the massless spectrum utilising the GGSO projection choice

$$c\begin{bmatrix} S \\ T_1 \end{bmatrix} = +1\,. \tag{4}$$

Additional constraints on the remaining GGSO phases need to be imposed in order to ensure compatibility with the string Scherk-Schwarz mechanism. It turns out that this requirement reduces significantly the number of acceptable models [16].

As mentioned in Section 1, an important issue concerns the possible presence of tachyon instabilities. Clearly, space-time supersymmetry automatically guarantees the absence of tachyons in the string spectrum. Different is the case of non-supersymmetric models, such as the ones considered here, where tachyons must be projected out by a proper choice of GGSO phases. A preliminary search shows that this constraint alone eliminates about 50% of the available configurations in the class of models under consideration.

We next employ a number of criteria associated with low energy phenomenology. In the Pati–Salam construction, Standard Model fermion generations are accommodated in $SU(4) \times SU(2)_L \times SU(2)_R$ representations as follows:

$$
\begin{aligned}
&F_L(\mathbf{4},\mathbf{2},\mathbf{1}) = Q(\mathbf{3},\mathbf{2},-1/6) + L(\mathbf{1},\mathbf{2},1/2)\,,\\
&\overline{F}_R(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) = u^c(\bar{\mathbf{3}},\mathbf{1},2/3) + d^c(\bar{\mathbf{3}},\mathbf{1},-1/3) + e^c(\mathbf{1},\mathbf{1},-1) + \nu^c(\mathbf{1},\mathbf{1},0)\,.
\end{aligned}
$$

In the string realisation these arise from the sectors[§] $\mathcal{S}^i_{pq} = S + b_i + pT_j + qT_k$, where $p,q = 0,1$, $(i,j,k) = \{(1,2,3),(2,1,3),(3,1,2)\}$ along with anti-generations $\overline{F}_L(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})$, $F_R(\mathbf{4},\mathbf{1},\mathbf{2})$. The net number of generations reads

$$
n_g = n_L - \overline{n}_L = n_R - \overline{n}_R\,, \tag{5}
$$

where $n_L, \overline{n}_L, n_R, \overline{n}_R$ denote the numbers of $F_L, \overline{F}_L, F_R, \overline{F}_R$ respectively. These can be expressed in terms of GGSO phases utilising the methods developed in [28–31]

$$
\begin{aligned}
n_L &= \sum_{i,p,q} P^i_{pq}\left(1 + X^{(i)SU(4)}_{pq}\right)\left(1 - c\left[{\mathcal{S}^i_{pq} \atop \alpha}\right]^*\right)\,,\\
\overline{n}_R &= \sum_{i,p,q} P^i_{pq}\left(1 - X^{(i)SU(4)}_{pq}\right)\left(1 + c\left[{\mathcal{S}^i_{pq} \atop \alpha}\right]^*\right)\,,\\
\overline{n}_L &= \sum_{i,p,q} P^i_{pq}\left(1 - X^{(i)SU(4)}_{pq}\right)\left(1 - c\left[{\mathcal{S}^i_{pq} \atop \alpha}\right]^*\right)\,,\\
n_R &= \sum_{i,p,q} P^i_{pq}\left(1 + X^{(i)SU(4)}_{pq}\right)\left(1 + c\left[{\mathcal{S}^i_{pq} \atop \alpha}\right]^*\right)\,,
\end{aligned} \tag{6}
$$

where P^i_{pq} is a projection operator determining whether states in the $\mathcal{S}^i_{pq}$ sector survive the GGSO projection

$$
P^i_{pq} = \frac{1}{2^3}\left(1 - c\left[{\mathcal{S}^i_{pq} \atop T_i}\right]^*\right)\left(1 - c\left[{\mathcal{S}^i_{pq} \atop z_1}\right]^*\right)\left(1 - c\left[{\mathcal{S}^i_{pq} \atop z_2}\right]^*\right)\,, \tag{7}
$$

and $X^{(i)SU(4)}_{pq} = -c\left[{\mathcal{S}^i_{pq} \atop R^i_q}\right]^*$, with $R^i_q = S + b_j + (1-q)T_3 + \alpha\,, j \neq i = 1,2$ and $R^3_q = S + b_1 + (1-q)T_2 + \alpha$. The existence of chiral generations can then be imposed as an additional constraint on the free phases defining a PS string model.

The breaking of the Pati–Salam symmetry to that of the Standard Model requires the presence of massless scalars in the $H\,(\mathbf{4},\mathbf{1},\mathbf{2})$ + h.c. representations. These come from the sectors $\mathcal{B}^i_{pq} = b_i + pT_j + qT_k$ with $p,q = 0,1$, $(i,j,k) = \{(1,2,3),(2,1,3),(3,1,2)\}$. Similarly, the required Standard Model breaking scalars accommodated in $h\,(\mathbf{1},\mathbf{2},\mathbf{2})$ representation arise from $\mathcal{V}^i_{pq} = b_i + x + pT_j + qT_k$. Again, the number of massless states of the type H, h are expressible in terms of GGSO phases and additional constraints associated with the presence of these bosonic states in the massless spectrum are imposed.

Altogether, the aforementioned criteria define a set of consistent non-supersymmetric PS models that satisfy some basic phenomenological requirements, such as chirality as well as the presence of PS breaking scalars and the existence of Higgs doublets responsible for recovering the Standard Model at low energies. Moreover, these models are compatible with the spontaneous breaking of supersymmetry via the Scherk–Schwarz mechanism. The existence of string models satisfying all the above criteria is a non-trivial question. To this end, we have performed a randomised computer search over the parameter space of models and find that approximately one in 1500 models meets all the above criteria.

[§]Here we define $b_3 = b_1 + b_2 + x$ with $x = \mathbb{1} + S + T_1 + T_2 + T_3 + z_1 + z_2$.

3 Moduli dependence and the Cosmological Constant

The models defined in the free-fermionic construction of the previous section are restricted to live on a special locus of the moduli space in which all internal coordinates are consistently fermionised. Albeit very useful for the study of the mass spectra and the characterisation of a range of phenomenological properties of the models, the fermionic description misses the dependence on the compactification moduli. What is more, whether the supersymmetry is broken spontaneously or explicitly cannot be decided by an analysis restricted to the fermionic point alone.

These questions may be addressed by deforming the theory away from the fermionic point, by means of marginal deformations of the current-current type. At the generic point, the theories we construct admit a reinterpretation by means of the orbifold formulation. The moduli dependence is then restored and it becomes possible to study the structure of one-loop effective potential and the dynamic fate of the no-scale moduli.

Following the orbifold description and notation of [16], the partition function of the vacua introduced in Section 2 is given by:

$$
\begin{aligned}
Z = \frac{1}{\eta^{12}\bar{\eta}^{24}} \frac{1}{2^4} \sum_{\substack{h_1,h_2,H,H' \\ g_1,g_2,G,G'}} \frac{1}{2^3} \sum_{\substack{a,k,\rho \\ b,\ell,\sigma}} \frac{1}{2^3} \sum_{\substack{H_1,H_2,H_3 \\ G_1,G_2,G_3}} (-1)^{a+b+HG+H'G'+\Phi} \\
\times \vartheta[{}^a_b]\, \vartheta[{}^{a+h_1}_{b+g_1}]\, \vartheta[{}^{a+h_2}_{b+g_2}]\, \vartheta[{}^{a-h_1-h_2}_{b-g_1-g_2}]\, \bar{\vartheta}[{}^k_\ell]^3\, \bar{\vartheta}[{}^{k+H'}_{\ell+G'}]\, \bar{\vartheta}[{}^{k-H'}_{\ell-G'}] \\
\times \bar{\vartheta}[{}^{k+h_1}_{\ell+g_1}]\, \bar{\vartheta}[{}^{k+h_2}_{\ell+g_2}]\, \bar{\vartheta}[{}^{k-h_1-h_2}_{\ell-g_1-g_2}]\, \bar{\vartheta}[{}^{\rho+H'}_{\sigma+G'}]\, \bar{\vartheta}[{}^{\rho-H'}_{\sigma-G'}]\, \bar{\vartheta}[{}^\rho_\sigma]^2\, \bar{\vartheta}[{}^{\rho+H}_{\sigma+G}]^4 \\
\times \Gamma^{(1)}_{2,2}[{}^{H_1}_{G_1}|{}^{h_1}_{g_1}](T^{(1)},U^{(1)})\; \Gamma^{(2)}_{2,2}[{}^{H_2}_{G_2}|{}^{h_2}_{g_2}](T^{(2)},U^{(2)})\; \Gamma^{(3)}_{2,2}[{}^{H_3}_{G_3}|{}^{h_1+h_2}_{g_1+g_2}](T^{(3)},U^{(3)}).
\end{aligned} \tag{8}
$$

The spin structure $a = 0,1$ distinguishes between spacetime bosonic and fermionic states respectively, whereas summation over $b = 0,1$ imposes the standard GSO projection. The boundary conditions of the right-moving fermions realizing the Kac–Moody algebra at level $k = 1$ and generating the gauge symmetry, are labeled by $k, \rho, H, H' = 0, 1$, along with the corresponding projections $\ell, \sigma, G, G' = 0, 1$. Similarly, twisted sectors of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold generating $\mathcal{N} = 1$ supersymmetry are labeled by $h_i, g_i = 0, 1$. The three additional freely-acting $\mathbb{Z}_2$ orbifold twists, associated with the Scherk–Schwarz breaking of $\mathcal{N} = 1 \to 0$, are parametrized by $H_i, G_i = 0, 1$. The partition function of the three compactified 2-tori arises as the product of three twisted/shifted (2,2) lattices $\Gamma^{(i)}_{2,2}[{}^{H_i}_{G_i}|{}^{h_i}_{g_i}](T^{(i)}, U^{(i)})$, defined as in [16]. With these conventions, it is straightforward to identify the fermionic point as $T = i$ and $U = (1 + i)/2$. Finally, the modular invariant phase $(-1)^\Phi$ can be matched to the GGSO projections of the fermionic construction, defined through $c[{}^{\beta_i}_{\beta_j}]$, while additionally implementing the Scherk-Schwarz mechanism by coupling the R-symmetry charges to the Scherk-Schwarz lattice shifts.

The one-loop effective potential is obtained by integrating the string partition function over the moduli space $\mathcal{F}$ of the worldsheet torus Σ_1:

$$
V_{\text{one-loop}}(t_I) = -\frac{1}{2(2\pi)^4} \int_{\mathcal{F}} \frac{d\tau^2}{\operatorname{Im}\tau^3} Z(\tau, \bar{\tau}; t_I), \tag{9}
$$

where τ is the complex structure on Σ_1, $\mathcal{F} = SL(2;\mathbb{Z})\backslash\mathbb{H}^+$ is a fundamental domain under the modular group, and t_I are the compactification moduli.

In the simplest case where supersymmetry is broken à-la Scherk-Schwarz coupled to a circle of radius $R \gg 1$, the potential reduces to the generic asymptotic form [32]:

$$
V_{\text{one-loop}}(t_I) \sim -\frac{n_B - n_F}{R^4} + \dots \tag{10}
$$

where n_F and n_B correspond, respectively, to the degeneracies of massless fermions and bosons, and the ellipses represent exponentially suppressed terms. Assuming that non-perturbative effects eventually stabilize the Scherk-Schwarz radius at sufficiently low Kaluza-Klein scales (c.f. [21] for a recent discussion), the polynomial suppression in (10) by far overshoots the observed value of the cosmological constant. This discrepancy may be resolved by imposing the additional condition $n_B = n_F$ at the massless level, leading to the exponential suppression of the effective potential at large volume, and hence of the one-loop backreaction against the classical background. Models satisfying this constraint have been recently termed "super no-scale" models [32–37].

Calculating the one-loop partition function of these models becomes a tedious, but tractable task. For simplicity, we identify the Scherk-Schwarz breaking with the first 2-torus, while fixing all moduli to their fermionic point values, except the volume T_2 controlling the supersymmetry breaking. A preliminary investigation reveals that large numbers of models share the same partition function, allowing us to reduce the search to a few cases displaying the salient features. As expected, most models exhibit a negative potential with a minimum at the fermionic point, leading to a negative cosmological constant and supersymmetry breaking close to the string scale. Remarkably, it is possible to identify also cases in which the potential is semi-positive definite, including models exhibiting a local minimum at the fermionic point.

We now focus on a particular model obtained using computer assisted search over a random sample of 10^8 GGSO configurations, utilising the criteria mentioned in Section 2. This is defined by the GGSO matrix:

$$c\begin{bmatrix} \beta_i \\ \beta_j \end{bmatrix} = \begin{pmatrix} +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 \\ +1 & +1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 \\ -1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 \\ -1 & +1 & -1 & +1 & +1 & +1 & +1 & +1 & -1 & +1 \\ -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 \\ +1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 \\ +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 \\ +1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 \\ +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 \\ -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 \end{pmatrix} \tag{11}$$

The massless spectrum includes four chiral generations arising from the sectors $S + b_1 + T_2 + T_3$, $S + b_3 + T_1$, PS breaking scalars coming from the sector b_3 and Standard Model scalar doublets in the sector $b_1 + T_2 + x$. The partition function at the fermionic point is given by:

$$\begin{aligned} Z = & \frac{2q_i}{q_r} - \frac{16q_i}{\sqrt{q_r}} + \left(40 + \frac{48}{q_i} + 144q_i + 56q_i^2\right) + \left(1248 + \frac{6528}{q_i} - 1152q_i - 416q_i^2\right)\sqrt{q_r} \\ & + \left(8192 + \frac{18816}{q_i^2} + \frac{105600}{q_i} + 4960q_i + 2688q_i^2 + 792q_i^3\right) q_r + \dots, \end{aligned} \tag{12}$$

where $q_r = e^{-2\pi\tau_2}$, $q_i = e^{2\pi i\tau_1}$.

The partition function at the generic point can be deduced from Eq. (8) using

$$\begin{aligned}\Phi =& ab + aG_1 + bH_1 + H_1G_1 + k(G_1 + G_3) + \ell(H_1 + H_3)\\ &+ \rho(g_2 + G' + G_1 + G_2) + \sigma(h_2 + H' + H_1 + H_2)\\ &+ H(g_1 + g_2 + G_1 + G_2) + G(h_1 + h_2 + H_1 + H_2)\\ &+ H'G_3 + G'H_3 + H'G' + h_1G_1 + g_1H_1\\ &+ h_2(G_1 + G_3) + g_2(H_1 + H_3) + h_2g_2 + H_1G_2 + H_2G_1\,.\end{aligned} \tag{13}$$

We first notice that the constant term in the partition function expansion, that corresponds to the difference between the bosonic n_B and fermionic n_F degrees of freedom, vanishes at the generic point. This is remarkable because it leads to the conclusion that the model exhibits super no-scale behaviour at generic points and especially for large values of the T_2 modulus, although $n_B - n_F = 40$ at the fermionic point. Next, we calculate the one loop potential utilising (9). The results are presented in Figure 1 where we plot the one-loop effective potential as a function of T_2.

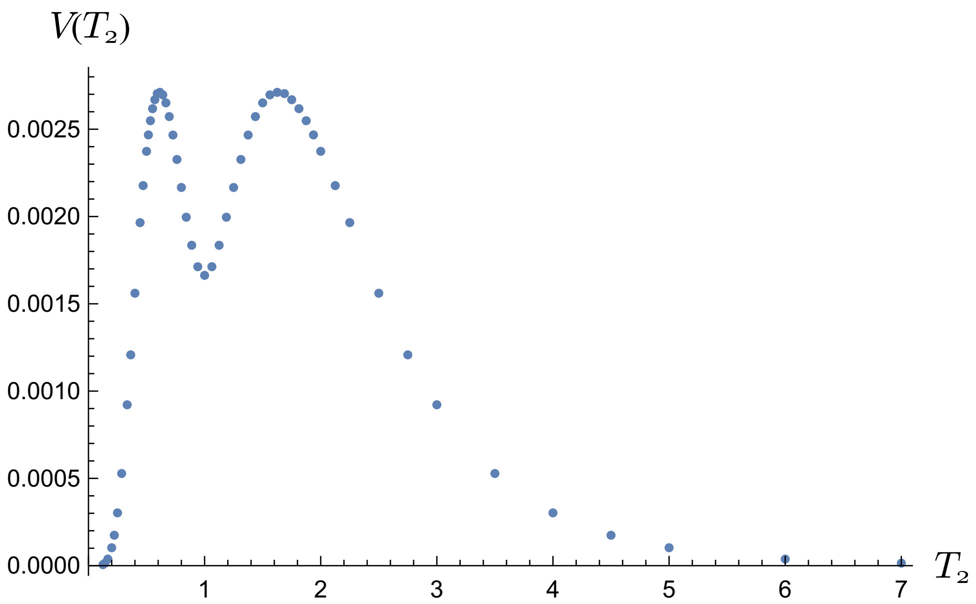

Figure 1: The one-loop effective potential as a function of T_2 for the model defined in Eq. (11) which satisfies all phenomenological constraints.

It is interesting to note that the massless level-matched term of the partition function, $n_B - n_F = 40$, would naively lead to the conclusion that the potential is negative, exhibiting a global minimum at the fermionic point. Careful calculation can, however, show that this is not true [16]. While a minimum does exist at the fermionic point, it turns out to be only a local minimum. The potential is, in fact, positive semi-definite, vanishing asymptotically as $T_2 \to \infty$ (and in the T-dual limit $T_2 \to 0$), where supersymmetry is effectively restored.

4 Conclusions

We have presented a class of chiral heterotic vacua with Pati–Salam gauge symmetry, exhibiting supersymmetry breaking realized via the Scherk–Schwarz mechanism. Utilising the free-fermionic in conjunction with the orbifold formulation, we are able to identify tachyon-free models with certain appealing phenomenological characteristics. These include the presence of chiral matter, the existence of PS symmetry breaking Higgs scalars, as well as the Standard Model Higgs doublets. We further secure an exponential suppression of the cosmological constant at large volume, by imposing the super no-scale condition at the massless level.

Among the vacua satisfying all the imposed phenomenological criteria, we are able to identify a class of models with a positive semi-definite one-loop effective potential, dynamically driving the theory to the large volume regime. These preliminary results necessitate further analysis of the Pati-Salam vacua and the classification of their massless spectra and effective potentials, which is already underway [38].

Acknowledgments

The research of K.V. is co-financed by Greece and the European Union (European Social Fund - ESF) through the Operational Programme "Human Resources Development, Education and Lifelong Learning" in the context of the project "Strengthening Human Resources Research Potential via Doctorate Research" (MIS-5000432), implemented by the State Scholarships Foundation (IKY).

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