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Analysis of Methods for Estimating the Uncertainty of the Relative Error of Electromagnetic Flowmeters Reservoirs

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Abstract

To create conditions for the recognition of Kazakhstan certificates of conformity and the results of product tests, an assessment of measurement uncertainty is required. In this regard, there has been observed the activation of the practical application in Kazakhstan of the concept of measurement uncertainty. The authors developed a physical stand for a mobile complex designed to verify electromagnetic flowmeters at the place of operation. To obtain verification results, programs were developed to calculate the measurement uncertainty of an electromagnetic flowmeter using the NI LabView graphical programming environment. In addition, a model for estimating the uncertainty of the relative error of flowmeters was proposed, and the measurement uncertainty was estimated using three methods: standard, Monte Carlo, and Kragten. Finally, a comparative analysis was conducted on the results of the estimation of the uncertainty of the relative error of a flowmeter. All methods give standard uncertainty values that do not exceed the acceptable range of relative error (± 1%). However, Monte Carlo method gives better results for sufficiently large number of simulations. No significant differences between the results obtained using standard and Kragten method were discovered.

Keywords: measurement uncertainty; standard GUM method; Monte Carlo method; Kragten method; verification; electromagnetic flowmeter.

Introduction

In the Republic of Kazakhstan, uranium mining is carried out by underground leaching – one of the most cost-effective and environmentally friendly mining methods. Hereby, a leach solution, is pumped through injection wells, and a uranium-containing productive solution is pumped through the pumping well. The subsoil is barely affected, and it can even be completely restored within a few years [1]. At the nodes receiving and distributing the leach solutions, a large number of industrial electromagnetic flowmeters (EFM) are used to measure the quantities involved; these must be metrologically verified at the end of the calibration interval.

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The generally accepted method for calibrating flowmeters using exemplary measuring instruments or calibration facilities accredited to ISO / IEC 17025-2017 can be costly and infeasible, mainly due to the staff and logistics costs associated with removing the flowmeters from the piping system. However, modern flowmeters are equipped with hardware and software that allow on-site verification that meets ISO 9001 requirements. Studies have already been conducted on the calibration of flow meters in the field of water supply and wastewater discharge [2].

The concept of measurement uncertainty has become the only – and most importantly, internationally recognized – measure of confidence in terms of measurement results over the past decade. Measurement uncertainty is a parameter associated with a measurement result that characterizes the scattering of the measured value. The standard method for estimating uncertainty is described in [3], including a comparison with the Monte Carlo method.

The issues of the estimation of measurement uncertainty have been widely covered in terms of analytical measurements [4], the calibration of measuring instruments [5-7] and other studies [8-10]. International organizations have developed and prepared basic documentation for the international harmonization of approaches to solving metrological problems. These include the ISO / IEC 17025: 2017 standard; the JCGM 100: 2008 Joint Metrology Guidelines document, as the latest revision of GUM: 1995, which provides guidance on measuring uncertainty in measurement; the International Dictionary of Metrology JCGM 200: 2012, which presents the terms and concepts used in the field of metrology; and JCGM 101: 2008 (Supplement 1 to GUM: 1995), which provides practical guidance on using Monte Carlo simulations to estimate uncertainty.

The following considers the main methods for assessing measurement uncertainty.

Standard Method

The uncertainty estimate presented in JCGM 100: 2008 is based on the law of the propagation of uncertainties. This technique, described in GUM (The Guide to the Expression of Uncertainty in Measurement, hereinafter referred to as the GUM method), has been successfully applied worldwide for different measuring systems and is currently the standard procedure for estimating uncertainty in metrology. In the 2013 edition [11-13], the terms of the concept of uncertainty [8] are aligned with the current International Dictionary of Metrology and with new additions to the GUM. It considers aspects such as a new Bayesian approach, the redefinition of coverage intervals, and the exclusion of the Welch-Satterth Waite formula for estimating effective degrees of freedom [11].

The main stages of uncertainty estimation include the formulation of the measurement problem and calculation. In formulating the measurement problem, the following is performed: determining the output (measured) value; identifying the input quantities on which the output quantity depends; and developing a measurement model.

The GUM estimation of measurement uncertainty in analytical measurements has been widely reported in the literature [10, 14, 15]. Examples of GUM estimation of the uncertainty of temperature, AC voltage, and pressure measurements are given in the work by [16]. Meanwhile, [7] describes uncertainty assessment during the metrological certification of means of measuring the moment of inertia of electric motors.

The GUM method has several limitations. It may not be usable if the uncertainty of one or more input sources is significantly greater than the rest or when the distribution of input quantities is asymmetric. To calculate the extended uncertainty in the GUM, the Welch-Satterth

Waite formula is used to calculate the effective number of degrees of freedom, which is needed to determine the quantile of the Student's *t*-distribution. Due to the complexity of the calculations using this formula, the analytical estimate of the effective number of degrees of freedom remains an unresolved problem [9].

One approach to overcoming these limitations is to use a convolution of the probability distributions of input quantities, for example, using the Monte Carlo simulation method [17].

Monte Carlo Method

The essence of the Monte Carlo method is as follows: each time the measurement function is calculated, it generates randomly generated input values that vary around its nominal value within the uncertainty interval in accordance with the distribution law.

Modeling does not require significant computational costs. Typically, a simulation for a model with 1,000,000 iterations, which generates reasonable results for a coverage probability of 95%, can be performed on a computer in a few minutes, depending on the software and hardware used.

In the work by [17], examples of the application of the Monte Carlo simulation method for estimating the measurement uncertainty of various practical problems are given: evaluating the real efficiency of a fuel cell, measuring torque, preparing a standard cadmium solution, and measuring the Brinell hardness.

Kragten Method

The Kragten method (spreadsheet method) is recommended for complex expressions to simplify calculations. This procedure uses an approximate numerical method of differentiation and requires only knowledge of the calculations used to obtain the final result (including any necessary correction factors), the numerical values of the parameters, and their uncertainties [18]. It assumes either that the measurement model is linear in the input variables or that the uncertainty of the corresponding input quantity is small compared to its value. These assumptions are not always fully observed. However, the method provides acceptable accuracy for practical purposes when it is considered with the necessary approximations made in estimating the uncertainties. The advantage of the Kragten method is that the correlation between the variables can be easily included by adding suitable additional members to the spreadsheet.

The analysis showed that in most cases, the GUM, Kragten and Monte Carlo methods give almost the same value for the standard uncertainty associated with the estimation of the measured value. The differences become apparent when the distributions are far from normal and the measurement result nonlinearly depends on one or more input quantities. Where there is significant non-linearity, the standard GUM method is not recommended. However, nonlinearity can be taken into account in the GUM by including higher order terms in the calculations [19].

Where the distributions differ significantly from normal, the Kragten and standard GUM methods give a distorted estimate of the standard uncertainty, while the Monte Carlo method allows a determination of the distribution law of the output quantity and, accordingly, displays the real "coverage interval" [21-22].

Based on the review, the above methods have not yet been applied to the estimation of the uncertainty of flow measurement. This problem is the subject of research in this article.

The aim of the work is to study methods for estimating uncertainty during the calibration of flowmeters on-site without removing them from their place of operation.

Methods

Standard Method (GUM)

The main stages of uncertainty assessment in accordance with the GUM: 1995 guidelines have been described above.

There are two types of standard uncertainty calculation:

- calculation by type A: the standard deviation (standard) of the distribution law of the measurement result with a classical frequency interpretation;
- calculation by type B: using other methods.

The uncertainty of the relative EFM error is estimated based on the standard of the Republic of Kazakhstan ST RK 2.328-2015 "Electromagnetic flowmeters. Verification Technique." This standard proposes the following measurement model:

$$\delta_Q = \frac{Q_r - Q_p}{Q_p} 100 \tag{1}$$

where Q_r is the value of the flow rate according to the metering values of EFM, and Q_p is the flow rate according to the indications of the reference Coriolis flowmeter (CFM).

However, this standard regulates the estimation of the uncertainty of the relative error of electromagnetic flowmeters using only type B.

The authors substantiate and propose calculating the uncertainty using not only type B but also type A [22]. The calculation of type A includes statistically processing the results of multiple measurements, namely the calculation of the mathematical expectation, variance, and standard deviation.

An estimate of the flow rate Q is the arithmetic mean of n = 11 observations Q_i (i = 1,2, ..., n) for each point being verified (j = 4):

$$\bar{Q} = \frac{1}{n} \sum_{i=1}^{n} Q_i. \tag{2}$$

Let us estimate the uncertainty of the input quantities Q_r and Q_p.

The arithmetic mean value of the mass flow rate is calculated by the formula:

- for a reference flowmeter:

$$\bar{Q}_p = \frac{\sum_{i=1}^{n} Q_{ip}}{n}; \tag{3}$$

- for a verified EFM:

$$\bar{Q}_r = \frac{\sum_{i=1}^{n} Q_{ir}}{n}. \tag{4}$$

For each point being verified, the arithmetic mean of the relative error of the EFM is determined according to formula (1), taking into account the results of formulas (3) and (4):

$$\bar{\delta} = \frac{\overline{Q_r} - \overline{Q_p}}{\overline{Q_p}} 100\%.$$

The standard measurement uncertainties of the electromagnetic and reference flowmeters using type A are calculated by the formulas:

$$u_{Aj}(Q_r) = \sqrt{\frac{\sum_{i=1}^{n}(Q_{ri} - \overline{Q_r})^2}{n(n-1)}}; \quad u_{Aj}(Q_p) = \sqrt{\frac{\sum_{i=1}^{n}(Q_{pi} - \overline{Q_p})^2}{n(n-1)}} \tag{5}$$

where Q_{ri}, Q_{pi} are the *i*-th readings at the *j* point being verified.

The standard uncertainty of the relative error of type A at each verified point (j = 1,2,3,4) is calculated by the formula:

$$u_{A_j}(\delta) = \sqrt{C_r^2 u_{A_j}(Q_r)^2 + C_p^2 u_{A_j}(Q_p)^2}$$

where C_r *and* C_p are sensitivity coefficients, which are defined as partial derivatives of equation (1) with respect to the corresponding variables and have the form:

$$C_r = \frac{100\%}{\bar{Q}_p}; \tag{6}$$

$$C_p = \frac{\bar{Q}_r * 100\%}{\bar{Q}_p^2}. \tag{7}$$

The final value of the standard uncertainty of the relative error of EFM type A is:

$$u_A(\delta) = \frac{\sum_{j=1}^{4} u_{Aj}(\delta)}{4}. \tag{8}$$

The type B uncertainty calculation includes:

1) The uncertainty of the readings of the electromagnetic flowmeter Q_r due to the discreteness of the readings of the flowmeter d_r under the assumption of a rectangular probability distribution:

$$u_{B1}(Q_r) = \frac{d_r}{2\sqrt{3}};$$

2) The uncertainty of the readings of the reference flowmeter is indicated in its document; if the document only indicates the error Δp of the reference flowmeter, assume its rectangular probability distribution:

$$u_{B2}(Q_p) = \frac{\Delta p}{\sqrt{3}}$$

where Δp is the accuracy of the reference flowmeter;

3) The uncertainty of the readings of the reference flowmeter based on the discreteness of its readings, assuming a rectangular probability distribution, is:

$$u_{B3}(Q_p) = \frac{d_p}{2\sqrt{3}}.$$

For the total uncertainty by type B of the relative error is calculated using the formula:

$$u_{B\Sigma}(\delta) = \sqrt{C_r^2 u_{B1}^2(Q_r) + C_p^2(u_{B2}^2(Q_P) + u_{B3}^2(Q_P))} \quad (9)$$

where C_{Qr} is the sensitivity coefficient of an electromagnetic flowmeter, which is calculated by formulas (6) and (7).

To calculate the total standard uncertainty of the relative EFM error, taking into account formulas (8) and (9), we have:

$$u_C(\delta) = \sqrt{u_A^2(\delta) + u_{B\Sigma}(\delta)}. \quad (10)$$

The calculation of the expanded uncertainty of the relative error of the EFM is performed according to the formula:

$$U(\delta) = k \cdot u_C(\delta). \quad (11)$$

The measurement result can be written as:

$$\delta \pm U(\delta)\ \%;\ p=0.95.$$

Monte Carlo Method

To apply the Monte Carlo method, it is necessary to choose the quantity m of the model estimation that needs to be done and the confidence level p. It is best to choose a sufficiently large value in comparison with 1 / (1-p), m (for example, exceeding it by 10^6 times).

The simulation of the process of estimating the uncertainty of the relative error of the electromagnetic flowmeter is performed as follows:

a) Two arrays of random numbers are generated, obeying uniform distribution laws, with a volume of m = 10^6 for input quantities: Q_r is the result of measuring the flow rate of magnetic resonance; Q_p is the result of measuring the flow with a reference flowmeter;

b) An array of an estimate of the output value is generated– the relative error of the EFM δ;

c) Estimates of the following parameters of the resulting distribution are calculated:

- expected value:

$$M(\delta) = \frac{\sum_{1}^{11} \delta_i}{11};$$

- total standard uncertainty:

$$u_c(\delta) = \sqrt{\frac{\sum_{1}^{11}(\delta_i - M(\delta))^2}{10}};$$

- expanded uncertainty:

$$U(\delta) = \frac{1}{2}[\delta_{975000} - \delta_{25000}];$$

- coverage ratio:

$$k = U(\delta)/u_c(\delta);$$

d) The obtained measurement result is written as: $\delta \pm U(\delta)$ %; P=0.95.

In general, uncertainty can be represented as total standard uncertainty u (y) or expanded uncertainty U (y) = ku (y), (k is the coverage factor). In some cases, it is convenient to represent uncertainty in relative values as a coefficient of variation or expanded uncertainty as a percentage of the recorded values of the results.

Kragten Method

The spreadsheet method is recommended for complex expressions in order to simplify calculations.

In the expression for the uncertainty of the relative error of the EFM:

$$u\left(\delta(Q_r, Q_p)\right) = \sqrt{\sum_{i=r,p}\left(\frac{\partial\delta}{\partial Q_i}u(Q_i)\right)^2}$$

the partial differentials ($\partial\delta/\partial Q_r$, ($\partial\delta/\partial Q_p$) are approximated by finite differences.
Multiplication by $u(Q_i)$ to obtain the uncertainty $u(\delta,Q_i)$ in δ due to the uncertainty in Q_i gives:

$$u(\delta,Q_r) \approx \delta\,(Q_r + u(Q_r), Q_p) - \delta\,(Q_r, Q_p)\ ;$$
$$u(\delta,Q_p) \approx \delta\,(Q_r, Q_p + u(Q_p)) - \delta\,(Q_r, Q_p)\ .$$

This method provides acceptable accuracy for practical purposes when it is considered taking into account the necessary approximations made when evaluating the value $su(Q_i)$. In (Kragten, 1994), this question is discussed more fully.

The total standard uncertainty of the EFM relative error is calculated by the formula:

$$u(\delta) = \sqrt{u^2(\delta, Q_r) + u^2(\delta, Q_p)}\ .$$

Expanded uncertainty of the relative EFM error:

$$U(\delta) = k \cdot u(\delta).$$

The measurement result is written as:

$$\delta \pm U(\delta)\ \%;\ P=0.95.$$

Results

The EFM calibration experiments were carried out on a physical model of the Geotechnological information and metrological complex (GIMC), developed by the authors, in the laboratory of the Department of Automation and Control of the Almaty University of Power Engineering and Telecommunications named after G. Daukeev.

The workstation interface for the verification developed in the graphical programming environment LabView (NI, USA) is shown in Figure 1.

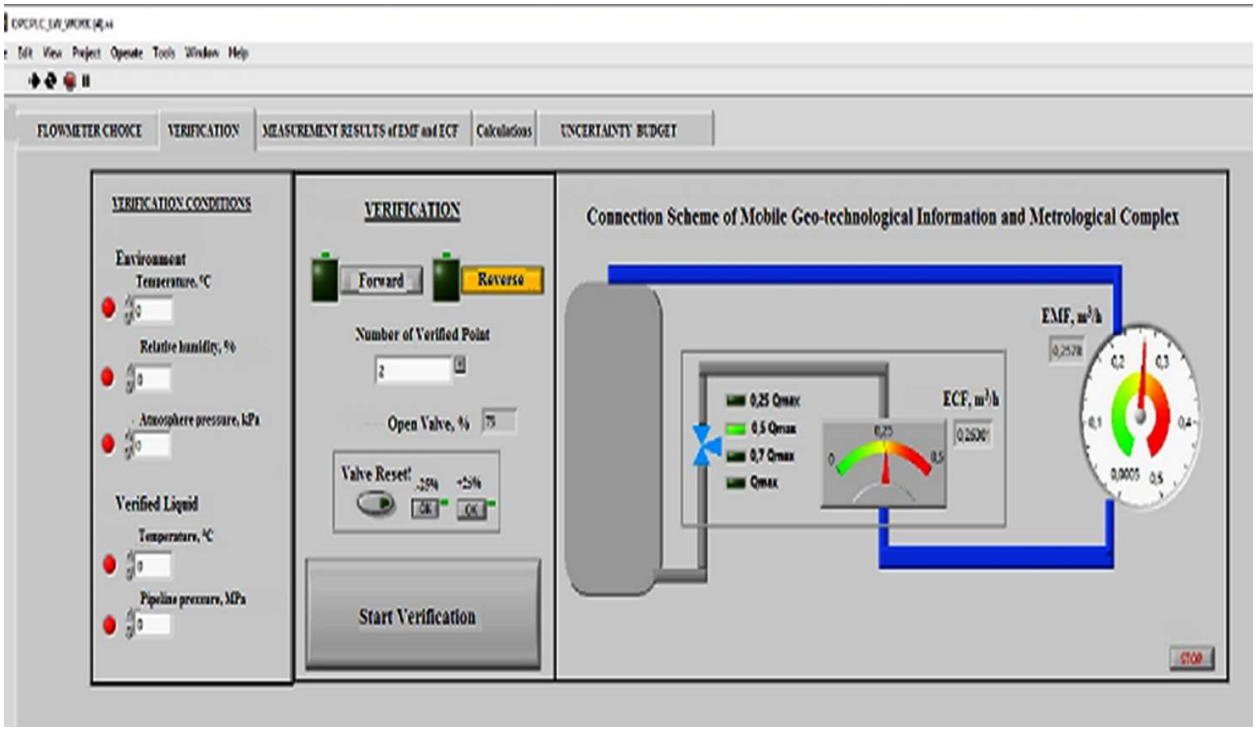

Figure 1. The interface of the metrologist's workstation for verification

The operator manually enters the environmental conditions and the parameters of the fluid being checked (leach solution). Next, they set the number of points being verified and the "direct" or "return" calibration course according to the measuring range of the calibrated EFM.

The verification process includes measurements at four points that are to be verified. At each point on the "forward" and "return" paths of the adjustable valve, eleven flow values are measured using the calibrated EFM and the reference. The experimental results are entered into the database of flowmeter readings, which are then used in the program to calculate the uncertainty of the relative error of the calibrated flowmeter, also developed in Software LabView.

Data from the database of flowmeter readings are used to calculate the uncertainty of the relative error of the calibrated flowmeter using three methods: GUM, Monte Carlo, and Kragten.

The GUM results of estimating the uncertainty of the relative error of the calibrated flowmeter for the 4th calibrated point (Q = 5700 dm^3 / h = 95% Q_{max}) are presented in the form of an "Uncertainty Budget" (Figure 2).

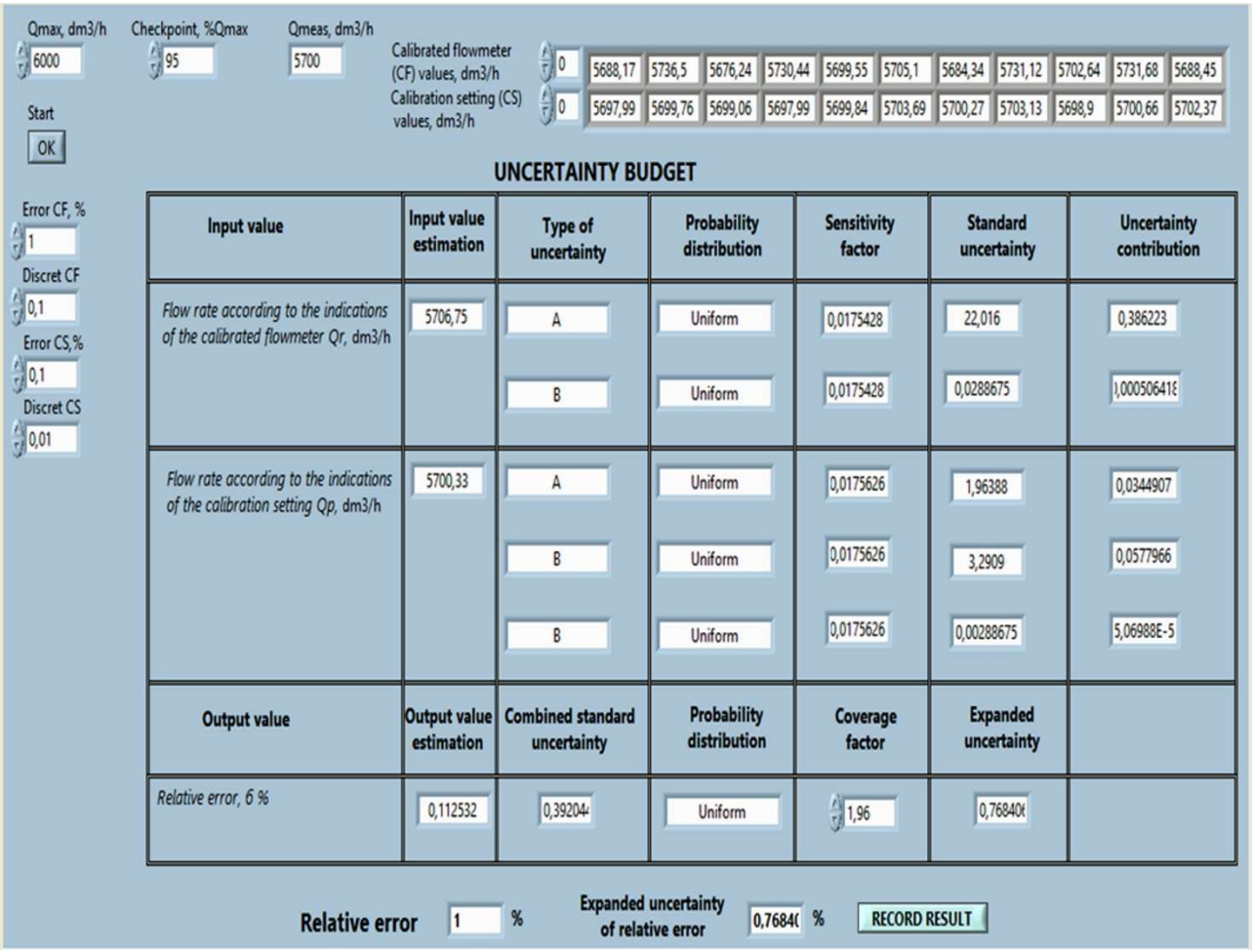

Figure 2. Calculation of the "Uncertainty Budget" using the GUM method (4th verified point)

The Monte Carlo results of estimating the uncertainty of the relative error of the calibrated flowmeter for the 4th calibrated point (Dimension = 1,000,000) are shown in Figure 3.

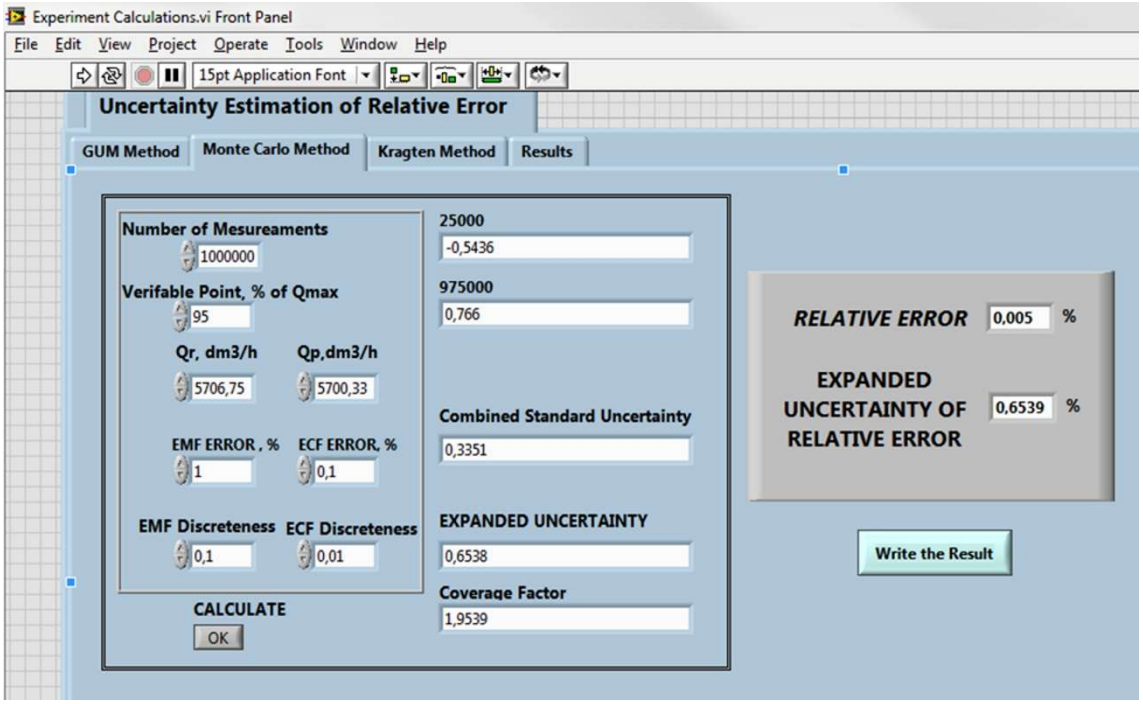

Figure 3. Calculation of the uncertainty of the relative error of the EFM Monte Carlo method (4th verified point)

The calculation results for each verified point obtained by the GUM and Monte Carlo methods are shown in Figure 4.

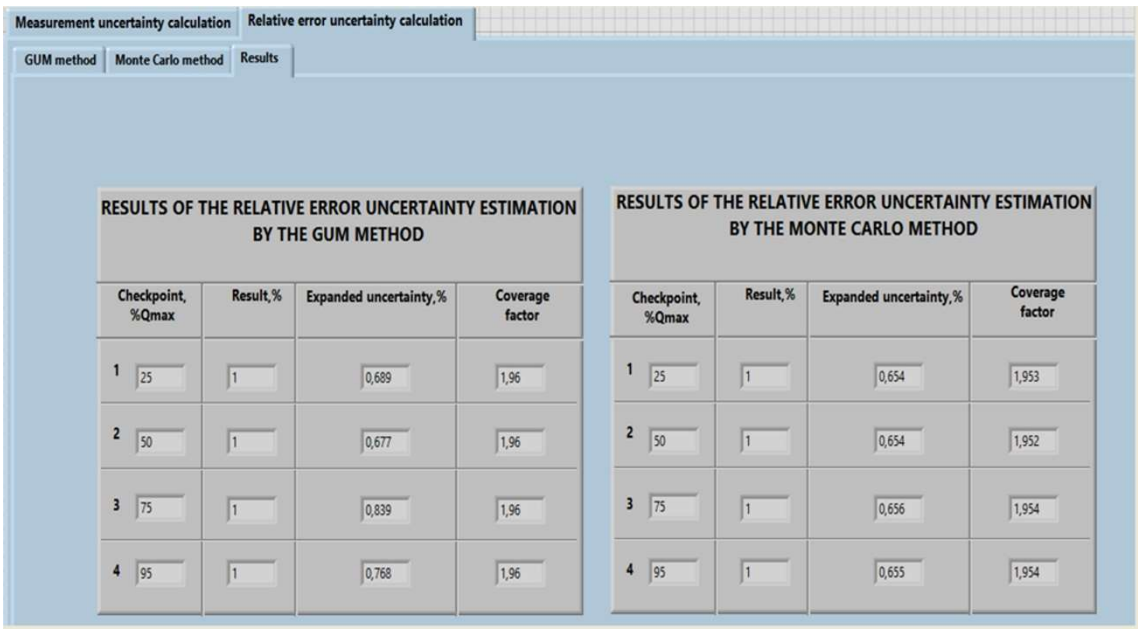

Figure 4. The results for the uncertainty of the relative error of EFM according to the GUM and Monte Carlo methods

The results of applying the Kragten method (matrix) when assessing the uncertainty of the relative error of the calibrated flowmeter for one calibrated point (Q = 5700 dm^3 / h) are shown in Table 1.

Table 1. Kragten matrix for calculating the uncertainty of the relative error of the EFM (4th verified point)

Name of Parameter	Initial Value	$u_A(Q_r)$	$u_A(Q_p)$	$u_{B1}(Q_r)$	$u_{B2}(Q_p)$	$u_{B3}(Q_p)$	Estimation Value
		22.02	**1.96**	**0.029**	**3.29**	**0.0029**	
Q_r	**5706.75**	**5728.77**	5706.75	5706.75	5706.75	5706.75	
Q_p	**5700.33**	5700.33	**5702.29**	5700.33	5700.33	5700.33	
$d_r/2$	**0.05**	0.05	0.05	**0.079**	0.05	0.05	
Δp	**0.1**	0.1	0.1	0.1	**3.39**	0.1	
$d_p/2$	**0.005**	0.005	0.005	0.005	0.005	**0.0079**	
δ	**0.11**	0.50	0.08	0.11	0.05	0.11	
$\delta - \delta_i$		-0.39	0.03	0.00	0.06	0.00	
$(\delta - \delta_i)^2$		0.15	0.00	0.00	0.00	0.00	
$u_c(\delta)$							**0.39**
$U(\delta)$							**0.78**

Results Analysis

The results of applying the GUM, Kragten and Monte Carlo methods for four control points to calibrate an electromagnetic flowmeter are shown in Table 2 (estimates of relative error –δ; expanded uncertainty –U; limit of repeatability (convergence) –u_r; coverage interval –CI).

Table 2. Comparative table of the calculated values for the four control points using the GUM, Monte Carlo, and Kragten methods

Verifiable point	Estimated parameters, %	GUM method	Kragten method	Monte Carlo method	D1, %	D2, %
25% *of* Q_{max}	δ	0.007	0.007	0.007	0	0
	U	0.689	0.701	0.654	1.8	5.0
	u_r	0.352	0.358	0.337	1.6	
	CI	[-0.682; 0.696]	[-0.694; 0.708]	[-0.651; 0.665]		
50% *of* Q_{max}	δ	-0.003	-0.003	-0.003	0	0
	U	0.677	0.667	0.654	1.5	3.4
	u_r	0.345	0.340	0.333	1.4	4.2
	CI	[-0.680; 0.674]	[-0.670; 0.664]	[-0.654; 0.648]		
75% *of* Q_{max}	δ	0.015	0. 015	0.01	0	33.3
	U	0.839	0.855	0.656	2	21.8

Verifiable point	Estimated parameters, %	GUM method	Kragten method	Monte Carlo method	D1, %	D2, %
	u_r	0.428	0.437	0.335	2	21.7
	CI	[-0.734; 0.944]	[-0.750; 0.960]	[-0.645; 0.665]		
95% *of* Q_{max}	δ	0.011	0.0112	0.005	0.4	54.5
	U	0.768	0.783	0.655	1.9	14.7
	u_r	0.392	0.391	0.334	0.26	14.8
	CI	[-0.655; 0.881]	[-0.655; 0.881]	[-0.653; 0.658]		

Based on Table 2, the dependencies of the uncertainty of the relative error are plotted against the experiment number (on the x-axis) obtained using all methods (Figure 5).

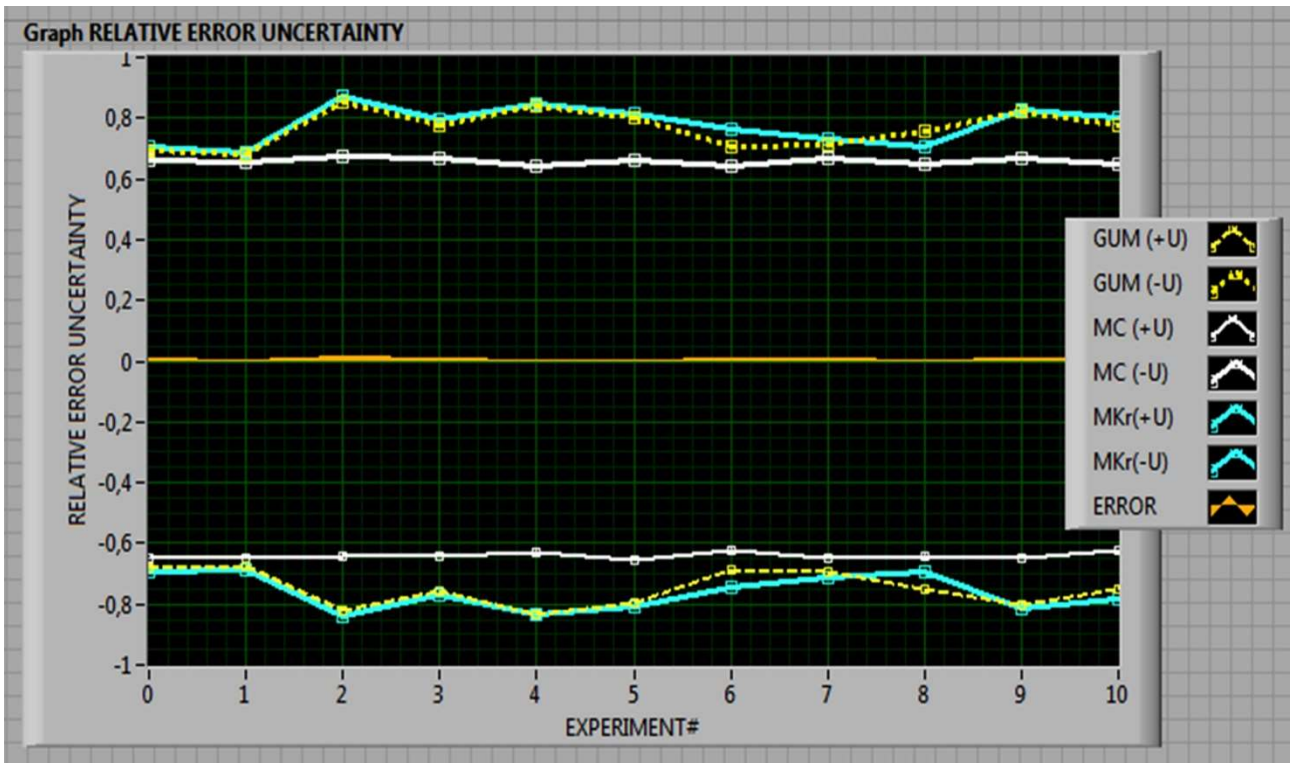

Figure 5. Uncertainty of the relative error vs the number of experiment

The graphs obtained using the Monte Carlo method (MC/solid white lines) show a constant value of the dispersion of the uncertainty of the relative error within ± 0.65%. The graphs obtained using the GUM method (GUM/dashed yellow lines) show the changing values of the spread of uncertainty of the relative error within ± 0.9%. In this case, the limit of the permissible relative error of the EFM is ± 1%. The graphs obtained using the Kragten method (MKr/solid blue lines) essentially repeat the GUM graphs.

Discussion

The authors performed a comparative analysis of the considered Monte Carlo and Kragten uncertainty estimation methods and the recommended standard GUM method. In a comparative analysis of the data from Table 2 and Figure 5, we can draw the following conclusions.

A comparison of the GUM and Kragten methods (data from column D1 in Table 2) showed that there are no differences between the estimates of the measured value (relative error), the differences between the expanded uncertainty and repeatability are less than 2%, and the coverage interval does not exceed ± 1% for each verified point in both methods.

The comparison of the GUM and Monte Carlo methods (data of column D2 in Table 2) showed that the differences between the estimates of the measured value (relative error) reach 55%, the differences between the expanded uncertainty and repeatability are not more than 21.8%, and the coverage interval does not exceed ± 1% for each verified point in both methods.

The large differences between the Kragten or GUM methods on the one hand and the Monte Carlo method on the other hand indicate significant deviations from the normality of the distribution of input quantities.

The calculations showed that all three methods (GUM, Kragten, and Monte Carlo) give values of standard uncertainty that do not exceed the permissible range of the relative error of EFM (± 1%).

The Monte Carlo method with a sufficiently large number of simulations gives a better approximation. However, Monte Carlo calculations take longer (due to the sorting and processing of large arrays), although they can be performed by less qualified personnel (no in-depth knowledge of mathematics is required). The Monte Carlo method can be considered as a practical alternative to the GUM uncertainty estimation method.

The Kragten method gives results similar to the GUM method. No significant differences between the results obtained by the GUM and the Kragten methods were noted. The Kragten method is recommended as a less time-consuming tool for calculations.

Conclusion

The Joint Committee for Guides in Metrology (JCGM) publishes and maintains background documentation on general aspects of metrology. The JCGM Committee Working Group 1 is responsible for evaluating the measurement data of a series of documents that provide information for evaluating and expressing measurement uncertainty.

To create conditions for the recognition of Kazakhstan certificates of conformity and test results for products, the implementation of clause 5.4.6 “Expression of measurement uncertainty” of normative document ISO / IEC 17025-2017 is required. In this regard, there is an increase in the practical application in Kazakhstan of the concept of measurement uncertainty.

The authors of the study conducted a study of methods for assessing measurement uncertainty as applied to the calibration of electromagnetic flowmeters.

The combined use of the classical GUM method, the Kragten spreadsheet and the Monte Carlo method is useful in developing a suitable strategy since each of the three approaches illuminates a different side of the problem.

References

[1] Golik, V. I., Zaalishvili, V. B., & Razorenov, Yu. I. (2014). Opyt dobychi urana vyshchelachivaniyem [Uranium leaching experience]. *Mining Informational and Analytical Bulletin*, *7*, 97–103. Retrieved from https://cyberleninka.ru/article/n/opyt-dobychi-urana-vyschelachivaniem (in Russian)

[2] Possolo, A. (2014). Statistical models and computation to evaluate measurement uncertainty. *Metrologia*, *51*(4), S228–S236. https://doi.org/10.1088/0026-1394/51/4/S228

[3] Sediva, S., & Havlikova, M. (2013). Comparison of GUM and Monte Carlo method for evaluation measurement uncertainty of indirect measurements. *Proceedings of the 14th International Carpathian Control Conference (ICCC)*, 325–329. https://doi.org/10.1109/CarpathianCC.2013.6560563

[4] Ellison, Stephen L R. (2014). Implementing measurement uncertainty for analytical chemistry: The *Eurachem Guide* for measurement uncertainty. *Metrologia*, *51*(4), S199–S205. https://doi.org/10.1088/0026-1394/51/4/S199

[5] Khan, S. G., Tashibayeva, A. E., & Bukayeva, G. (2018). Development of program for estimating of corolis flowmeters' measurement uncertainty. *Bulletin of the Almaty University of Power Engineering and Telecommunications*, *1*(40), 34–39. Retrieved from https://aues.kz/magazine/18_1.pdf

[6] Khan, S., & Tashibayeva, A. (2017). Razrabotka programmy otsenivaniya neopredelennosti izmereniya pri poverke elektromagnitnykh raskhodomerov [Development of program for estimating the measurement uncertainty at the calibration of electromagnetic flowmeters]. *Bulletin of the Almaty University of Power Engineering and Telecommunications*, *2*(37), 27–34. (in Russian)

[7] Vasilevskyi, O. M. (2014). Calibration method to assess the accuracy of measurement devices using the theory of uncertainty. *International Journal of Metrology and Quality Engineering*, *5*(4), 403. https://doi.org/10.1051/ijmqe/2014017

[8] Ehrlich, C. (2014). Terminological aspects of the *Guide to the Expression of Uncertainty in Measurement* (GUM). *Metrologia*, *51*(4), S145–S154. https://doi.org/10.1088/0026-1394/51/4/S145

[9] Lepek, A. (2003). A computer program for a general case evaluation of the expanded uncertainty. *Accreditation and Quality Assurance*, *8*(6), 296–299. https://doi.org/10.1007/s00769-003-0649-1

[10] Traple, M. A. L., Saviano, A. M., Francisco, F. L., & Lourenço, F. R. (2014). Measurement uncertainty in pharmaceutical analysis and its application. *Journal of Pharmaceutical Analysis*, *4*(1), 1–5. https://doi.org/10.1016/j.jpha.2013.11.001

[11] Bich, W. (2014). Revision of the '*Guide to the Expression of Uncertainty in Measurement*'. Why and how. *Metrologia*, *51*(4), S155–S158. https://doi.org/10.1088/0026-1394/51/4/S155

[12] Bich, W., Cox, M. G., Dybkaer, R., Elster, C., Estler, W. T., Hibbert, B., Wöger, W. (2012). Revision of the '*Guide to the Expression of Uncertainty in Measurement*.' *Metrologia*, *49*(6), 702–705. https://doi.org/10.1088/0026-1394/49/6/702

[13] Bich, W., Cox, M., & Michotte, C. (2016). Towards a new GUM—an update. *Metrologia*, *53*(5), S149–S159. https://doi.org/10.1088/0026-1394/53/5/S149

[14] Massart, D. L. (Ed.). (1997). Handbook of chemometrics and qualimetrics (1st ed., Version 1st ed.). Amsterdam ; New York: Elsevier.

[15] Meyer, V. R. (2007). Measurement uncertainty. *Journal of Chromatography A*, *1158*(1–2), 15–24. https://doi.org/10.1016/j.chroma.2007.02.082

[16] NASA. (2010, July 13). Measurement uncertainty analysis principles and methods: Nasa measurement quality assurance handbook – Annex 3. Measurement System Identification: Metric. NASA-HDBK-8739.19-3. Retrieved from https://standards.nasa.gov/file/2724/download?token=fJf4dbXX

[17] Guimaraes Couto, P. R., Carreteiro, J., & de Oliveir, S. P. (2013). Monte Carlo simulations applied to uncertainty in measurement. In W. K. V. Chan (Ed.), *Theory and Applications of Monte Carlo Simulations*. https://doi.org/10.5772/53014

[18] Kragten, J. (1994). Tutorial review. Calculating standard deviations and confidence intervals with a universally applicable spreadsheet technique. *The Analyst*, *119*(10), 2161. https://doi.org/10.1039/an9941902161

[19] Ellison, S. L. R, Williams, A., & Eurachem Working Group on Uncertainty in Chemical Measurement. (2012). Eurachem/CITAC guide: Quantifying uncertainty in analytical measurement (3rd ed., Version 3rd ed.). Retrieved from https://www.eurachem.org/images/stories/Guides/pdf/QUAM2012_P1.pdf

[20] González, A. G., Herrador, M. Á., Asuero, A. G., & Martín, J. (2018). A practical way to ISO/Gum measurement uncertainty for analytical assays including in-house validation data. In G. S. Zaman (Ed.), *Quality Control in Laboratory*. https://doi.org/10.5772/intechopen.72048

[21] Stant, L. T., Aaen, P. H., & Ridler, N. M. (2016). Comparing methods for evaluating measurement uncertainty given in the JCGM 'Evaluation of Measurement Data' documents. *Measurement*, *94*, 847–851. https://doi.org/10.1016/j.measurement.2016.08.015

[22] Zhusupbekov, S. S., Ibrayeva, L. K., Khan, S. G., & Komada, P. (2019). The experience of application of measurement uncertainty evaluation methods in calibration. *NEWS of National Academy of Sciences of the Republic of Kazakhstan*, *4*(436), 79–85.