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Viscous Modified Chaplygin gas in theories of gravitation

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Abstract

In this paper we have proposed Viscous modified chaplygin gas with homogeneous hypersurface space time in the context of general relativity, Brans-Dicke Theory, Self-Creation theory and modified $f(R,T)$ theories of gravitation.It is observed that the universe is accelerating and expanding. The model behave like ΛCDM model.

Keywords: Viscosity, Chaplygin gas, General relativity, Self creation, Brans-Dicke, $f(R,T)$ gravity

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1 Introduction

The recent impressive amount of observational data from type Ia supernovae [1], cosmic microwave backgound radiation [2]and large scale structure [3] indicates that the universe is pervade of 76 percent dark energy and 24 percent by other matter [4]. This is hypothetical form of energy which is assumed to be reason of expansion of the universe.This prime candidate do not have laboratory evidences. There are several models of dark energy found in the literature. Quintessence [5], Phantom [6], Tachyon [7], Holographic dark energy [8], Anisotropic dark energy [9] etc.are some of the suggested models of dark energy. Chaplygingas is another candidate of the dark energy which describe the evolution of the universe from the radiation era to the ΛCDM model [10]. The chaplygin gas model and its modification have been active field of research. Chattopadhyay and Debnath [11] have donsidered holographic dark energy and variable modified chaplygin gas. Chakraborty and Debnath [12] have studied the model of modified chaplygin gas in anisotropic universe. The generalized chaplygin gas model is associated with d-branes [13] and provide explanation of phantom divide crossing [14].

It is believed that inflation is important in the evolution of the universe. In the inflationary phase, there could be viscous type fluid arised due to decoupling of neutrinos. Eckart [15] has described a viscous fluid models in cosmology. Hiscock and Lindblom [16] have shown that Eckart model posseses cousality and stability problem. Lateron Muller

[17] has solved these issues by including higher order derivation terms in the transport equation. Ratra and Peebles [18] have obtained solutions of Einstein equation by using extended irreverseible thermodynamics. Bulk viscous cosmological models with string in the $f(R,T)$ theory is studied by Naidu et al.[19]. Singh et al.[20] have obtained power law and exponential expansion model for viscous fluid coupled with zero mass scalar field. Ram and Singh [21] have considered viscous fluid distribution in the general relativity. Zimdahl et al.[22] have shown that coincident problem can be solved by considereing interaction between dark energy and dark matter. Keeping this in view, we have considered viscous chaplygin gas in the framework of general relativity, Brans-Dicke theory , Self creation theory and $f(R,T)$ theory of gravitation.

The paper is organized as follows:section 2, deals with basic equation, section 3,4,5 and 6 devoted to solutions respectively in general relativity, Brans-Dicke theory, Self-Creation theory, and $f(R,T)$ theory of gravitation. In section 7, we conclude our results.

2 Basic Equations

Homogeneous hypersurface are LRS spaces with a group of motion G_4 on V_3. Einsteins general relativity field equations for homogenous hypersurface in case of perfect fluid are solved by Stewart and Ellis [23]. we consider the homogeneous hypersurface in the following form

$$ds^2 = -dt^2 + A^2dx^2 + B^2[dy^2 + \Sigma^2(y,k)dz^2] \tag{1}$$

where $\Sigma^2(y,k) = siny, y, sinhy$ respectively for $k = 1, 0, -1$. Varma and Ram [24] have investigated bulk viscous homogeneous hypersurface cosmological models in general relativity. Singh and Beesham [25] have studied anisotropic dark energy for homogeneous hypersurface space time in general relativity. The energy momentum tensor for bulk viscous fluid is given by

$$T_{ij} = (\rho + \bar{P})u_i u_j - \bar{P}g_{ij} \tag{2}$$

with $\bar{P} = P - \xi u^i_{;i}$. Here P, ρ, ξ being respectively the pressure, density and Coefficient of Bulk viscosity. u^i is the four veleocity vector of the fluid. The equation of modified Chaplygin gas is given by [26]

$$P = \gamma\rho - \frac{\beta}{\rho^\alpha}, 0 \leq \gamma \leq 1, 0 \leq \alpha \leq 1 \tag{3}$$

so that viscous modified chaplygin gas is defined as

$$\bar{P} = \gamma\rho - \frac{\beta}{\rho^\alpha} - \xi\theta \tag{4}$$

where θ is expansion scalar defined by

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{5}$$

here and elsewhere overdot denote differentiation with respect to t.Very recently, Jawad and Iqbal [26] have studied non-linear viscous modified variable chaplygin gas and viscous generalized cosmic chaplygin gas. Zhai et al.[27] have discous viscous generalized chaplygin gas.

3 Solution in General Relativity

The Einsteins general theory of relativity is most successful, complete and consistent theory of gravitation. It is fundamental for models of the universe. Various gravitational phenomenon have been explained in the framework of general relativity (GR). The field equations of General Relativity are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{6}$$

Reddy and Naidu [28] have investigated higher dimensional model with massless scalar field. Tripathy and Behera [29] have studied Bianchi type VI metric in the context of general relativity. By the use of the co-moving co-ordinates the field equations (6) with the help of (1), (2) and (4) can be explicitly written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} = -\gamma\rho + \frac{\beta}{\rho^\alpha} + \xi\theta \tag{7}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} = -\gamma\rho + \frac{\beta}{\rho^\alpha} + \xi\theta \tag{8}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} = \rho \tag{9}$$

Using equations (8) and (9), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{k}{B^2} = 0 \tag{10}$$

Equation (10) further reduced to

$$\frac{d}{dt}(-B^2\dot{A} + AB\dot{B}) = -kA \tag{11}$$

On integration of (11), we get

$$(-B^2\dot{A} + AB\dot{B}) = -k\int Adt + c_1 \tag{12}$$

where c_1 is constant. For simplicity we take $c_1 = 0$, then equation (12) read as

$$2B\ddot{B} - 2(\frac{\dot{A}}{A})B^2 = F(t) \tag{13}$$

where $F(t) = -\frac{2k}{A}\int Adt$ then equation (13) leads to

$$B^2A^{-2} = \int F(t)A^{-2}dt + c \tag{14}$$

The set of field equations (7)-(9) reduced to the single equation (14). In order to solve the integration we should require integrable form of the scale factor A. So that we assume the following simple form of A as

$$A = e^{mt} \tag{15}$$

where m is a real number.Using (15) in (14) and performing the integration we obtain

$$B^2 = \frac{k}{m^2} + ce^{2mt} = k_1 + ce^{2mt} \tag{16}$$

where $k_1 = \frac{k}{m^2}$. The volume is obtained as

$$V = AB^2 = e^{mt}(k_1 + ce^{2mt}) \tag{17}$$

The expansion scalar for the model is given by

$$\theta = \frac{k_2 + 3mce^{2mt}}{k_1 + ce^{2mt}} \tag{18}$$

The shear scalar is found to be

$$\sigma = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{\sqrt{3}}(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}) = \frac{km}{3(k_1 + cm^2e^{2mt})} \tag{19}$$

The deceleration parameter is obtained as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = 2 - \frac{3(k_2e^{mt} + ce^{3mt})(ke^{mt} + 3cm^2e^{3mt})}{(k_2e^{mt} + 3cme^{3mt})^2} \tag{20}$$

where $k_2 = \frac{k}{m}$ From figure (1) it is clear that the universe is expanding with constant rate of expansion. The universe is decelerating or accelerating according as sign of deceleration parameter is positive or negative. Here, we see that the sign of deceleration parameter is negative. Hence the universe is accelerating. . The shear scalar is non negative for $k = 1, -1$ i.e. the model shows shearing universe for $k = 1, -1$. The ratio of expansion to shear then becomes non-negative so that the universe is anisotropic for $k = 1, -1$. In case of $k = 0$, the shear scalar is vanished i.e. the universe is isotropic for $k = 0$. The energy density and coefficient of bulk viscosity are found to be

$$\rho = \frac{(k + 2cm^2e^{2mt})}{(k_1 + ce^{2mt})} + \frac{(c^2m^2e^{4mt})}{(k_1 + ce^{2mt})^2} \tag{21}$$

$$\xi\theta = \frac{(\gamma k + (2\gamma + 3)cm^2e^{2mt})}{(k_1 + ce^{2mt})} + \frac{(\gamma - 1)c^2m^2e^{4mt}}{(k_1 + ce^{2mt})^2} + m^2 - \frac{\beta}{\rho^\alpha} \tag{22}$$

4 Solution in Brans-Dicke theory

Scalar field models are improtant in the study of inflationary cosmology. In 1961, Brans and Dicke [30] have proposed a gravitational theory involving a scalar function. Brans-Dicke theory (BDT) is simple and reduced to general relativity in some extend. Therefore it is natural choice as alternative to general relativity. In BDT, it is possible to generate acceleration in the matter dominated epoch in the absence of any Q-field [31]. Matter and non-gravitational fields posses a long range scalar field in the BDT [32]. Therefore cosmological models in BDT attrating the researcher. The BDT equation with respect to scalar and tensor field are as follows:

$$\begin{aligned} R_{ij} - \frac{1}{2}Rg_{ij} + \omega\phi^{-1}(\phi_{,i}\phi_j + \frac{1}{2}g_{ij}\phi_{,\mu}\phi^{,\mu}) \\ +\phi^{-1}(\phi^i_{;j} - g_{ij}\phi^{;\mu}_{;\mu}) = -8\pi T_{ij}\phi^{-1} \end{aligned} \tag{23}$$

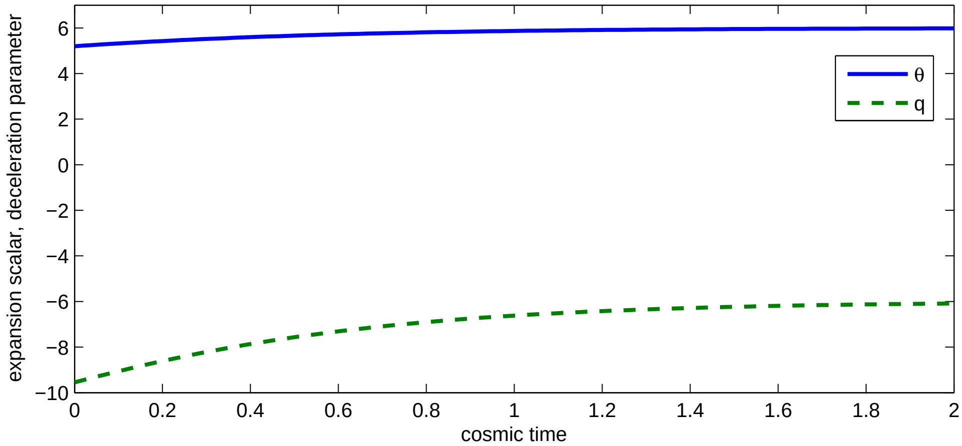

Figure 1: Plot of expansion scalar and deceleration parameter versus cosmic time

The scalar field is defined as

$$\Box\phi = \frac{8\pi\lambda}{\phi^{-1}}(3 + 2\omega^{-1}T) \tag{24}$$

where $\Box\phi = \phi^{\mu}_{;\mu}$ is the inverent D'Alembertion and T is the trace of the energy momentum tensor. Other symbols have their usual meaning as in Riemmanian geometry. The Einstein's field equations of general relativity could be recovered for $\phi = \phi_0, \omega \to 0$ and $T \neq 0$ [33]. It then follows form equation (1),(2),(4) and (23) that the field equations of BDT takes the following form:

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} + \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 + 2\frac{\dot{\phi}\dot{B}}{\phi B} + \frac{\ddot{\phi}}{\phi} = -8\pi\phi^{-1}(\gamma\rho - \frac{\beta}{\rho^{\alpha}} - \xi\theta) \tag{25}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 + \frac{\dot{\phi}}{\phi}(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}) + \frac{\ddot{\phi}}{\phi} = -8\pi\phi^{-1}(\gamma\rho - \frac{\beta}{\rho^{\alpha}} - \xi\theta) \tag{26}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} - \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 - \frac{\dot{\phi}}{\phi}(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}) = 8\pi\phi^{-1}\rho \tag{27}$$

BDT provided solutions to cosmological problems like cosmic acceleration, inflation, coincidence problem [34].From equations (25) and (26) we have

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{k}{B^2} + \frac{\dot{\phi}}{\phi}(\frac{|.B}{B} - \frac{\dot{A}}{A}) = 0 \tag{28}$$

Equation (28) can be written as

$$\frac{d}{dt}(\phi(-B^2\dot{A} + AB\dot{B})) = -kA\phi \tag{29}$$

On integration of (29), we yield

$$2B\dot{B} - 2 - B^2\frac{\dot{A}}{A} = -\frac{k}{A\phi}\int A\phi dt + c_2 \tag{30}$$

where c_2 is constant. For simplicity we take $c_2 = 0$, we see that equation (30) has three unknowns namely A, B, ϕ, so we need two additional conditions. We assume the relation between scalar field ϕ and metric potential A as $\phi = A^l$, where l is constant [35] and $A = e^{mt}$ as in section 3. Now Evaluating the integration we obtain

$$B^2 = \frac{k}{m^2} + ce^{2mt} = k_3 + ce^{2mt} \tag{31}$$

where $k_3 = \frac{k}{m^2(l+1)}$. We find that solutions for GR and BDT are approximately similar. The energy density, and coefficient of bulk viscosity can be expressed as

$$\begin{aligned} 8\pi\phi^{-1}\rho = \frac{(k + 2cm^2(1-l)e^{2mt})}{(k_3 + ce^{2mt})} &+ \frac{(c^2m^2e^{4mt})}{(k_3 + ce^{2mt})^2} \\ &- \frac{\omega m^2 l^2}{2} - m^2 l \end{aligned} \tag{32}$$

$$\xi\theta = \frac{e^{mlt}}{8\pi}[\frac{(k + cm^2(5-l)e^{2mt})}{(k_3 + ce^{2mt})} + m^2(l^2+1)] - \frac{\beta}{\rho^\alpha} \tag{33}$$

5 Solution in Self Creation Theory

Self-Creation Theory (SCT) is alternative scalar-tensor theory of gravitation proposed by Barber [36]. Barbers first theory is not consistent, but his second theory is well established. It is modification of BDT. In SCT, matter is continuously created within the limit of observations. Recently, Caglar and Aygun [37] have considered higher dimensional flat FRW universe with string cloud and domain walls. Reddy and Naidu [38] studied Kaluza-Klein model with perfect fluid in self-Creation theory. The field equations in Barber's self creation theory are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij}\phi^{-1} \tag{34}$$

The scalar field is defined as

$$\Box\phi = \frac{8\pi\lambda}{3}T \tag{35}$$

where $\Box\phi = \phi^{\mu}_{;\mu}$ is the inverent D'Alembertion and T is the trace of the energy momentum tensor. λ is a coupling constant, the semicolon denoted covariant differentiation. The SCT tends to GR in every respect when $\lambda \to 0$. The Field equations (34) for the metric (1) with the helf of (2) and (4) written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} = -8\pi\phi^{-1}(\gamma\rho - \frac{\beta}{\rho^\alpha} - \xi\theta) \tag{36}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} = -8\pi\phi^{-1}(\gamma\rho - \frac{\beta}{\rho^\alpha} - \xi\theta) \tag{37}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} = 8\pi\phi^{-1}\rho \tag{38}$$

Proceeding as in the section 3 and assuming the relation $\phi \ltimes a^n$, where n is an arbitrary constant. Here, the scalar field is depends on the scale factor. We have expanding universe for $n > 0$ [35]. Then,the expression of scalar field, energy density and coefficient of bulk viscosity are obtained as

$$\phi = N(k_1 e^{mt} + ce^{3mt})^{\frac{n}{3}} \tag{39}$$

$$\rho = \frac{N(k_1 e^{mt} + ce^{3mt})^{\frac{n}{3}}}{8\pi}\left[\frac{(k + 2cm^2 e^{2mt})}{(k_1 + ce^{2mt})} + \frac{(c^2 m^2 e^{4mt})}{(k_1 + ce^{2mt})^2}\right] \tag{40}$$

$$\xi\theta = \frac{N(k_1 e^{mt} + ce^{3mt})^{\frac{n}{3}}}{8\pi}\left[\frac{(\gamma k + (2\gamma + 3)cm^2 e^{2mt})}{(k_1 + ce^{2mt})} + \frac{(\gamma - 1)c^2 m^2 e^{4mt}}{(k_1 + ce^{2mt})^2} + m^2\right] - \frac{\beta}{\rho^\alpha} \tag{41}$$

6 Solution in $f(R, T)$ theory

Recently, Harko et al.[39] have proposed new modified theory of gravitation known as $f(R, T)$ theory of gravitation. It is developed by introducing an arbitrary function of Ricci scalar R and scalar torsion T in the standard Einstein-Hilbert action. There may be extra acceleration due to geometry and matter coupling. Many researcher have studied cosmological models in $f(R, T)$ theory of gravitation. Sahoo et al.[40] have explored string cosmological models in $f(R, T)$ theory of gravitation. Singh and Singh [41] have presented the reconstruction of modified $f(R, T)$ gravity in the presence of perfect fluid. Following to Harko et al.[39], and taking the matter Lagrangian $L_m = P$ for the specific function $f(R, T) = R + 2\mu T$, here we write the field equations of $f(R, T)$ gravity as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2\mu T_{ij} + (2\mu P + \mu T)g_{ij} \tag{42}$$

The trace of energy momentum tensor is given by

$$T = 3\bar{P} - \rho \tag{43}$$

Now for the metric (1) and using equations (2),(4),(5),(42) and (43) the set of field equations is obtained as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} = (8\pi\gamma + 7\mu\gamma - \mu)\rho - \frac{(8\pi + 7\mu)\beta}{\rho^\alpha} - (8\pi + 5\mu)\xi\theta \tag{44}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} = (8\pi\gamma + 7\mu\gamma - \mu)\rho - \frac{(8\pi + 7\mu)\beta}{\rho^\alpha} - (8\pi + 5\mu)\xi\theta \tag{45}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{k}{B^2} = (5\mu\gamma - 8\pi - 3\mu)\rho - \frac{(5\mu)\beta}{\rho^\alpha} - (3\mu)\xi\theta \tag{46}$$

From equation (44) and (45) we arrive at equation (10), therefore, we get the same solutions as in GR. For particular choice of $\gamma = \frac{8\pi+3\mu}{5\mu}$ we found energy density as

$$\begin{aligned}\rho = \left[\frac{\beta(16\pi + 4\mu)}{3}\right]^{\frac{1}{\alpha}}\Bigg(m^2 &- \frac{(16\pi + \mu)cm^2 e^{2mt} + (8\pi + 5\mu)k}{(3\mu)(k_1 + ce^{2mt})} \\ &- \frac{8(\pi + \mu)c^2 m^2 e^{2mt}}{(3\mu)(k_1 + ce^{2mt})^2}\Bigg)^{-\frac{1}{\alpha}}\end{aligned} \tag{47}$$

Now for $4\pi^2 + 5\pi + \mu + \mu^2 = 0$ the coefficient of bulk viscosity is given by

$$\begin{aligned}\xi\theta = &-\frac{5m^2}{16\pi + 4\mu} + \frac{(48\pi - 3\mu)cm^2 e^{2mt} + (8\pi + 5\mu)5k}{3\mu(16\pi + 4\mu)(k_1 + ce^{2mt})} \\ &-\frac{k}{(3\mu)(k_1 + ce^{2mt})} + \frac{(24\pi - 36\mu)c^2 m^2 e^{4mt}}{(3\mu)(16\pi + 4\mu)(k_1 + ce^{2mt})^2}\end{aligned} \tag{48}$$

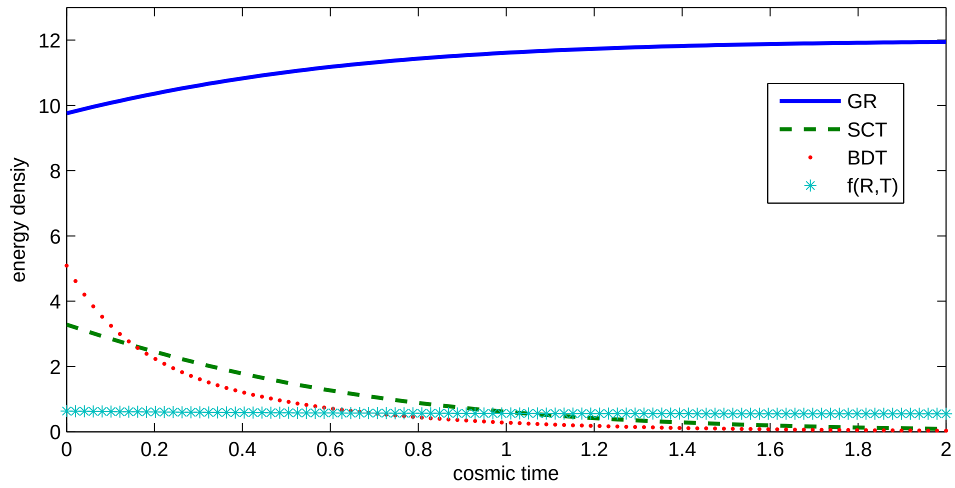

Figure 2: Plot of energy densities versus cosmic time

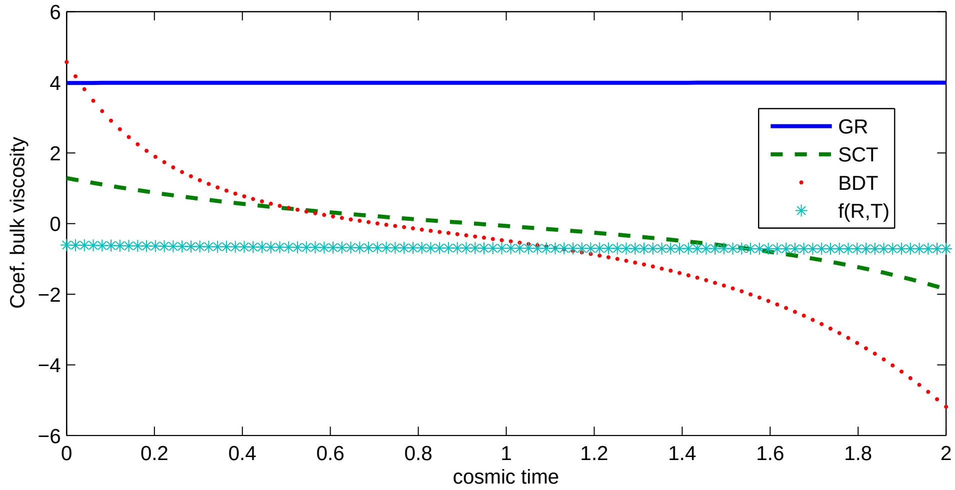

Figure 3: Plot of coefficient of bulk viscosity versus cosmic time

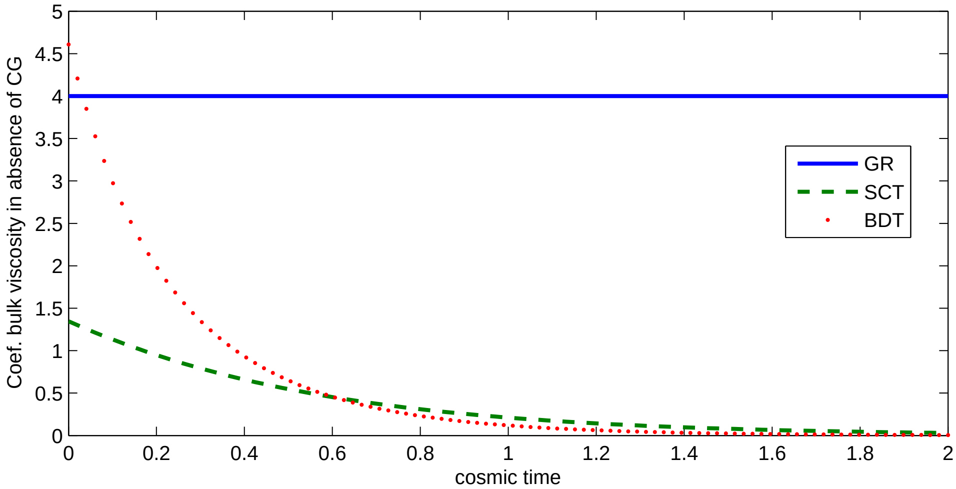

Figure 4: Plot of coefficient of bulk viscosity in absence of chaplygin gas versus cosmic time

Figure (2) reveals that the energy density in general relativity is larger than that of BDT, SCT and $f(R,T)$. For positive value of n, l the scalar field becomes increasing function of cosmic time as well as the energy density of SCT and BDT increases with incresing time. Thus, we have collapsing universe for $n, l > 0$. For $l, n < 0$ the energy densities of BDT and SCT are decreasing with increasing time. The energy density of $f(R,T)$ is smaller than that of GR, BDT,SCT showing the effect of matter source added in the Einstein-Hilbert action.

Figure (3) and (4) shows that the coefficient of bulk viscosity remain constant throughout the evolution of the universe in case of GR whereas in BDT and SCT it is decreasing with increasing time. In $f(R,T)$ it is negative. In case of BDT and SCT, it tends to negative in the far future. In the absence of Chaplygin gas the behavior of coefficient of bulk viscosity in GR is same as in the presence of chaplgyin gas whereas in BDT and SCT it tends to zero as $t \to \infty$.

In the earlier studies of bulk viscosity for homogeneous hypersurface in GR it is observed that $\rho \to 0$ as $t \to \infty$ [42,43,44]. It is important to note here that in the present analysis we found that $\rho \to constant$ as$t \to \infty$ It may be due to presence of Chaplygin gas [45]. Thus, the model describes ΛCDM model.

In case of BDT and SCT, the model describes the universe from stiff era ($\gamma = 1$) and (ρ is large) to the ΛCDM model(ρ is small) i.e. there is energy transfer occurs from former to later.

In $f(R,T)$ gravity, the condition $4\pi^2 + 5\pi + \mu + \mu^2 = 0$ gives us negative value of μ approximately -3.15. For this value of constant μ the parameter γ becomes negative. Thus Chaplygin gas behave like Phantom field which is compatible with the result obtained by Jawad and Iqbal [26].

7 Conclusion

In this work we have explored homogeneous hypersurface space time with viscous chaplygin gas in the context of GR, BDT, SCT and $f(R,T)$. we found that
1. The universe is expanding and accelerating.
2. The universe is shearing and anisotropic for $k = 1, -1$ and isotropic for $k = 0$.
3. The energy density is constant throughout the evoulution of the universe in GR whereas in BDT, SCT and $f(R,T)$ it is decreasing with increasing time.
4. The coefficient of bulk viscosity is constant for GR and decreasing with increasing time for BDT and SCT. In case of $f(R,T)$ it is negative.

References

[1] S. Perlmutter et al., Astrophys. Space Sci. 483, 565, 1997.
[2] R.R. Caldwell, M.Doran., Phys. Rev. D 69, 103517, 2004.
[3] S.F. Daniel, Phys. Rev. D 77, 103513, 2008.
[4] G. Hinshaw et al., Astrophys. J. Suppl.180, 225, 2009.
[5] V. Sahni, A.A. Starobinsky, Int. J. Mod. Phys. D 9, 373, 2000.
[6] R.R. Caldwell, Phys. Lett. B 545, 23, 2002.
[7] A. Sen, J. High Energy Phys.48, 204, 2002.
[8] O. Akarsu, C.B. Kilinc, Astrophys. Space Sci. 326, 315, 2010.
[9] M.R. Setare, Phys. Lett. B 442, 1, 2006.

[10] A.K. Singha, U. Debnath, Int. J .Theor. Phys.
[11] S. Chattopadhyay, U. Debnath, Astrophys Space Sci. 319, 2, 2009.
[12] S. Chakroborty, U. Debnath, Astrophys. Space Sci. 312, 53-56, 2009.
[13] H. Zhang, Z.H. Zhu, arXiv.0104.3121, 2007.
[14] M. Bordemann, J. Hoppe, Phys. Lett. B 317, 315, 1993.
[15] C. Eckart,Phys. Rev. 58, 919, 1940.
[16] W.P. Hiscock, I. Lindblom, Phys. Rev. D 31, 725, 1985.
[17] I. Muller, Z. Phys. 198, 329, 1967.
[18] B. Ratra, P.J.E. Peebles, Phys. Rev. D 37, 3406, 1988.
[19] R.L. Naidu, D.R.K. Reddy, T. Ramprasad, Astrophys. Space Sci. 348, 247-252, 2013.
[20] N.I. Singh, S.S. Singh, S.R. Devi, Astrophys. Space Sci. 334, 187-191, 2011.
[21] S. Ram, C.P. Singh, Astrophys. Space Sci. 260, 541-549, 1999.
[22] L.P. Chimento, A.S. Jakubi, D. Pavan, W. Zimdahl, Phys. Rev. D 67, 8, 2003.
[23] J.M. Stewart, G.F.R. Ellis, J. Math. Phys.9, 1072, 1968.
[24] M.K. Varma, S. Ram, Astrophys.Space Sci.326, 299-304, 2010.
[25] C.P. Singh, A. Beesham, Grav. Cosmol. 17, 3, 284-290, 2011.
[26] A. Jawad, A. Iqbal, arXiv:1610.09961v1, 2016.
[27] X.H. Zhai, Y.D. Xu, X. Z.Li, Int. J. Mod. Phys. D.15, 8, 1151-1161, 2006.
[28] D.R.K. Reddy, R.L. Naidu, Int. J. Theor. Phys. 47, 2339-2343, 2008.
[29] S.K. Tripathy, D. Behera, Astrophys. Space Sci.
[30] C.H. Brans, R.H. Dicke, Phys. Rev. 124, 925, 1961.
[31] S.P. Hatkar, C.D. Wadale, S.D. Katore, Astrophys. Space Sci. 365, 7, 2020
[32] A. Zrazi, C. Simeonc, Eur. Phys. J. plus, 126, 11, 2011.
[33] N. Banerjee, S. Sen, Phys. Rev. D.56, 1334, 1997.
[34] O. Betolami, P.J. Martins, Phys. Rev. D.61, 064007, 2000.
[35] M. Sharif, S. Waheed, arXiv.1207.7262v1, 2012.
[36] G.A. Barber, Gen. Relative. Gravit. 14, 117, 1982.
[37] H. Caglar, S. Aygun, Chin. Phys. C. 40, 4, 045103, 2016.
[38] D.R.K. Reddy, R.L. Naidu, Int. J. Theor. Phys. 48, 10-13, 2009.
[39] T. Harko, F.S.N. Lobo, S. Nojiri, S.D. Odintsov, Phys. Rev. D. 84, 024020, 2011.
[40] P.K. Sahoo, B. Mishra, P. Sahoo, S.K.J. Pacif, Eur. Phys. J. Plus, 131, 333, 2016.
[41] C.P. Singh, V. Singh, Gen. Relativ. Gravit. 46, 1696, 2014.
[42] S. Chandel, M.K. Singh, S. Ram, Adv. Stud. Theor. Phys. 6, 24, 1189-1198, 2012.
[43] S. Ram, M.K. Verma, Astrophys. Space Sci.330, 1, 151-156, 2010.
[44] M.M. Sancheti, S.D. Katore, S.P. Hatkar, Int. J. Math. Sci. Eng. Appl. 7(IV), 391-402, 2013.
[45] J.C. Fabric, S.V.B. Goncalves, P.E. de Souza, Gen. Rev. Gravit, 34, 53-63, 2002.