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Evaluation of fidelity for the interaction of a three-level atom with a two-mode quantized field in the cavity optomechanics

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Abstract

In quantum information theory, fidelity is a measure of the closeness of the two quantum states. In this paper we investigate the fidelity for a tripartite system includes on a V type three-levels atom and the two modes of field and a cavity optomechanics. We define an effective Hamiltonian for interaction in this system and then, using the time dependent Schrodinger equation, we write the time dependent wave function of the system. We determine the initial and final density matrix using time dependent wave function of the system. Eventually, we compute fidelity in a time dependent manner for this system numerically. At first, for all cases, fidelity is equal to one. Our observation shows that, for this system fidelity oscillate with time. The quality of these oscillations depends on the initial quantum state of the tripartite system.
Keywords: Fidelity; Density matrix; Three-levels atom; Cavity optomechanics

1 Introduction

The optomechanical systems are widely used today. They can also be used to understand quantitative schemes [1, 2]. Optomechanical systems are used for mass measurement and low force measurement [3, 4, 5], gravitational wave detection [6, 7] and quantum information processing [8] and mechanical oscillation cooling near ground state [9, 10, 11]. Entanglement is also an important quantum mechanics phenomenon that plays a special role in quantum computing and communication [12], sensitive measurements [13], quantum swapping and quantum teleportation [14, 15, 16] and so on.

In addition, it is not possible to observe entenglement in the macroscopic world due to the coupling of systems with the environment and the presence of decoherence. More recently, optical systems, as macroscopic systems, which are initially formed from an

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optical cavity with a moving mirror at one end, allow the analysis of quantum aspects such as entenglement [17]. In these systems, the radiation pressure force causes the mirror to move around the equilibrium point, thereby leading to mechanical-optical coupling, and thus entanglement occurs between them.

In recent years, in a mechanical optical system, a three-dimensional entanglement consisting of a two levels atom and a single-state quantum field is investigated in an optical cavity with a mirror capable of moving that atoms first considered with a optional superposition of two levels and the cavity field in a photon state [18]. Also in two other studies, a three-level atom in a mechanical optical system has recently been analyzed in [19, 20].

The authors considered the feasibility of strong coupling between the quantized motion of a mechanical system and the trapped atom motion. They exhibited that the coupling can be increased by the cavity finesse. The scheme is used for the transformation of the squeezed or Fock states from atom to the membrane. They eventually produced and appraised the entanglement between atom and membrane.

2 The proposed model and its analytic solution

This optomechanical system is illustrated in FIG 1. The system consists of a three levels V-type atom that interacts with a two-dimensional quantum radiation field in the cavity optomechanics, with one of its mirrors having mobility. The atomic levels are as $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$ and $|\tilde{3}\rangle$ and their energies as $\tilde{\omega}_3 < \tilde{\omega}_2 < \tilde{\omega}_1$ that we get $\hbar = 1$ where permitted transmissions are $|\tilde{1}\rangle \to |\tilde{3}\rangle$ and $|\tilde{2}\rangle \to |\tilde{3}\rangle$, $|\tilde{1}\rangle \to |\tilde{2}\rangle$ transmission is prohibited in the electric dipole approximation. The moving mirror can now be considered as a quantum harmonic oscillator, and we write the whole Hamiltonian system as follows

$$\hat{H} = \hat{H}_0 + \hat{H}_I, \qquad \hat{H}_0 = \hat{H}_A + \hat{H}_F + \hat{H}_M, \qquad \hat{H}_I = \hat{H}_{AF} + \hat{H}_{MF} \tag{1}$$

in which the Hamiltonian free atom, field and mirror may be defined as follows

$$\hat{H}_0 = \sum_{j=1}^{3} \tilde{\omega}_j \hat{\sigma}_{jj} + \sum_{j=1}^{2} \Omega_j \cdot_j^\dagger \cdot_j + \omega_M \hat{b}^\dagger \hat{b} \tag{2}$$

and the interaction Hamiltonians of the atom-field and the mirror-field are considered respectively [21]

$$\hat{H}_{AF} = \lambda_1(\cdot_1 \hat{\sigma}_{13} + \cdot_1^\dagger \hat{\sigma}_{31}) + \lambda_2(\cdot_2 \hat{\sigma}_{23} + \cdot_2^\dagger \hat{\sigma}_{32}) \tag{3}$$

$$\hat{H}_{MF} = -\sum_{j=1}^{2} G \cdot_j^\dagger \cdot_j (\hat{b}^\dagger + \hat{b}) \tag{4}$$

where $\Omega_j (j = 1, 2)$ are the frequencies of the two-mode field, ω_M is the frequency of the mirror and also $\hat{\sigma}_{ij} = |i\rangle \langle j| \, (i, j = 1, 2, 3)$ represents the atomic operators and $\cdot_j (\cdot_j^\dagger)$ is the annihilation (creation) operator of the jth mode of field $(j = 1, 2)$ and also λ_1 and λ_2 are the atom-field coupling constants and G indicates the optomechanical coupling strength to the field modes. Now we get the Hamiltonian interaction picture using the relationship $\hat{H}_{int} = e^{i\hat{H}_0 t} \hat{H}_I e^{-i\hat{H}_0 t}$ we write [21]

$$\hat{H}_{int} = -\sum_{j=1}^{2} G \cdot_j^\dagger \cdot_j (\hat{b}^\dagger e^{i\omega_M t} + \hat{b} e^{-i\omega_M t}) + \sum_{j=1}^{2} \lambda_j (\cdot_j \hat{\sigma}_{j3} e^{-i\omega_M t} + \cdot_j^\dagger \hat{\sigma}_{3j} e^{i\omega_M t}) \tag{5}$$

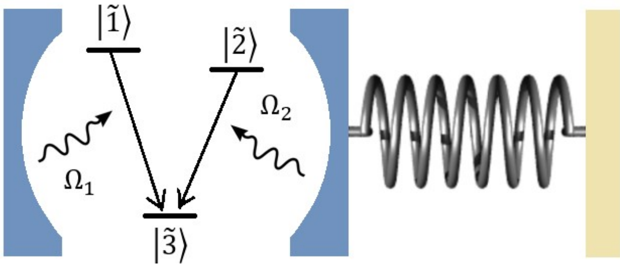

Figure 1: The V type three level atom interacting with a two modes field in an optomechanical cavity.

The effective Hamiltonian of system with the coarse grained method can be obtained that become [21]

$$\begin{aligned}\hat{H}_{eff} = \sum_{j=1}^{2} \frac{G\lambda_j}{\omega_M}(\cdot_j \hat{b}^\dagger \hat{\sigma}_{j3} + \cdot_j^\dagger \hat{b} \hat{\sigma}_{3j}) - \frac{G^2}{\omega_M}\sum_{j=1}^{2}(\cdot_j^\dagger \cdot_j)^2 - 2\frac{G^2}{\omega_M}(\cdot_1^\dagger \cdot_1)(\cdot_2^\dagger \cdot_2) \\ - \sum_{j=1}^{2} \frac{\lambda_1^2}{\omega_M}(\cdot_j^\dagger \cdot_j |j\rangle\langle j| - \cdot_j^\dagger \cdot_j |3\rangle\langle 3| + |j\rangle\langle j|) - \frac{\lambda_1\lambda_2}{\omega_M}(\cdot_1 \cdot_2^\dagger \hat{\sigma}_{12} + \cdot_1^\dagger \cdot_2 \hat{\sigma}_{21})\end{aligned}$$

Now we define the system in initial state as follows

$$|\psi(0)\rangle = |1,1;0\rangle \left(\cos(\theta) |\tilde{1}\rangle + \sin(\theta) |\tilde{3}\rangle\right) \tag{6}$$

As is clear, the atom is at first prepared in an arbitrary superposition of the ground ($|\tilde{3}\rangle$) and excited ($|\tilde{1}\rangle$) states, the cavity field modes are each in the single-photon mode and the mechanical mode is in the vacuum state. So, we suggest the time dependent wave function of system under the effect of Hamiltonian (**??**) as follows

$$\begin{aligned}|\psi(t)\rangle = M_1(t) |1,1;0;\tilde{1}\rangle + M_2(t) |1,1;0;\tilde{3}\rangle + M_3(t) |0,1;1;\tilde{1}\rangle \\ + M_4(t) |1,0;1;\tilde{2}\rangle + M_5(t) |2,0;0;\tilde{2}\rangle\end{aligned} \tag{7}$$

where $M_i(t)(i = 1,2,3,4,5)$ are the probability amplitudes and satisfy the normalization condition below

$$\sum_{i=1}^{5} M_i(t) M_i^*(t) = 1 \tag{8}$$

we use the relation $i\partial_t |\psi(t)\rangle = \hat{H}_{eff} |\psi(t)\rangle$ to calculate the probability amplitudes $M_i(t)(i = 1,2,3,4,5)$ that after a long calculation, we will have

$$M_1(t) = \frac{\cos(\theta)\left(2\lambda_1^2 e^{\frac{it\left(4G^2+2\lambda_1^2+\lambda_2^2\right)}{\omega_M}} + \lambda_2^2 e^{\frac{4iG^2 t}{\omega_M}}\right)}{2\lambda_1^2 + \lambda_2^2} \tag{9}$$

$$M_2(t) = \frac{\sin(\theta) e^{-\frac{it\left(\xi - 5G^2\right)}{2\omega_M}} \left(\left(3G^2 - 2\lambda_1^2 - 2\lambda_2^2\right)\left(-1 + e^{\frac{i\xi t}{\omega_M}}\right) + \xi e^{\frac{i\xi t}{\omega_M}} + \xi\right)}{2\xi} \tag{10}$$

where $\xi = \sqrt{9G^4 - 4\left(\lambda_1^2+\lambda_2^2\right)\left(2G^2-\lambda_1^2-\lambda_2^2\right)}$ and

$$M_3(t) = -\frac{2iG\lambda_1 \sin(\theta) e^{\frac{5iG^2 t}{2\omega_M}} \sin\left(\frac{t\sqrt{9G^4-4\left(\lambda_1^2+\lambda_2^2\right)\left(2G^2-\lambda_1^2-\lambda_2^2\right)}}{2\omega_M}\right)}{\sqrt{9G^4-4\left(\lambda_1^2+\lambda_2^2\right)\left(2G^2-\lambda_1^2-\lambda_2^2\right)}} \tag{11}$$

$$M_4(t) = -\frac{2iG\lambda_2 \sin(\theta) e^{\frac{5iG^2 t}{2\omega_M}} \sin\left(\frac{t\sqrt{9G^4-4\left(\lambda_1^2+\lambda_2^2\right)\left(2G^2-\lambda_1^2-\lambda_2^2\right)}}{2\omega_M}\right)}{\sqrt{9G^4-4\left(\lambda_1^2+\lambda_2^2\right)\left(2G^2-\lambda_1^2-\lambda_2^2\right)}} \tag{12}$$

$$M_5(t) = \frac{\sqrt{2}\lambda_1\lambda_2 \cos(\theta)\left(e^{\frac{it\left(4G^2+2\lambda_1^2+\lambda_2^2\right)}{\omega_M}} - e^{\frac{4iG^2 t}{\omega_M}}\right)}{2\lambda_1^2+\lambda_2^2} \tag{13}$$

it is clear that we can write

$$\begin{aligned} M_1(0) = \cos\theta, \quad M_2(0) = \sin\theta \\ M_3(0) = M_4(0) = M_5(0) = 0 \end{aligned} \tag{14}$$

3 Fidelity of the tripartite system

Fidelity is an criterion for measuring the closeness of states. This criterion is given as

$$F(\rho(0), \rho(t)) = \left[Tr\left(\sqrt{\sqrt{\rho(t)}\rho(0)\sqrt{\rho(t)}}\right)\right]^2 \tag{15}$$

here we can write using Equation (7)

$$\rho(0) = \begin{pmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) & 0 & 0 & 0 \\ \sin(\theta)\cos(\theta) & \sin^2(\theta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{16}$$

and it is obvious that $Tr\rho(0) = 1$ and also

$$\rho(t) = \begin{pmatrix} M_1(t)M_1^*(t) & M_1(t)M_2^*(t) & M_1(t)M_3^*(t) & M_1(t)M_4^*(t) & M_1(t)M_5^*(t) \\ M_2(t)M_1^*(t) & M_2(t)M_2^*(t) & M_2(t)M_3^*(t) & M_2(t)M_4^*(t) & M_2(t)M_5^*(t) \\ M_3(t)M_1^*(t) & M_3(t)M_2^*(t) & M_3(t)M_3^*(t) & M_3(t)M_4^*(t) & M_3(t)M_5^*(t) \\ M_4(t)M_1^*(t) & M_4(t)M_2^*(t) & M_4(t)M_3^*(t) & M_4(t)M_4^*(t) & M_4(t)M_5^*(t) \\ M_5(t)M_1^*(t) & M_5(t)M_2^*(t) & M_5(t)M_3^*(t) & M_5(t)M_4^*(t) & M_5(t)M_5^*(t) \end{pmatrix} \tag{17}$$

and it is obvious that $Tr\rho(t) = 1$.

Using the relationships in this section, we can evaluate and estimate fidelity numerically. In the next section, we do this evaluation for fidelity criteria and analyze the results.

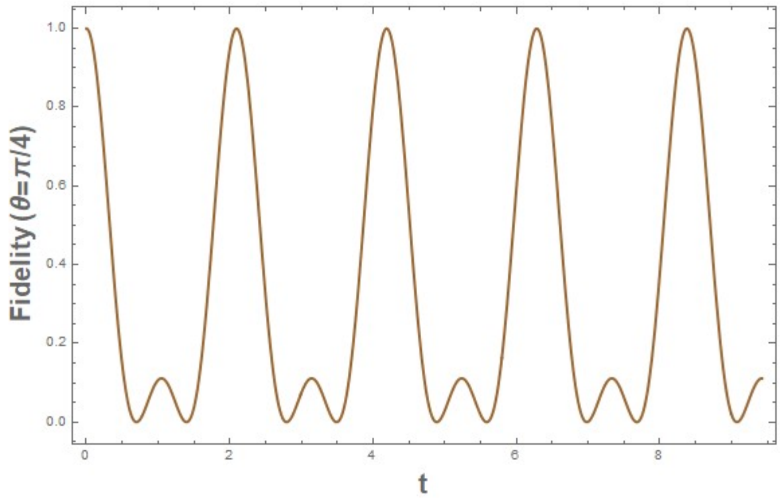

Figure 2: Fidelity of the tripartite system with respect to t for $\theta = \pi/4$.

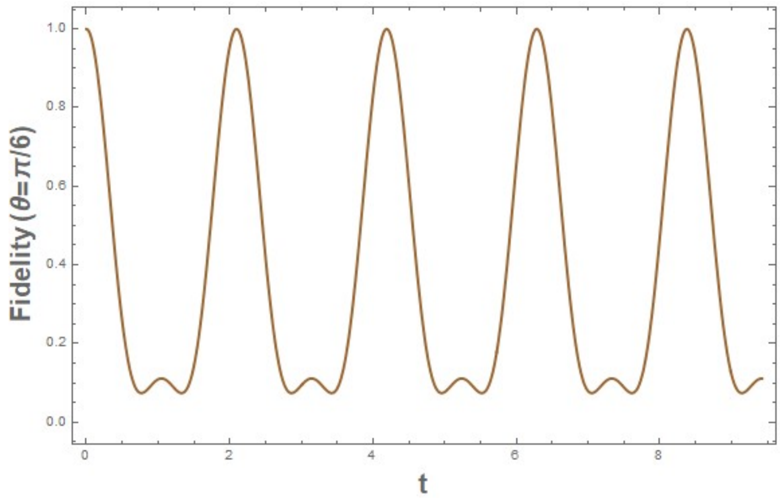

Figure 3: Fidelity of the tripartite system with respect to t for $\theta = \pi/6$.

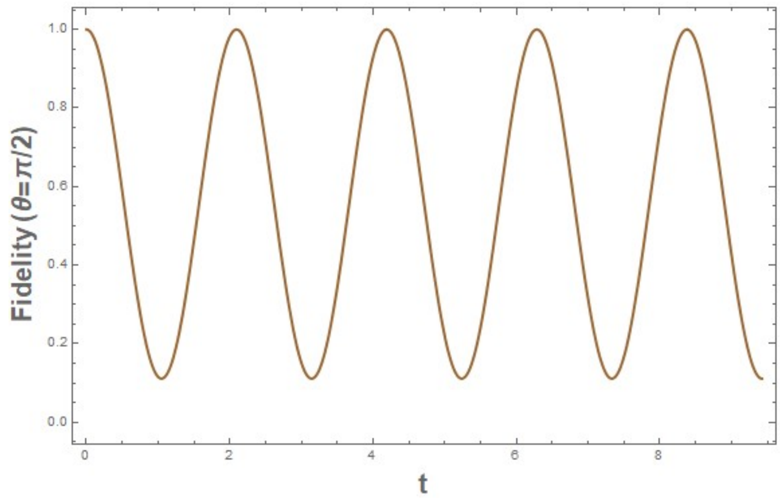

Figure 4: Fidelity of the tripartite system with respect to t for $\theta = 0$ and $\pi/2$.

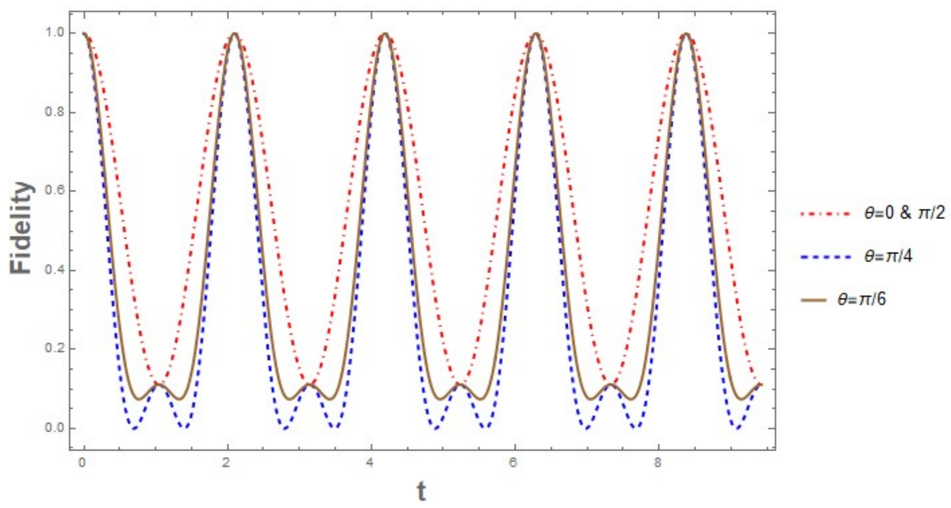

Figure 5: Diagram of fidelity of the tripartite system with respect to t for $\theta = 0, \pi/4, \pi/6, \pi/2$, together.

Table 1: The minimum value of fidelity tor $\theta = 0, \pi/4, \pi/6, \pi/2$.

θ (rad)	Minimum fidelity
0	0.111
$\frac{\pi}{4}$	0
$\frac{\pi}{6}$	0.074
$\frac{\pi}{2}$	0.111

4 Results

In this section, we present our results for evaluation and calculation of fidelity of our the tripartite system. All calculations presented in this section are only numerically obtainable. Therefore, the computer must be used to perform these calculations. We have examined this criterion (fidelity) for four different value of θ that are $\theta = 0, \pi/4, \pi/6, \pi/2$. In all cases at first fidelity is equal to one. The fidelity, except for the $\theta = \pi/4$, in all other cases it cannot be zero. The minimum value of fidelity for these cases are shown in TABLE 1. The comparison of these minimums for these cases is as

$$Min(\theta = \pi/4) < Min(\theta = \pi/6) < Min(\theta = 0, \pi/2) \tag{18}$$

In the cases $\theta = \pi/4$ and $\theta = \pi/6$ we have a relative maximum, both the same value 0.111 that is equal to the same minimum fidelity for $\theta = 0, \pi/2$.

In FIG 2 the fidelity with respect to t for case $\theta = \pi/4$ and in FIG 3 the fidelity with respect to t for case $\theta = \pi/6$. Also in FIG 4 the fidelity with respect to t for case $\theta = 0, \pi/2$. Finally in FIG 5 to compare between these three cases, we have drawn them together. In all of these three cases fidelity, oscillates with time.

5 Conclusion

In this work we considered the fidelity for a tripartite system includes on a V type three-level atom and the two modes of field and a cavity optomechanics. We wrote an effective Hamiltonian for interaction in this system and then, using the time dependent Schrodinger equation, we write the time dependent wave function of the system. We also defined the initial wave function. Using them, we determine the initial and final density matrix. Finally, we evaluated fidelity in a time dependent manner for this system numerically. Our consideration showed that, for this system fidelity oscillate with time as shown in the figures. In the cases $\theta = \pi/4$ and $\theta = \pi/6$ we have a relative maximum, both the same value 0.111 that is equal to the same minimum fidelity for $\theta = 0, \pi/2$. In all cases at first fidelity is equal to one. The fidelity, except for the $\theta = \pi/4$, in all other cases it cannot be zero.

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