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Investigation of fidelity and concurrence for a moving light beam in a turbulent atmosphere

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Abstract

In this paper, we studied the fidelity and concurrence for a moving light beam in a turbulent atmosphere using the classical entanglement. We using the classical entanglement of the radial-spatial degrees of freedom of a light beam found that the fidelity and the concurrence of this beam depends on the parameters of the turbulent atmosphere and the amount of distance the beam moves. In the absence of turbulence, fidelty and concurrence, both are equal to *1* and for maximum turbulence, these two critorions are both equal to *0*. Also we realized this subject that given the spatial degree of freedom of light that is related to its orbital angular momentum, for the larger orbital angular momentum of the light beam, the effect of atmospheric perturbation on concurrence is reduced. Therefore, we conclude that in order to increase the information capacity and increase the security of the communication channel one should use a higher orbital angular momentum light beam.

Keywords: Turbulent atmosphere; Classical entanglement; Fidelity; Concurrence

1 Introduction

In quantum information processing, entanglement plays an important role. Entanglement is a nonlocal correlation between the subsystems of a mixed system [1]. For many years, entanglement was a quantum effect but recently it has also been produced in the field of classical optics [2]. Classical entanglement has most of the quantum entanglement properties including Bell's inequality rejection [3].

In year 1998, local entanglement between different degrees of freedom of the particle was investigated [4]. Despite the fundamental differences between the classical entanglement and quantum entanglement, classical entanglement is designed to simulate the

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quantum information processing [2, 5, 6]. In quantum information theory, transportation is one of the importance protocols which is based on quantum entanglement [7].

One way of communicating in the future is quantum transportation in the free space. High dimensional entanglement with spatial degrees of freedom oflight is the perfect basis for this communication because using light degrees of freedom increases information capacity and increases security of quantum channels. These degrees of freedom can be Orbital Angular Momentum (OAM) [8]. In this paper we use the classical entanglement of the radial-spatial degrees of freedom of a light beam.

2 Classical entanglement of the radial-spatial degrees of freedom of a light beam

The state vector in the classical Hilbert space is called "*cebits*". One notation similar to Dirac notation is used to describe cebits. We use the Dirac notation for qubit, that is, $|qubit\rangle$. We use the Dirac like notation, namely, $|cebit)$ for cebits. We can, for a light beam, show the entanglement between the radial and spatial degrees of freedom by the classical entanglement Bell state that is

$$|\zeta) = \frac{1}{\sqrt{2}}\left[|r_1)\,|\ell_0) + |r_2)\,|-\ell_0)\right] \tag{1}$$

where $|r_1)$ and $|r_2)$ are radial degrees of freedom of cebit and $|\ell_0)$ and $|-\ell_0)$ are spatial degrees of freedom of cebit. We use the classical radial and spatial entanglement as the source of quantum communication in a turbulent atmosphere. The effect of atmospheric perturbation on a beam of light is characterized by the alteration in the refractive index of the turbulent atmosphere. Hence the photon that has a OAM, pass through the turbulent atmosphere, destroys the wavefront due to the oscillations of the refractive index of the turbulent atmospher and thus destroys the encoded quantum information [9].

For the Bell state defined in the equation (1), the density matrix is

$$\rho_0 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2}$$

as the entanglement state passes through the poorly turbulent atmosphere, the OAM degree of freedom is affected by this poorly disruptive environment. However, due to the ignition of diffraction effects on the weak turbulence, the radial degree of freedom does not change. To investigate the effects of weak atmospheric turbulence on the OAM degree of freedom we use a single phase plate model [9]. In this model, the effect of atmospheric perturbation on each photon is considered as a linear mapping Λ [10]. After the photon passes through the environment, the final density matrix becomes

$$\rho_f = (I \otimes \Lambda)\rho_0 \tag{3}$$

If we define two parameters, a, survival amplitude, and b, perturbtion amplitude, then we can write [10]

$$a = \Lambda^{\ell_0,\ell_0}_{\ell_0,\ell_0} = \Lambda^{-\ell_0,-\ell_0}_{-\ell_0,-\ell_0} = \Lambda^{-\ell_0,\ell_0}_{-\ell_0,\ell_0} = \Lambda^{\ell_0,-\ell_0}_{\ell_0,-\ell_0} \tag{4}$$

and

$$b = \Lambda^{\ell_0,\ell_0}_{-\ell_0,-\ell_0} = \Lambda^{-\ell_0,-\ell_0}_{\ell_0,\ell_0} \tag{5}$$

where for any $\Lambda^{\ell_0,\ell_0'}_{\ell,\pm\ell}$ we can write [9]

$$\Lambda^{\ell_0,\ell_0'}_{\ell,\pm\ell} = \frac{\delta_{\ell_0-\ell_0',\ell\mp\ell}}{2\pi}\int_0^\infty dr R_{0\ell_0}(r)R^*_{0\ell_0'}(r)r\int_0^{2\pi} d\vartheta \times e^{-i\vartheta[\ell\pm\ell-(\ell_0+\ell_0')]}e^{-0.5D_\phi(2r|\sin(\vartheta/2)|)} \tag{6}$$

where $R_{0l_0}(r)$ is the radial portion of the input Laguerre-Gaussian light beam at $z = 0$ (z coordinate in a cylindrical system), with radial and azimuthal quantum numbers 0 and l_0, respectively, for a light beam.

$D_\phi(r) = 6.88(r/r_0)^{5/3}$ is the phase structure function and r is radial coordinate in a cylindrical system. We will explain about r_0 in more detail in the next section. The effect of atmospheric perturbation on the light beam is determined by a and b.

The specific status of these two parameters are, $a = 1, b = 0$ (Absence of perturbation) and $a = 0, b = 1$ (Existence of maximum perturbation) [10].

Finally for final density matrix we can write

$$\rho_f = \frac{1}{2(a+b)}\begin{pmatrix} b & 0 & 0 & 0 \\ 0 & a & a & 0 \\ 0 & a & a & 0 \\ 0 & 0 & 0 & b \end{pmatrix} \tag{7}$$

where $Tr(\rho_f) = 1$ is confirmed.

3 Single Phase Plate model

In the single phase plate model, the effect of atmospheric perturbation on each photon is considered as a linear mapping Λ. For this model the amount of atmospheric turbulence is explained by dimensionless quantity W as

$$W = \frac{w_0}{r_0} \tag{8}$$

where w_0 is the radius of the light beam and r_0 is the Fried parameter, which is defined as [10]

$$r_0 = 0.185\left(\frac{\lambda^2}{C_n^2 z}\right)^{3/5} \tag{9}$$

where λ is the wavelength, z is the propagation distance and C_n^2 is refractive index structure constant which is $10^{-17}m^{-2/3}$ for weak perturbation and is $3.2\times10^{-13}m^{-2/3}$ for strong perturbation [11].

4 Fidelity and concurrence of classical entanglement state

Fidelity is an criterion for measuring the closeness of states. This criterion is given as

$$F(\rho_0, \rho_f) = \left[Tr\left(\sqrt{\sqrt{\rho_f}\rho_0\sqrt{\rho_f}}\right)\right]^2 \tag{10}$$

according to the definition of ρ_f, we will have

$$\sqrt{\rho_f} = \begin{pmatrix} \frac{\sqrt{\frac{b}{a+b}}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\sqrt{\frac{a}{a+b}} & \frac{1}{2}\sqrt{\frac{a}{a+b}} & 0 \\ 0 & \frac{1}{2}\sqrt{\frac{a}{a+b}} & \frac{1}{2}\sqrt{\frac{a}{a+b}} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{\frac{b}{a+b}}}{\sqrt{2}} \end{pmatrix} \tag{11}$$

and finally we will have

$$\sqrt{\sqrt{\rho_f}\rho_0\sqrt{\rho_f}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\sqrt{\frac{a}{a+b}} & \frac{1}{2}\sqrt{\frac{a}{a+b}} & 0 \\ 0 & \frac{1}{2}\sqrt{\frac{a}{a+b}} & \frac{1}{2}\sqrt{\frac{a}{a+b}} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{12}$$

by combining these calculations we can write

$$F = \frac{a}{a+b} \tag{13}$$

But for concurrence, this criterion can be written according to [9] as

$$C = max\left[0, \frac{(1-2b/a)}{(1+b/a)^2}\right]. \tag{14}$$

5 Results

In this section first, the results of our calculation for fidelity of classical entenglement state, will be provided. As shown in figure 1, we provide the diagram of fidelity with respect to variables a and b. As mentioned earlier according to the this figure, we can write

$$\lim_{(a,b)\to(1,0)} F = 1 \tag{15}$$

$$\lim_{(a,b)\to(0,1)} F = 0 \tag{16}$$

Also according to the concurrence formula we can write

$$\lim_{(a,b)\to(1,0)} C = max[0,1] = 1 \tag{17}$$

$$\lim_{(a,b)\to(0,1)} C = max[0,0] = 0 \tag{18}$$

We can take a and b as the functions of ℓ_0 and write

$$a(\ell_0) = \Lambda^{\ell_0,\ell_0}_{\ell_0,\ell_0} = \frac{1}{2\pi}\int_0^\infty dr R_{0\ell_0}(r) R^*_{0\ell_0}(r) r \int_0^{2\pi} d\vartheta \times e^{-0.5\times 6.88((2r|\sin(\vartheta/2)|)/r_0)^{5/3}} \tag{19}$$

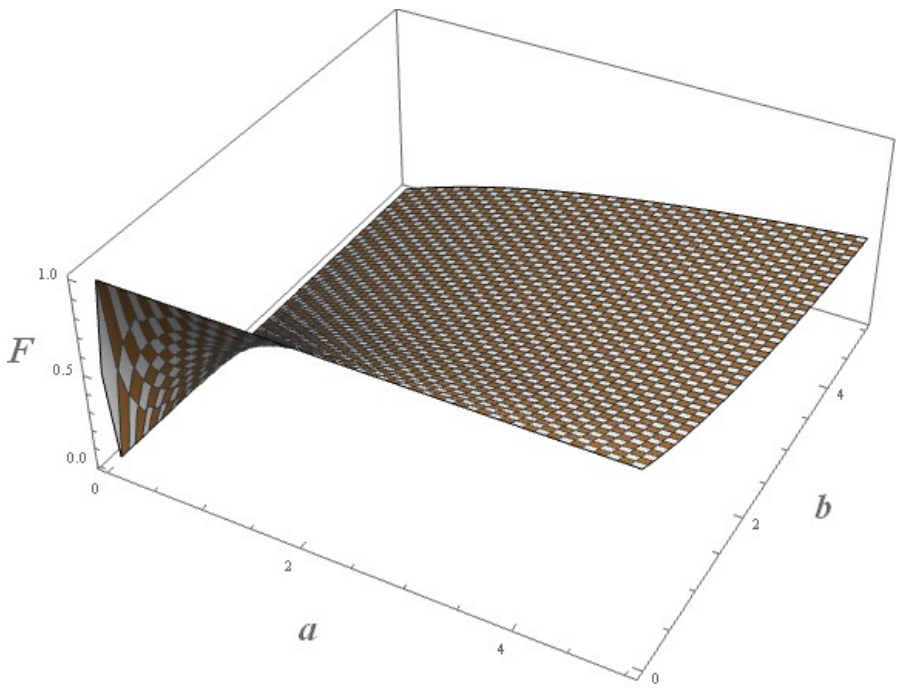

Figure 1: Three dimensional diagram of fidelity with respect to variables a and b.

and

$$b(\ell_0) = \Lambda^{\ell_0,\ell_0}_{-\ell_0,-\ell_0} = \frac{1}{2\pi}\int_0^\infty dr R_{0\ell_0}(r) R^*_{0\ell_0}(r) r \int_0^{2\pi} d\vartheta \times e^{4i\ell_0\vartheta} e^{-0.5\times 6.88((2r|\sin(\vartheta/2)|)/r_0)^{5/3}} \tag{20}$$

using these two formulas, we can calculate the concurrence for each a and b. In figure 2 the diagram of the concurrence with respect to variable W, for $\ell_0 = 1$ is presented and in figure 3 the diagram of the concurrence with respect to variable W, for $\ell_0 = 2$ is presented. Also in figure 3 the diagram of the concurrence with respect to variable W, for $\ell_0 = 3$ is presented.

Finally in figure 4 the diagrams of the concurrence with respect to variable W, for $\ell_0 = 1, 2, 3$ are plotted together.

6 Conclusion

We have shown for a classical entenglement state that in fact is a light beam with the radial and spatial degrees of freedom and, in other words, with an orbital angular momentum, that the effects of environmental perturbations on information transmission capacity and communication channel security are important. According on the calculations made in this paper, we can briefly conclude that in the absence of turbulence, fidelity and concurrence, both are equal to 1 and for maximum turbulence, these two critorions are both equal to 0. Also we realized this subject that given the spatial degree of freedom of light that is related to its orbital angular momentum, for the larger orbital angular momentum of the light beam, the eï¬€ect of atmospheric perturbation on concurrence is reduced. Therefore, we conclude that in order to increase the information capacity and increase the security of the communication channel one should use a higher orbital angular momentum light beam.

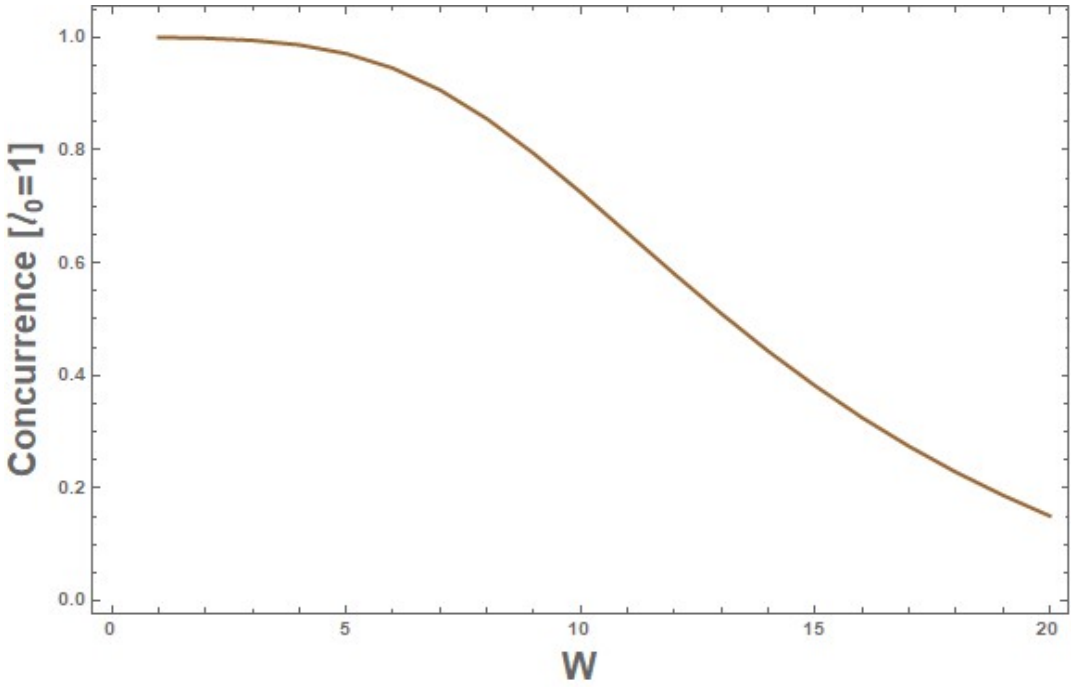

Figure 2: The diagram of concurrence with respect to variable W for $\ell_0 = 1$.

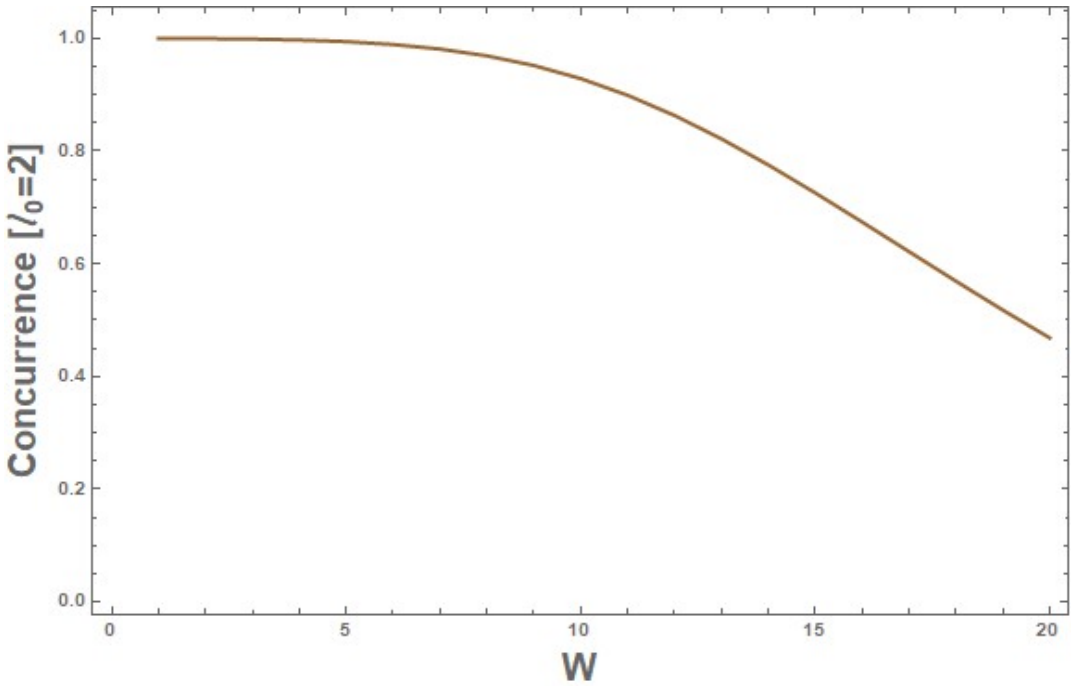

Figure 3: The diagram of concurrence with respect to variable W for $\ell_0 = 2$.

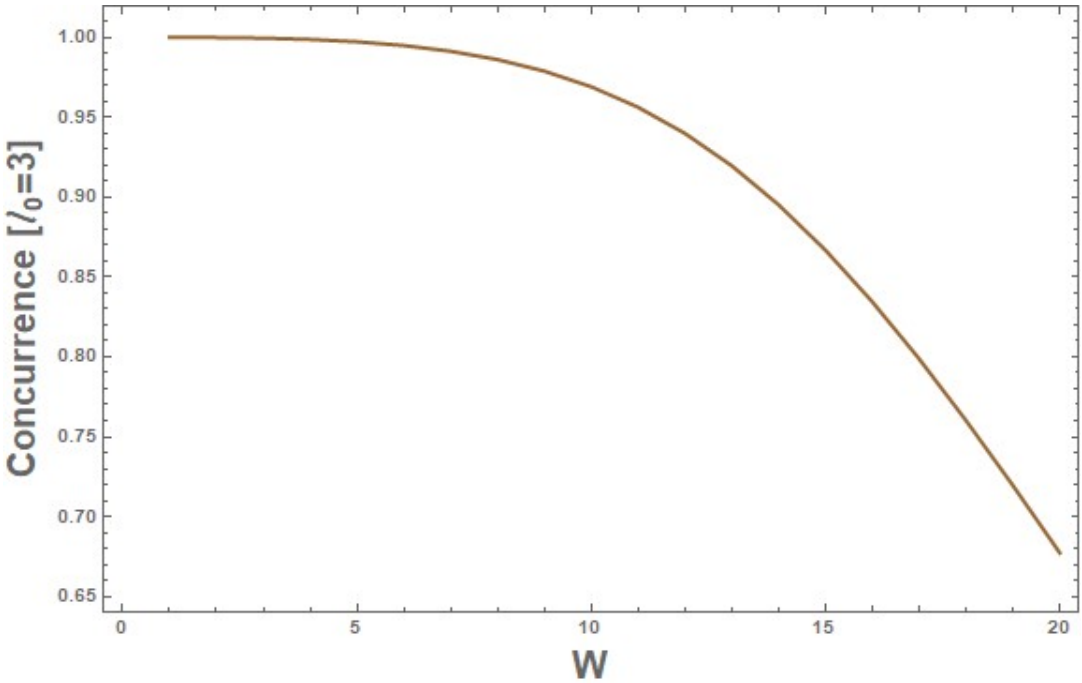

Figure 4: The diagram of concurrence with respect to variable W for $\ell_0 = 3$.

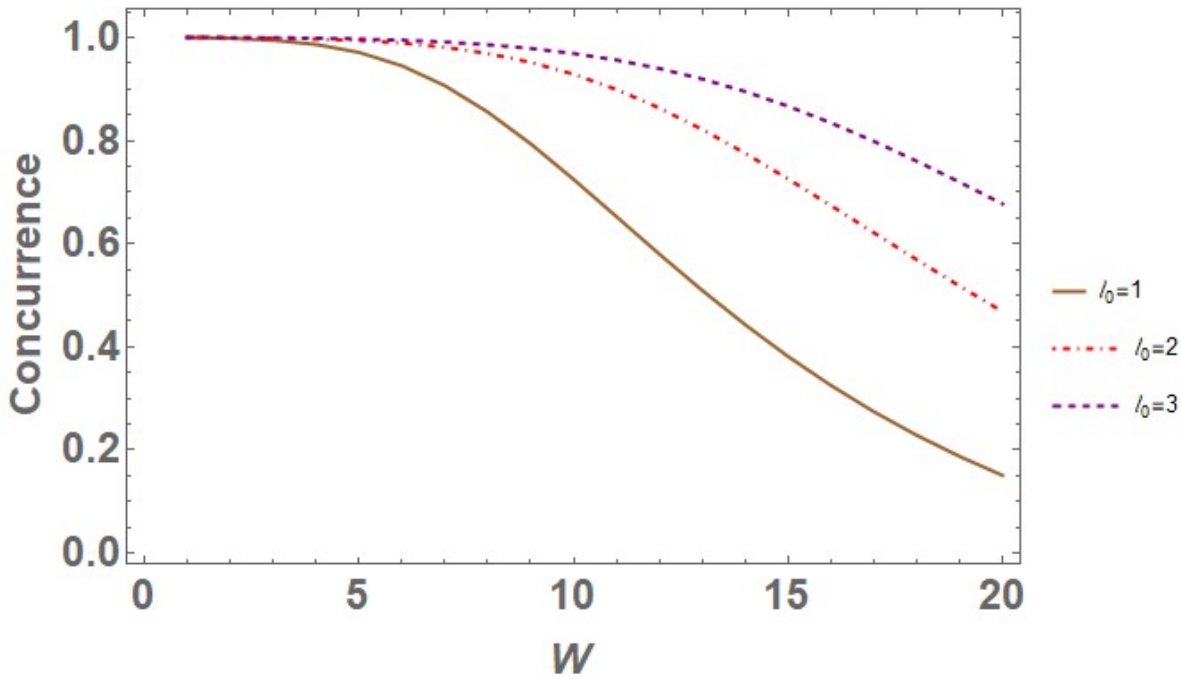

Figure 5: The diagrams of the concurrence with respect to variable W, for $\ell_0 = 1, 2, 3$ together.

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