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Solution of Einstein's equation in case of Photon

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Abstract

A brief discussion of the characteristics of photon has been done in this work. On the basis of these discussions a photon is considered to be feebly charged and is a gravitating mass point which is rotating about an axis of rotation. On this consideration R-N metric has been applied to the case of photon and Einstein's tensor in the mixed tensor form has been found out. Using Einstein's tensor Schwarzchild external and internal solutions as well as the solution in vacuum have been obtained.

Introduction

Solution of Einstein's equation is an important subject of study for physicists. Different workers have found different forms of metric as the solution for different cases. In this work the authors are interested about the solution of Einstein's equation for the case of photon.

The characteristics of photon have been considered by several authors [1-12] which speak about the charge, mass, spin and angular momentum etc. of the system of photon. It is known that a photon has spin and linear motion simultaneously [11-14]. Again, it has electromagnetic field accompanying a gravitational field [15] and Electromagnetic Gravitational Interaction in the system of photon has been introduced in several works [16-18].

According to different workers the small mass of the photon is considered to be concentrated in a ring of radius r rotating with angular velocity ω so that $r\omega$ is comparable to the velocity of light (c). It is, also, having a linear motion with velocity $\vec{v}$ comparable to c, along the axis of rotation. Hence, there are two simultaneous superimposed velocities present in the system of photon. Again, photon is assumed to be feebly charged (charge Q, say) [5-10] having mass $h\nu / c^2 (= m)$ [1, 4, 11, 12] which is also small and arises during motion. The photons also radiate while moving [19].

In this work, we shall try to find out solutions of Einstein's equation for photons on the basis of the above considerations.

The Solution

It is known that the solution for a radially symmetric rotating mass (m) having charge (Q) was obtained by Newman and others [20] using null tetrad method. They started with Reissneir - Nordström metric [21]

$$ds^2 = -(1-\frac{2mr-Q^2}{r^2})^{-1}dr^2 - r^2d\theta^2 - r^2Sin^2\theta d\phi^2 + (1-\frac{2mr-Q^2}{r^2})c^2dt^2 \tag{1}$$

where $m =$ gravitating mass point and Q is the charge. Now, we may consider photons to be feebly charged gravitating mass points. So, for solution of Einstein's equation for photons we may, also, start from the same R—N metric considering m to be the mass of the photon and Q its charge. Let us put $(1-\frac{2mr-Q^2}{r^2})^{-1} = A$ and $c^2(1-\frac{2mr-Q^2}{r^2}) = B$ for ease of handling where A and B are functions of r only and

$$AB = c^2 \tag{2}$$

Thus,

$$ds^2 = -Adr^2 - r^2d\theta^2 - r^2Sin^2\theta d\phi^2 + Bdt^2 \tag{3}$$

From (3) the fundamental tensor is given by

$$g_{\mu\nu} = \begin{pmatrix} -A & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2Sin^2\theta & 0 \\ 0 & 0 & 0 & B \end{pmatrix} \tag{4}$$

Hence,

$$g^{\mu\nu} = \begin{pmatrix} -\frac{1}{A} & 0 & 0 & 0 \\ 0 & -\frac{1}{r^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2Sin^2\theta} & 0 \\ 0 & 0 & 0 & \frac{1}{B} \end{pmatrix} \tag{5}$$

Let us find out three index Γ functions using the following popular formulae (with no summation convention) [22]

$$a)\Gamma^{\mu}_{\mu\mu} = \frac{1}{2}g^{\mu\mu}\frac{\partial g_{\mu\mu}}{\partial x^{\mu}},\ b)\Gamma^{\nu}_{\mu\mu} = -\frac{1}{2}g^{\nu\nu}\frac{\partial g_{\mu\mu}}{\partial x^{\nu}},\ c)\Gamma^{\nu}_{\mu\nu} = \frac{1}{2}g^{\nu\nu}\frac{\partial g_{\nu\nu}}{\partial x^{\mu}} \text{ for } \mu \le \nu,\ d)\Gamma^{\sigma}_{\mu\sigma} = 0 \text{ for } \mu > \sigma, \text{ and } e)\Gamma^{\nu}_{\mu\nu} = \Gamma^{\nu}_{\nu\mu} \tag{6}$$

where μ, ν, σ are different suffixes.

Using (6) we obtain following widely used independent 3-index symbols which would be required in our computations.

$$\Gamma^1_{11}=\frac{\dot{A}}{2A},\Gamma^1_{22}=-\frac{r}{A},\Gamma^1_{33}=-\frac{rSin^2\theta}{A},\Gamma^1_{44}=\frac{\dot{B}}{2A},\Gamma^1_{21}=0=\Gamma^1_{12},\ \Gamma^1_{31}=0=\Gamma^1_{13},\ \Gamma^1_{41}=0=\Gamma^1_{14},$$

$$\Gamma^2_{12}=\frac{1}{r}=\Gamma^2_{21},\Gamma^2_{33}=-Sin\theta Cos\theta,\Gamma^3_{13}=\frac{1}{r}=\Gamma^3_{31},\Gamma^3_{23}=Cot\theta,\ \Gamma^4_{11}=0,\Gamma^4_{14}=\frac{\dot{B}}{2B},\Gamma^4_{44}=0 \quad (7)$$

At this stage we shall find out $R_{11},R_{22},R_{33},R_{44}$, the Ricci tensors using the formula

$$R_{\mu\nu}=\frac{\partial}{\partial x^\nu}\Gamma^\beta_{\mu\beta}-\frac{\partial}{\partial x^\beta}\Gamma^\beta_{\mu\nu}+\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\alpha\nu}-\Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\alpha\beta} \quad (8)$$

Thus, we obtain

$$R_{11}=\frac{\ddot{B}}{2B}-\frac{\dot{B}^2}{4B^2}-\frac{\dot{A}}{Ar}-\frac{\dot{A}\dot{B}}{4AB} \quad (9)$$

$$R_{22}=-1+\frac{1}{A}-\frac{r\dot{A}}{2A^2}+\frac{r\dot{B}}{2AB} \quad (10)$$

$$R_{33}=Sin^2\theta R_{22} \quad (11)$$

and

$$R_{44}=-\frac{\ddot{B}}{2A}+\frac{\dot{A}\dot{B}}{4A^2}-\frac{\dot{B}}{Ar}+\frac{\dot{B}^2}{4AB} \quad (12)$$

It is to be remembered that for the line element (3) all the off-diagonal elements of $R_{\mu\nu}$ are identically zero. Now, we have the mixed tensor

$$R^\nu_\mu=g^{\nu\alpha}R_{\alpha\mu} \quad (13)$$

Using this we could obtain

$$R^1_1=g^{11}R_{11}=-\frac{1}{A}\left(\frac{\ddot{B}}{2B}-\frac{\dot{B}^2}{4B^2}-\frac{\dot{A}}{Ar}-\frac{\dot{A}\dot{B}}{4AB}\right) \quad (14)$$

$$R^2_2=g^{22}R_{22}=-\frac{1}{r^2}\left(-1+\frac{1}{A}-\frac{r\dot{A}}{2A^2}+\frac{r\dot{B}}{2AB}\right) \quad (15)$$

$$R^3_3=g^{33}R_{33}=R^2_2 \quad (16)$$

$$R^4_4=g^{44}R_{44}=\frac{1}{B}\left(-\frac{\ddot{B}}{2A}+\frac{\dot{A}\dot{B}}{4A^2}-\frac{\dot{B}}{Ar}+\frac{\dot{B}^2}{4AB}\right) \quad (17)$$

Hence, the scalar curvature

$$R=R^\mu_\mu=R^1_1+R^2_2+R^3_3+R^4_4=-\frac{\ddot{B}}{AB}+\frac{\dot{B}^2}{2AB^2}+\frac{\dot{A}\dot{B}}{2A^2B}+\frac{2\dot{A}}{A^2r}-\frac{2\dot{B}}{ABr}+\frac{2}{r^2}-\frac{2}{Ar^2} \quad (18)$$

Now, for static spherically symmetric line element we get Einstein's tensor $R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$ in the mixed tensor form to be $R^\nu_\mu-\frac{1}{2}g^\nu_\mu R$. Thus,

$$R_1^1 - \frac{1}{2} g_1^1 R = \frac{\dot{B}}{ABr} - \frac{1}{r^2} + \frac{1}{Ar^2} \tag{19}$$

$$R_2^2 - \frac{1}{2} g_2^2 R = \frac{\ddot{B}}{2AB} + \frac{\dot{B}}{2ABr} - \frac{\dot{A}}{2A^2 r} - \frac{\dot{B}^2}{4AB^2} - \frac{\dot{A}\dot{B}}{4A^2 B} = R_3^3 - \frac{1}{2} g_3^3 R \tag{20}$$

$$R_4^4 - \frac{1}{2} g_4^4 R = -\frac{\dot{A}}{A^2 r} - \frac{1}{r^2} + \frac{1}{Ar^2} \tag{21}$$

Again, Einstein's field equations, neglecting cosmological constant $\wedge$, may be written as

$$R_\mu^\nu - \frac{1}{2} g_\mu^\nu R = -8\pi T_\mu^\nu \tag{22}$$

Hence, using (19), (20) and (21) we get from (22) the non-zero energy-momentum tensors respectively to be

$$-8\pi T_1^1 = \frac{\dot{B}}{ABr} - \frac{1}{r^2} + \frac{1}{Ar^2} \tag{23}$$

$$-8\pi T_2^2 = -8\pi T_3^3 = \frac{\ddot{B}}{2AB} + \frac{\dot{B}}{2ABr} - \frac{\dot{A}}{2A^2 r} - \frac{\dot{B}^2}{4AB^2} - \frac{\dot{A}\dot{B}}{4A^2 B} \tag{24}$$

and

$$-8\pi T_4^4 = -\frac{\dot{A}}{A^2 r} - \frac{1}{r^2} + \frac{1}{Ar^2} \tag{25}$$

Let us put $u = -8\pi T_1^1$, $v = -8\pi T_2^2 = -8\pi T_3^3$ and $w = 8\pi T_4^4$. Then,

$$u = \frac{\dot{B}}{ABr} - \frac{1}{r^2} + \frac{1}{Ar^2} \tag{26}$$

$$v = \frac{\ddot{B}}{2AB} + \frac{\dot{B}}{2ABr} - \frac{\dot{A}}{2A^2 r} - \frac{\dot{B}^2}{4AB^2} - \frac{\dot{A}\dot{B}}{4A^2 B} \tag{27}$$

and

$$w = \frac{\dot{A}}{A^2 r} - \frac{1}{r^2} + \frac{1}{Ar^2} \tag{28}$$

Now, we have $A = (1 - \frac{2mr - Q^2}{r^2})^{-1}$. Therefore,

$$\dot{A} = A^2 \left(-\frac{2m}{r^2} + \frac{2Q^2}{r^3}\right) \tag{29}$$

Hence, using (29) equation (28) becomes $w = -\frac{2m}{r^3} + \frac{2Q^2}{r^4} + \frac{1}{r^2} - \frac{1}{Ar^2}$ which leads to

$$A = \left[1 - \frac{2mr - 2Q^2}{r^2} - \frac{r^2}{R_0{}^2}\right]^{-1} \tag{30}$$

where $w = \frac{1}{R_0^2}$ and R_0 depends on mass density ρ_0 (say).

It is to note that A in (2) and that in (30) are not same as some effects of medium have been included in the latter. Thus, Schwarzchild external solution of Einstein's equation is

$$ds^2 = -[1 - \frac{2mr - 2Q^2}{r^2} - \frac{r^2}{R_0^2}]^{-1} dr^2 - r^2 d\theta^2 - r^2 Sin^2\theta d\phi^2 + c^2[1 - \frac{2mr - 2Q^2}{r^2} - \frac{r^2}{R_0^2}] dt^2 \qquad (31)$$

It is seen from (1) and (31) that the line element in (31) is dependent upon certain characteristic of the medium.

Let us proceed to find out the line element for the static interior solution of Einstein's equation for the case of photon. Let us assume that photon is composed of a perfect homogeneous fluid of regular shape as predicted in [22]. Then under static conditions we obtain (26), (27) and (28) where the values of u, v and w would be $u = -8\pi p_0, v = -8\pi p_0$ and $w = 8\pi\rho_0$. Differentiating both sides of (26) with respect to r one gets

$$\frac{du}{dr} = \frac{\ddot{B}}{ABr} - \frac{\dot{B}}{ABr^2} - \frac{\dot{B}^2}{AB^2 r} - \frac{\dot{A}\dot{B}}{A^2 Br} + \frac{2}{r^3} - \frac{2}{Ar^3} - \frac{\dot{A}}{A^2 r^2} \qquad (32)$$

Now, since the photons have small mass in motion hence, they will exert small pressure which includes radiation pressure also. Let it be p_0. Then, we may write $T_1^1 = p_0 = T_2^2$.
Thus, from (26) and (27) we shall have $u = v$ which means that the total pressure at the surface of a sphere is zero. Thus, $v - u = 0$ i.e.

$$\frac{\ddot{B}}{2AB} - \frac{\dot{B}}{2ABr} - \frac{\dot{B}^2}{4AB^2} - \frac{\dot{A}\dot{B}}{4A^2 B} - \frac{\dot{A}}{2A^2 r} + \frac{1}{r^2} - \frac{1}{Ar^2} = 0 \qquad (33)$$

Multiplying (33) by $\frac{2}{r}$ and rearranging we get

$$\frac{\ddot{B}}{ABr} - \frac{\dot{B}^2}{AB^2 r} - \frac{\dot{A}\dot{B}}{A^2 Br} + \frac{2}{r^3} - \frac{2}{Ar^3} - \frac{1}{r}(\frac{\dot{B}}{ABr} + \frac{\dot{A}}{A^2 r}) + \frac{\dot{B}^2}{2AB^2 r} + \frac{\dot{A}\dot{B}}{2A^2 Br} = 0 \qquad (34)$$

Again, adding the corresponding sides of (26) and (28) we get

$$u + w = \frac{\dot{B}}{ABr} + \frac{\dot{A}}{A^2 r} \qquad (35)$$

Using (35) equation (34) becomes $\frac{du}{dr} + \frac{\dot{B}}{2B}(u + w) = 0$ which on integration gives

$$(u + w)B^{\frac{1}{2}} = s \qquad (36)$$

Here, w has been taken equal to $8\pi T_4^4 = 8\pi\rho_0$ which is constant and s is an integration constant.
Using (35) relation (36) becomes

$$(\frac{\dot{B}}{ABr} + \frac{\dot{A}}{A^2 r})B^{\frac{1}{2}} = s \qquad (37)$$

Again, on differentiation of A with respect to r we obtain from (30)

$$\dot{A} = -A^2\left(\frac{2m}{r^2} - \frac{4Q^2}{r^3} - \frac{2r}{R_0^2}\right) \tag{38}$$

Using (30) and (38) equation (37) becomes

$$\frac{1}{2}B^{-\frac{1}{2}}\dot{B} - \left(\frac{m}{r^2} - \frac{2Q^2}{r^3} - \frac{r}{R_0^2}\right)\left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{-1} B^{\frac{1}{2}} = \frac{s}{2} r \left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{-1} \tag{39}$$

To solve this equation for B let us put $B^{\frac{1}{2}} = \psi(r)$. Hence, $\psi'(r) = \frac{1}{2}B^{-\frac{1}{2}}\dot{B}$ and we get from (39):

$$\psi'(r) - \left(\frac{m}{r^2} - \frac{2Q^2}{r^3} - \frac{r}{R_0^2}\right)\left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{-1}\psi(r) = \frac{s}{2} r \left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{-1} \tag{40}$$

This equation is of the form

$$\psi' + P(r)\psi = Q(r) \tag{41}$$

where

$$P(r) = -\left(\frac{m}{r^2} - \frac{2Q^2}{r^3} - \frac{r}{R_0^2}\right)\left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{-1} \quad \text{and} \quad Q(r) = \frac{s}{2} r \left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{-1} \tag{42}$$

The solution of (42) would be

$$\psi \exp\left(\int P dr\right) = \int Q \exp(P dr) dr + \text{Integration constant} \tag{43}$$

Now,

$$\exp\left(\int P dr\right) = \left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{-\frac{1}{2}} \tag{44}$$

Hence,

$$\int Q\left\{\exp\left(\int P dr\right)\right\} dr = \frac{s}{2}\int \frac{r dr}{\left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{\frac{3}{2}}} \tag{45}$$

It is very difficult to integrate it in the present form. However, for a photon both m and Q are very small. Also, $r \ll R_0$. Hence we may write:

$\left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{-\frac{3}{2}} = 1 + \frac{3m}{r} - \frac{3Q^2}{r^2} + \frac{3}{2}\frac{r^2}{R_0^2}$. So, (45) could be written as

$$\int Q\left\{\exp\left(\int P dr\right)\right\} dr = \frac{s}{2}\left[\frac{r^2}{2} + 3mr - 3Q^2 \log r + \frac{3}{8}\frac{r^4}{R_0^2}\right] \tag{46}$$

Thus, (43) becomes

$$\psi = \left[\frac{s}{2}\left(\frac{r^2}{2} + 3mr - 3Q^2 \log r + \frac{3}{8}\frac{r^4}{R_0^2}\right) + s_1\right]\left(1 - \frac{2m}{r} + \frac{2Q^2}{r^2} - \frac{r^2}{R_0^2}\right)^{\frac{1}{2}} \tag{47}$$

Hence, similar to (1) the Schwarzchild's interior solution for a photon in motion would be

$$ds^2 = -(1-\frac{2m}{r}+\frac{2Q^2}{r^2}-\frac{r^2}{R_0^2})^{-1}dr^2 - r^2 d\theta^2 - r^2 Sin^2\theta d\phi^2 +$$
$$[\frac{s}{2}(\frac{r^2}{2}+3mr-3Q^2\log r+\frac{3}{8}\frac{r^4}{R_0^2})+s_1]^2(1-\frac{2m}{r}+\frac{2Q^2}{r^2}-\frac{r^2}{R_0^2})dt^2 \tag{48}$$

The solution arrived at (48) is similar to Schwarzchild's interior solution. Here A is similar to that of a charged particle of mass m but B is not the same as that of the particle mentioned (photon). Another point to note that c has been implicitly included within s and s_1 in (48). Now, to evaluate the integration constants s and s_1 one should use the appropriate boundary conditions applicable to the particular case of photon. Since, (48) is valid for a photon hence we may substitute $\frac{h\nu}{c^2}$ in place of m and obtain

$$ds^2 = -(1-\frac{2h\nu}{rc^2}+\frac{2Q^2}{r^2}-\frac{r^2}{R_0^2})^{-1}dr^2 - r^2 d\theta^2 - r^2 Sin^2\theta d\phi^2 +$$
$$[\frac{s}{2}(\frac{r^2}{2}+3\frac{h\nu}{c^2}r-3Q^2\log r+\frac{3}{8}\frac{r^4}{R_0^2})+s_1]^2(1-\frac{2h\nu}{rc^2}+\frac{2Q^2}{r^2}-\frac{r^2}{R_0^2})dt^2 \tag{49}$$

Let us study the case of empty space now. Here, $R_{\mu\nu}=0$, which means that in empty space outside the static sphere of perfect fluid all the components of the energy – momentum tensor are zero. So, we have $R_{11}=0=R_{22}=R_{33}=R_{44}$. Thus, from (9), (10), (11) and (12) we can write

$$\frac{\ddot{B}}{2B}-\frac{\dot{B}^2}{4B^2}-\frac{\dot{A}}{Ar}-\frac{\dot{A}\dot{B}}{4AB}=0 \tag{50}$$

$$\frac{1}{A}-\frac{r\dot{A}}{2A^2}+\frac{r\dot{B}}{2AB}-1=0 \tag{51}$$

$$\frac{\ddot{B}}{2A}-\frac{\dot{A}\dot{B}}{4A^2}+\frac{\dot{B}}{Ar}-\frac{\dot{B}^2}{4AB}=0 \tag{52}$$

Dividing (52) by $\frac{B}{A}$ we get

$$\frac{\ddot{B}}{2B}-\frac{\dot{A}\dot{B}}{4AB}+\frac{\dot{B}}{Br}-\frac{\dot{B}^2}{4B^2}=0 \tag{53}$$

Subtracting (50) from (53) and after rearrangement we obtain

$$\frac{\dot{B}}{B}+\frac{\dot{A}}{A}=0 \tag{54}$$

On integration it gives

$$AB = c_1 \tag{55}$$

Substituting (55) in (51) we get

$$\frac{\partial}{\partial r}(rB) = c_1 \tag{56}$$

Integrating we obtain

$$B = c_1 + \frac{c_2}{r} \tag{57}$$

Here, the constants of integration c_1 and c_2 in the solution for B are not arbitrary. Now,for convenience c_1 may taken to be c^2 as in (2) and $c_2 = -(2m + \frac{Q^2}{r})c^2$ keeping in mind the R-N metric so that Schwarzchild exterior solution in empty space would be

$$ds^2 = -(1 - \frac{2mr - Q^2}{r^2})^{-1} dr^2 - r^2 d\theta^2 - r^2 Sin^2\theta d\phi^2 + (1 - \frac{2mr - Q^2}{r^2})c^2 dt^2 \tag{58}$$

which is nothing but the R-N metric for a gravitating mass point m having charge Q considered in (1). Since, we are considering empty space so there is no term in (58) which signifies the effect of medium.

Conclusion

It is found from the above studies that Schwarzchild exterior solution of Einstein's equation is of a form similar to the forms of the line elements applicable to other cases. Here the form of the line element remains the same while the magnitudes of some co-efficients in the metric would change. Moreover, the metric for the internal solution is of a form quite different from that in general cases.

Again, the metric for the solution in vacuum is the same as that considered at the beginning. This is because of the fact that the parameters related to photon in vacuum does not change their forms so also their nature in this case.

References

[1] Saibal Roy (2007), Lorentz's Electromagnetic Mass: A Clue for Unification?, Apeiron, **14 (3),** p-155.

[2] C.V. Raman and S. Bhagavantam (1931), Experimental Proof of the Spin of the Photon, Indian Journal of Physics**, 6** , p- 353-366.

[3] M. Harwit (2003), Photon Orbital Angular Momentum in Astrophysics, The Astrophysical Journal, **597**, p- 1266-1270.

[4] D. Zu (2008), The classical structure model of single photon and classical point of view with regard to wave particle duality of photon, Progress in Electromagnetic Research Letters**, 1,**p-109-118.

[5] V.V. Kobychev and S.B. Popov (2005), Astronomy Letters, **31**, p- 147-151.

[6] C. Sivaram (1999), Constraints on the photon mass and charge and test of equivalence Principle from GRB 990123, Bull. Astr. Soc., India, **27**, p-627-630.

[7] V.V. Kobychev and S.B. Popov (2004), Constraints on the photon charge based on observation of extragalactic sources, arXiv: hep-ph/0411398 v1 30 Nov. 2004.

[8] L.B. Okun (2005), On the charge of the photon, arXiv: hep-ph/0505250v1 30 May 2005.

[9] L.B. Okun (2006), Photon: history,mass,charge, arXiv: hep-ph/0602036 v2 13 Feb 2006.

[10]C. Amsler, M. Doser, M. Antonelli, D. Asner, K. Babu, H. Baer, H. Band, R. Bornett et al (2008), Review of Particle Physics: Gauge and Higgs Bosons, Phys.Letters, **B 667.**

[11]M.A. Grado Caffaro, M.Grado Caffaro (2012), Gravitational Field from Photons, Optik—International Journal for Light and Electron Optics, **123 (9),** p- 814—815.

[12]W. B. Bonor (2009), The Gravitational Field of Photons, Gen. Relativ. Gravit., **41**, p-77—85.

[13]A. Einstein (1916), Ann. der Physik, **354 (7),** p- 769—822.

[14]A. Einstein (1954), Physics and Relativity (1936) in ideas and opinions, Crown, New York.

[15]R. Misra, M.C. Das, (2017), A Step Towards Gravitoelectromagnetisation, International Journal of Scientific Research in Science, Engineering and Technology, **6**, p -543-547

[16]D.F. Crawford (1975), Interaction of a photon with a gravitational field, Nature, **254**,p-313-314.

[17]Vadim Y. Kosyev, Electromagnetic Gravitational Interaction, Conference "Time Machine" Magazine, New Energy Technologies, May—June (2003).

[18]P.L. Saldanha and C.H. Monken (2011), Interaction between light and matter: a photon wave function approach, New Journal of Physics, **13**, 073015.

[19]O. Keller (2000), Space-time description of photon emission from a atom, Phys. Rev., **A 62**, 022111.

[20]E.T. Newman, E. Couch, K. Chinnapared, A. Extort, A Prakash (1965), Metric of a Rotating Charged Mass, J. Math. Phys., **6**, p-918.

[21]K.D. Krori (2010), Fundamentals of Special and General Relativity, PHI Learning Pvt. Ltd.,New Delhi, p-241.

[22]Satya Prakash (1995), Relativistic Mechanics, Pragati Prakashan, Meerut, p-375.