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Distribution function for plasma with RF heating from quasilinear Fokker-Planck equation

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Abstract

In the present paper we consider a magnetically confined plasma in the presence of ion cyclotron resonant heating (ICRH) in the minority scheme. The distribution function of the minority species was split into a steady state part corresponding to the time averaged part and the time harmonic perturbation corresponding to the rapid varying part driven by high-frequency field. The both distribution functions are supposed non-Maxwellian and written as Maxwellian multiplied by deformation functions, specific for each part. The goal of the paper is to study the radial profile of these deformation functions and their relation with power density absorbtion in the particular case of isotropic velocity space. The steady state part of the distribution function is obtained from the quasilinear Fokker-Planck equation. On the other hand, the kinetic equation is used to find the rapid varying part of the distribution function driven by radio-frequency field in terms of the steady state part and so the deformation functions from Maxwellian form. The both deformation functions and the diffusion coefficient are plotted for some specific situations using density and temperature profiles inspired by experiment data in JET. The obtained results show how the deformation functions depends both on the power density deposition profile and the value of power density deposition.

1 Introduction

Among of different range of RF heating, the ion cyclotron resonance heating (ICRH) represents a certain option for heating plasma in future devices as ITER and DEMO. Spotlight on particular aspects of the characteristics of ICRH will help us to better understand this process.

In the present paper we consider a magnetically confined plasma in the presence of ICRH in the minority scheme. The expression of the distribution function in the presence of ICRH for various plasma constituents are largely discussed in literature, see for example [1] - [8]. Often, for the facility of calculation, the equilibrium distribution function is taken in Maxwellian form, but actually the equilibrium distribution function is not a Maxwellian. The question is: how much the Maxwellian is deformed by heating process? To study this question, we start with an equilibrium distribution function written as a Maxwell distribution function multiplied by a deformation function. On the other hand, in the presence of ICRH the steady state part of the distribution function is written as a sum of two components: one is corresponding to the time averaged part, $\mathcal{F}^\alpha$, and the second one, g^α, to the rapid varying part driven by high-frequency field. These two components are in fact not independently of each other, but are respectively the zero order approximation and the first order approximation of the actual steady state distribution function. This aspect becomes clear in Sec II where the kinetic equation for plasma with ICRH is written in zero and first order approximation. Following the same steps in solving the system of equations as in [7], the spectral component of the rapid varying part of the distribution function g^α will be given in terms of the $\mathcal{F}^\alpha$.

The last one will be obtained in Sec III using the formalism of the quasilinear Fokker-Planck equation (see for example [5], [8]) applied in the case of isotropic velocity space. Then we derive the relation between the two first orders for distortion of the density distribution function (DDF) from a Maxwellian due to the ICRF heating in the simplified case of isotropic velocity space and use this to obtain the distribution function corresponding to the rapid varying scale.

In Sec IV, an estimation of the perpendicular electric field will be used in the differential equation which relates the deformation functions for $\mathcal{F}^\alpha$ and g^α in order to obtain an algebraic relation between the two deformation functions. In Sec V is given a numerical illustration of the deformation from Maxwellian distribution function using values of plasma and heating parameters similar to a specific shot in JET [3]. A testing radial profile of the density power absorption and relevant density and temperature profiles are used to plot the radial profiles for quasilinear diffusion coefficient, deformation function for $\mathcal{F}^\alpha$, and for the spectral component of deformation function for g^α. Their dependence on the power density absorption will be also shown.

The last section is dedicated to discussion and conclusions, where the behaviour of the quasilinear diffusion coefficient and the deformation functions are analysed.

2 The kinetic equation

We consider a non-ohmic multi-component plasma heated at the ion cyclotron resonance for minority species i. The kinetic equation for the particles of species α ($\alpha = e, D, i$) that applies in such conditions is written in conservative form as :

$$\partial_t f^{\ \alpha} + \nabla \cdot (\mathbf{v} f^{\ \alpha}) + L_0^\alpha f^{\ \alpha} = C^\alpha \left(f^{\ \alpha}, f^{\ \beta}\right) - L^\alpha \ f^{\ \alpha} \tag{1}$$

where the force operator has been split into two contributions: one, L_0^α, due to the equilibrium magnetic field (equilibrium electric field $\mathbf{E}_0 = 0$)

$$L_0^\alpha \equiv \frac{e_\alpha}{m_\alpha}\left(\frac{1}{c}\mathbf{v} \times \mathbf{B}_0\right) \cdot \frac{\partial}{\partial \mathbf{v}} \tag{2}$$

and the second one, L^α, arising from the RF electromagnetic field

$$L^\alpha \equiv \frac{e_\alpha}{m_\alpha}\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right) \cdot \frac{\partial}{\partial \mathbf{v}} \tag{3}$$

with e_α and m_α the charge and mass of the particles of species α. The non-canonical phase-space variables are the particle's position $\mathbf{x}$ and velocity $\mathbf{v}$. The operator C^α is the collision operator.

The distribution function is also split into two contributions: one, $\mathcal{F}^{\alpha}$, corresponding to the time averaged part (secular behavior) and the second one, g^{α}, corresponding to the rapid varying part (due to RF waves) of the distribution function:

$$f^{\alpha} = \mathcal{F}^{\alpha} + g^{\alpha} \tag{4}$$

In the absence of the RF heating, the kinetic equation is written as

$$\partial_t {F_0}^{\alpha} + \nabla \cdot \left(\mathbf{v} {F_0}^{\alpha}\right) + L_0^{\alpha} {F_0}^{\alpha} = C^{\alpha}\left({F_0}^{\alpha}, F_0^{\beta}\right) \tag{5}$$

with the **stationary** solution ${F_0}^{\alpha}$ approximated as Maxwellian $\mathcal{F}_M^{\alpha}$,

$$\mathcal{F}_M^{\alpha} = n_{0\alpha}\left(\pi \mathsf{v}_{th\alpha}^2\right)^{-3/2} \exp\left(-\frac{\mathsf{v}^2}{\mathsf{v}_{th\alpha}^2}\right) \tag{6}$$

where $\mathsf{v}_{th\alpha} = \sqrt{2T_{\alpha}/m_{\alpha}}$ is the thermal velocity of the species α and T_{α} , m_{α} (with $\alpha = e, i, m$), denote the temperature and the mass of the species α, respectively.

The solution for the pair $(\mathcal{F}^{\alpha}, g^{\alpha})$ results from (1), by series expansion after g^{α}. Then, the equation (1) reads as

$$\partial_t^{\alpha} \mathcal{F}^{\alpha} + \nabla \cdot [\mathbf{v}\mathcal{F}^{\alpha}] + L_0^{\alpha} \mathcal{F}^{\alpha} = 0 \tag{7}$$

in the zero order approximation, and

$$\partial_t g^{\alpha} + \nabla \cdot [\mathbf{v} g^{\alpha}] = -L^{\alpha}\ \mathcal{F}^{\alpha} \tag{8}$$

in first order approximation.

The equilibrium distribution function $\mathcal{F}^{\alpha}$ corresponding to the slow scale for the plasma with RF heating will be assumed of the form

$$\mathcal{F}^{\alpha}(\mathbf{r}, t) = \chi_1^{\alpha}(\mathbf{r}, t)\, \mathcal{F}_M^{\alpha} \tag{9}$$

With Fourier transform defined as

$$g^{\alpha}(\mathbf{r}, t) = \int d\mathbf{k} \int d\omega\ g_{\mathbf{k},\omega}^{\alpha} \exp\left(i\mathbf{k}\cdot\mathbf{r} - i\omega t\right) \tag{10}$$

the gyrophase averaging of the spectral component $g_{\mathbf{k},\omega}^{\alpha}$ will be written as deformed Maxwellian

$$\left\langle g_{\mathbf{k},\omega}^{\alpha} \right\rangle_{\varphi} = \frac{1}{2\pi} \int_0^{2\pi} d\varphi\ g_{\mathbf{k},\omega}^{\alpha} \equiv \chi_{2\ \mathbf{k},\omega}^{\alpha}\ \mathcal{F}_M^{\alpha} \tag{11}$$

Following the reasoning indicated in standard books (see for example [7]), the solution of equation (8) for $\left\langle g_{\mathbf{k},\omega}^{m} \right\rangle_{\varphi}$ in the case of minority species heating ($\alpha = m$) is obtained as

$$\left\langle g_{\mathbf{k},\omega}^{m} \right\rangle_{\varphi} = \frac{e_m}{m_m}\ \mathcal{F}_M^m \chi_1^m E_y \left(\frac{m_m \mathsf{v}_{\perp}}{T_m} - \frac{\partial \ln \chi_1}{\partial \mathsf{v}_{\perp}} - \frac{k_{\parallel}}{\omega}\beta_1\right) \frac{J_1(z)\, J_0(z)}{\omega - k_{\parallel}\mathsf{v}_{\parallel}} \tag{12}$$

where we assumed $Im \left\langle g_{\mathbf{k},\omega}^{m} \right\rangle_{\varphi} = 0$. From (11) and (12) results the equation which relates χ_1 and $\chi_{2\ \mathbf{k},\omega}^{m}$ in the minority heating scheme, which read as

$$\chi_{2\ \mathbf{k},\omega}^{m} = \frac{cE_y}{B}\left(\frac{m_m \mathsf{v}_{\perp}}{T_m}\chi_1^m - \frac{\partial \chi_1^m}{\partial \mathsf{v}_{\perp}} - \frac{k_{\parallel}}{\omega}\beta_1\right) \frac{J_1\left(\xi_{\perp}\mathsf{v}_{\perp}/\mathsf{v}_{thm}\right) J_0\left(\xi_{\perp}\mathsf{v}_{\perp}/\mathsf{v}_{thm}\right)}{\overline{\omega} - \xi_{\parallel}\mathsf{v}_{\parallel}/\mathsf{v}_{thm}} \tag{13}$$

where

$$\beta_1 = \mathsf{v}_\perp \frac{\partial \ln \chi_1}{\partial \mathsf{v}_\parallel} - \mathsf{v}_\parallel \frac{\partial \ln \chi_1}{\partial \mathsf{v}_\perp} \tag{14}$$

$$\xi_\parallel = \frac{k_\parallel \mathsf{v}_{thm}}{\Omega_{cm}} \quad , \quad \xi_\perp = \frac{k_\perp \mathsf{v}_{thm}}{\Omega_{cm}} \ , \quad \overline{\omega} = \frac{\omega}{\Omega_{cm}} \ , \ \Omega_{cm} = \frac{Z_m e B_0}{m_m c} \tag{15}$$

with Ω_{cm} denoting the *Larmor gyro-frequency* of the minority, Z_m the *charge number* of the minority and v_{thm} the *thermal velocity* of the minority species.

3 Solution for distribution functions in isotropic case

A solution for χ_1 can be obtained as solution of a quasilinear Fokker-Planck equation (QLFPE) where it is taken into account only the contribution due to the perpendicular component of the electric field, $E_\perp^+$, (which is concordant to the direction of rotation of the minority). The contributions due to the perpendicular component of the electric field, $E_\perp^-$ and the one due to the parallel component of the electric field, $E_\parallel$ are neglected.

In the case of minority heating it is sufficient to regard the heated ions as test particles colliding with a Maxwellian background plasma. Owing to the weak nonlinearity of the Fokker-Planck operator this is usually an acceptable approximation. If the applied power is not too large we can (as a first approximation) simplify the QLFPE further by neglecting the anisotropy which develops in the ion distribution function, $\mathcal{F}^m(w)$, which should satisfy the steady-state, Fokker-Planck equation [8] with the solution given as (9) with

$$\chi_1^m \equiv \exp\left\{ w^2 - 2\int_0^w \frac{B(u)\ u du}{A(u) + 2u \frac{\langle D(\xi_\perp u)\rangle_{ql}}{\mathsf{v}_{thm}^2 \nu^{m/e}}} \right\} \tag{16}$$

where

$$A(w) \equiv \Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{the}} w\right) + \frac{\nu^{m/i}}{\nu^{m/e}} \Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{thi}} w\right) \tag{17}$$

$$B(w) \equiv \frac{T_m}{T_e} \Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{the}} w\right) + \frac{\nu^{m/i}}{\nu^{m/e}} \frac{T_m}{T_i} \Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{thi}} w\right) \tag{18}$$

$$\Psi(u) \equiv \frac{1}{u^2}\left[erf(u) - \frac{2}{\sqrt{\pi}} u \exp\left(-u^2\right)\right] \tag{19}$$

and $\langle D(\xi_\perp w)\rangle_{ql}$ is the normalized quasilinear diffusion operator. The isotropic part of the normalized quasilinear diffusion operator for minority heating, see for example [8], reads as

$$\langle D(\xi_\perp w)\rangle_{ql} = \frac{D_0^m}{2} \int_{-1}^{+1} d\lambda\ \left(1 - \lambda^2\right)\ J_p^2\left(\xi_\perp w \sqrt{1-\lambda^2}\right) \tag{20}$$

with $\lambda = \mathsf{v}_\parallel/\mathsf{v}$ the pitch angle and D_0^m given as

$$D_0^m = \frac{P_{abs}^{lin}}{4 n_m m_m \int_0^\infty w^3\ J_0^2(\xi_\perp w) \exp(-w^2)\ dw} \tag{21}$$

Here, $J_0(x)$ indicates the Bessel functions of the first kind, P_{abs}^{lin} is the power absorbed per unit volume by the heated species, n_m is the number density of the minority species, respectively. The coefficient D_0^m (with dimension of $\nu_m \mathsf{v}_{thm}^2$) is proportional to the power available per ion of the heated species (here the minority species m). When $\langle D(\xi_\perp w)\rangle_{ql} \equiv 0$ and all species have the same temperature, $\mathcal{F}^m(w)$ reduces to unperturbed Maxwellian, $\mathcal{F}_M^m(w)$.

As can be seen from (17) and (18), the collisions between minority species (seen as a test particle) and the background particles (electrons and majority ion species) are taking into account through the quantities $A(r,w)$ and $B(r,w)$.

In the isotropic case, $\mathsf{v}_\perp = \mathsf{v}_\parallel = \mathsf{v}$ and $\beta_1 = 0$, equation (13) reads as

$$\chi_{2\ \mathbf{k},\omega}^{m,is} = \frac{e_m E_y}{m_m \Omega_{cm} \mathsf{v}_{thm}} \left(2w\chi_1^m - \frac{\partial \chi_1^m}{\partial w} \right) \frac{J_1(\xi_\perp w)\, J_0(\xi_\perp w)}{\overline{\omega} - \xi_\parallel w} \tag{22}$$

where χ_1^m is given by (16). At the limit, in the absence of RF heating $E_y = 0$ and $\chi_{2\ \mathbf{k},\omega}^{m,is} = 0$.

The component electric field E_y can be estimated from the power density absorption, (see [7] or [10]),

$$P_{abs}^{lin} = 2\frac{n_m e Z_m c}{B_0} \frac{R_0}{r\,|\sin\theta|} \left[1 + \frac{r}{R_0}\cos\theta \right]^3 E_y^2 \tag{23}$$

where θ is the poloidal angle (angle between minor radius r and the horizontal midplane), and for estimation was assumed $|\mathbf{E}_\perp|^2 \simeq 2E_y^2$.

Introducing expression (16) for χ_1^m in (22) and E_y from (23), the function $\chi_{2\ \mathbf{k},\omega}^{m,is}$ can be written as

$$\chi_{2\ \mathbf{k},\omega}^{m,is} = H_{\mathbf{k},\omega}^m\ \chi_1^m \tag{24}$$

where

$$H_{\mathbf{k},\omega}^m \equiv Y^m\ Z_{\mathbf{k},\omega}^m\ Q^m \tag{25}$$

and

$$Y^m = \frac{1}{\mathsf{v}_{thm} B} \left[\frac{2cB_0}{n_m e Z_m} \left(\frac{R_0}{R} \right)^3 \left(\frac{R}{R_0} - 1 \right) |\tan\theta| \right]^{1/2} \tag{26}$$

$$Z_{\mathbf{k},\omega}^m = \frac{wB(w) J_1(\xi_\perp w)\, J_0(\xi_\perp w)}{\overline{\omega} - \xi_\parallel w} \tag{27}$$

$$Q^m = \frac{\sqrt{P_{abs}^{lin}}}{A(w) + 2w\frac{<D(w)>_{ql}}{\upsilon^{m,e}\mathsf{v}_{thi}^2}} \tag{28}$$

The factor Y^m is a constant in the velocity space and is also independent on the power of rf wave, but the factor Q^m depends on the heating power both through $\sqrt{P_{abs}^{lin}}$ and $< D(w) >_{ql}$ (which also contains the heating power P_{abs}^{lin}). The factor $Z_{\mathbf{k},\omega}^m$ depends both on the particles velocity and the characteristics of the wave $(\mathbf{k},\omega)$.

4 Illustration of the deformation from Maxwellian of the minority distribution function due to ICRH

In order to illustrate the behaviour of χ_1^m and $\chi_{2\ \mathbf{k},\omega}^{m,is}$ let us consider the case of a low concentration of ions ^3He colliding with a thermal background plasma, composed by Hydrogen and electrons. This corresponds to the situation in some experiments at JET, see for example details about shot # 79352 given in [11]. As has been discussed there, at low concentration of ${}^3He$ (less than 3%) the heating correspond to the ICRH minority heating scheme and for a concentration $X({}^3He)$ higher than 5%, to a MC (mode conversion) scheme. In the present paper we consider ICRH minority heating scheme with $X({}^3He)$ about 3% of the density of the majority ion species.

The frequency of the applied RF wave is $\omega = 32\ MHz$ and the toroidal magnetic field at the magnetic axis is $B_0 = 3.41\ Tesla$. The perpendicular wave number $k_\perp$ is assumed here as $k_\perp \approx 0.5/\rho_{Lm}$ (so $\xi_\perp = k_\perp \mathsf{v}_{thm}/\Omega_{cm} = 0.5 < 1$), and the parallel wave number $k_\parallel$ is assumed as $k_\parallel = 27/R$.

The initial electron density radial profile and presumed density radial profile of minority ion species are given in fig.1 and fig.2 , respectively.

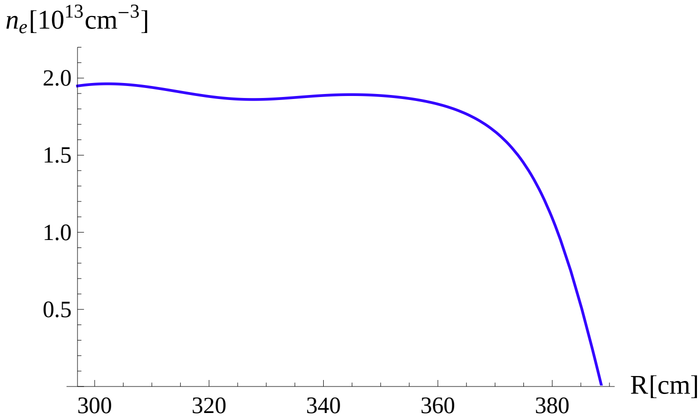

Figure 1. Electron density

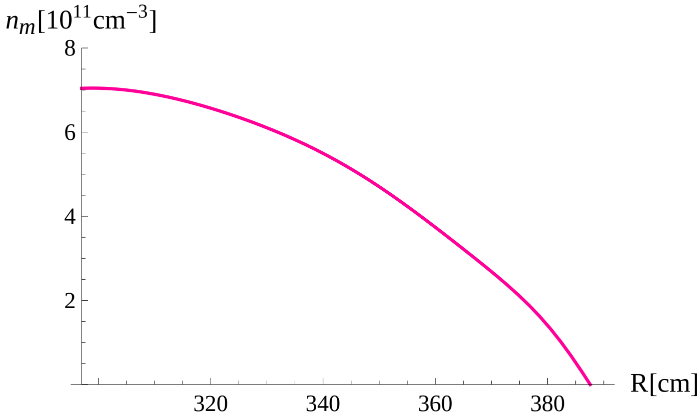

Figure 2. Radial variation of the minority ion species He^3 density.

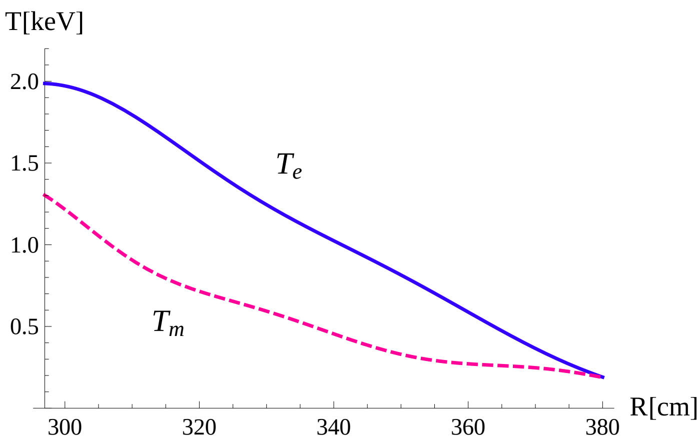

Figure 3. Electron temperature profile (solid blue line) and minority ions species temperature profile (dashed red line).

The initial temperatures (given in keV) of electrons and minority ion species, respectively, are plotted in fig.3. Before starting RF heating we assume $T_m = T_i$ (minority ion species have the same temperature as majority ion species).

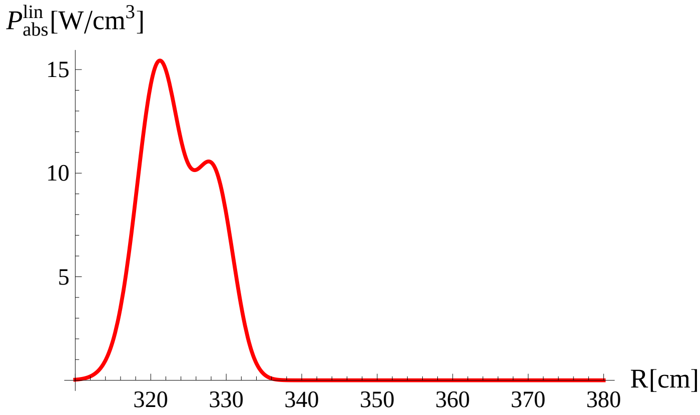

Figure 4. The density power profile.

Rigorously, the squared magnitude of the perpendicular electric field can be obtained by solving the dispersion equation. Here will be used an estimation of E_y given in [12] for typical JET case, where $|\mathbf{E}_\perp|^2 \sim 10 \div 50\ [kV^2/m^2]$ for 1 MW of coupled power. In this case, equation (23) leads to values $P_{abs}^{lin} \simeq 3 \div 16\ W/cm^3$ per 1 MW of coupled power. For the fundamental ($n = 1$) minority heating the cyclotron resonance layer for ^3He is located approximately at 0.24 m to the low field side of the plasma core, [11]. Given that, we choose the density power profile (as test form) sketched in fig.4.

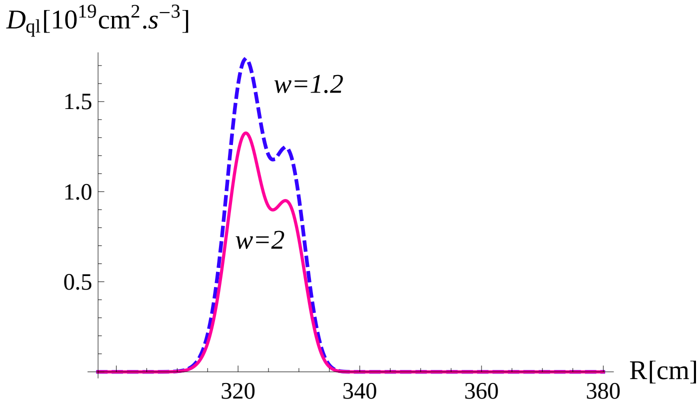

Figure 5. Quasilinear diffusion coefficient $\langle D\left(\xi_\perp w\right)\rangle_{ql}$ for $w = 1.2$ (dashed line) and $w = 2$ (solid line).

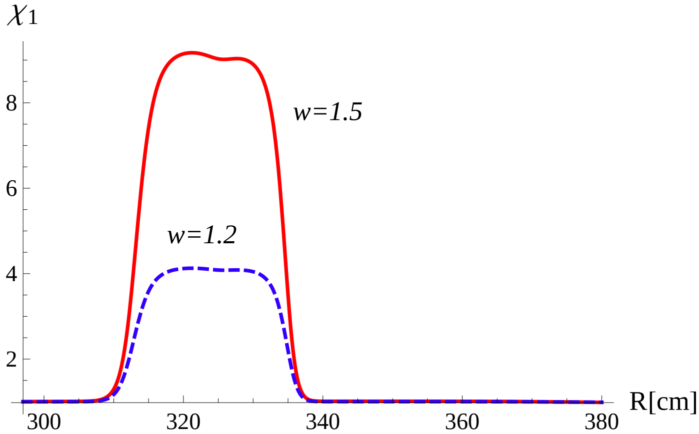

Figure 6. The function χ_1 as function of R at $w = 1.5$ (continuous line) and $w = 1.2$ (dashed line).

Then, the quasilinear diffusion coefficient $\langle D(\xi_\perp w)\rangle_{ql}$ as given by (20) is plotted for $w = 1.2$ and $w = 2$ in fig.5. We remark that the profile of $\langle D(\xi_\perp w)\rangle_{ql}$ follow the aspect of P_{abs}^{lin} and is continuously decreasing with increasing of w. The function $\chi_1^m(R, w)$, which is continuously increasing with w, has radial profiles like pedestals in the power deposition layer as is shown in fig.6, where the profiles are represented for two given values of the normalized velocity ($w = 1.5$ and $w = 1.2$) of the heated species.

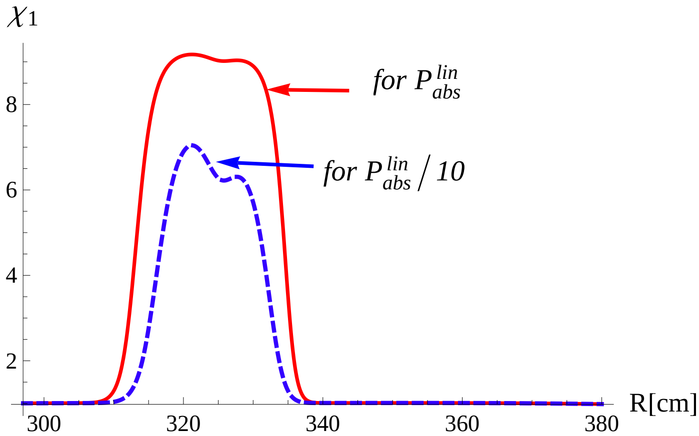

Figure 7. The function $\chi_1^m(R, w)$ plotted for $w = 1.5$ and two different values of the power density deposition, P_{abs}^{lin} (solid line) and $P_{1,\ abs}^{lin} = P_{abs}^{lin}/10$ (dot-dashed line).

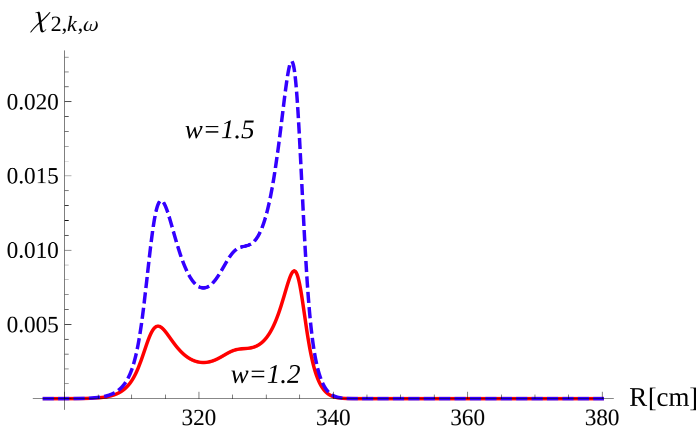

Figure 8. The function $\chi_{2\ \mathbf{k},\omega}^{m,is}$ plotted as function of R for $w = 1.5$ (dashed blue line) and $w = 1.2$ (solid red line).

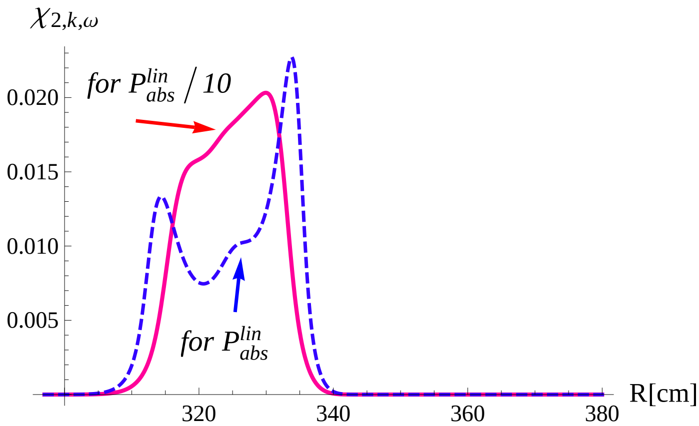

Figure 9. The function $\chi^{m,is}_{2\ \mathbf{k},\omega}(R)$ is plotted for $w = 1.5$ and two different values of the power density deposition, P^{lin}_{abs} (blue dashed line) and $P^{lin}_{1,\ abs} = P^{lin}_{abs}/10$ (red solid line).

We note that the radial variation of $\chi^{m}_{1}(R, w)$ is sensitive to the profile of the power density deposition, P^{lin}_{abs} , but less sensitive than $\langle D(\xi_\perp w)\rangle_{ql}$. For a power density deposition ten times smaller, $P^{lin}_{1,\ abs} = P^{lin}_{abs}/10$, (with P^{lin}_{abs} as given in fig.4, the quasilinear diffusion coefficient is now $\langle D_a(\xi_\perp w)\rangle_{ql} = \langle D(\xi_\perp w)\rangle_{ql}/10$. The values of the function $\chi^{m}_{1a}(R)$ which corresponds to $P^{lin}_{1,\ abs}$ is not so drastically reduced comparative with $\chi^{m}_{1}(R)$ which corresponds to P^{lin}_{abs}. Concerning the aspect of $\chi^{m}_{1a}(R)$, this is more sensitive to the aspect of the power deposition profile than $\chi^{m}_{1}(R)$ is, see fig.7.

The spectral function $\chi^{m,is}_{2\ \mathbf{k},\omega}$ given in (24) is plotted in fig.8 as function of R for two given values of the reduced velocity, $w = 1.2$ and $w = 1.5$.

5 Discussion and conclusions

The equilibrium distribution function for a given species in a non-ohmic multi-component plasma is distorted from Maxwellian when the plasma is heated using ion cyclotron resonance frequency waves. A particular case considered here (and very often met in the heating of plasma) is the using of the ion cyclotron resonance for minority species. The effect of the ICRH is observed on both the steady state part of the distribution function (described here by χ^{m}_{1}) and also to the rapid varying part driven by high-frequency field (described here by $\chi^{m,is}_{2\ \mathbf{k},\omega}$). So, χ^{m}_{1} and $\chi^{m,is}_{2\ \mathbf{k},\omega}$ are the "disturbances from the Maxwellian" form of the respective distribution functions.

The magnitude order and the shape of these "disturbances" was illustrated using numerical examples for the plasma and heating parameters similar to some experiments in JET. As can be seen from the plots, the radial profile of the quasilinear diffusion coefficient $\langle D(\xi_\perp w)\rangle_{ql}$ follows the aspect of P^{lin}_{abs} (see Fig.4 and Fig.5), but the radial profile of the function $\chi^{m}_{1}(R, w)$ has a pedestal in the power deposition layer (see fig.6). The aspect of the radial profile of $\chi^{m,is}_{2\ \mathbf{k},\omega}$ depends both on the profile and values of the power density deposition, as can be seen in fig.9. The difference between the profiles of $\chi^{m,is}_{2\ \mathbf{k},\omega}$ for P^{lin}_{abs} and $P^{lin}_{1,\ abs}$ is very drastically in the center region of the deposition layer. In the first case we remark that the maximum values of $\chi^{m,is}_{2\ \mathbf{k},\omega}$ are at the border of the resonance layer with the minimum values in the center. The relevance of results obtained in the present work are however limited by using the quasilinear theory and restricted to values of w so that $\xi_\perp w \lesssim 1$.

Finally, the gyrophase average of the distribution function reads now as

$$\left\langle f^{\ m,is} \right\rangle_{\varphi} = (1 + \Delta)\, \chi_1^m \mathcal{F}_M^m$$

$$\Delta = \int d\mathbf{k} \int d\omega\ H_{\mathbf{k},\omega}^m \exp\left(i\mathbf{k} \cdot \mathbf{r} - i\omega t\right)$$

where $H_{\mathbf{k},\omega}^m$ is given by (25)-(28). The quantity Δ contains the influence of the ICRF wave spectrum on the distribution function. We remark the presence of non-linear effects even in the case of isotropic velocity space.

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