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Test particles transport in two-dimensional turbulent plasma

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Abstract

The transport of charged test particles in two-dimensional turbulent plasma is studied using numerical simulations. We have calculated the mean squared spatial and velocity displacements and the particle trajectories for different values of the parameter alpha (see below). The plasma is considered in the magnetohydrodynamic approximation and the turbulence level is maintained stationary by the use of an external mechanical force. The self-consistent electromagnetic field generated by the plasma acts on the test particle via the Lorentz force while the interaction between the test particles and the plasma constituents is modeled as a drag force.

1 Introduction

The transport of charged test particles in magnetohydrodynamic plasma is influenced by two main effects. First we have the interaction between test particles and the plasma constituents trough collisions and second we have the influence of the electromagnetic field on the test particles by means of the Lorentz force. Assuming that the mass of the test particles is much higher then that of the plasma constituents, the interaction of the two can be seen as a drag force that acts on the test particles. The particle transport behavior will be due to a combination of plasma effects: magnetic trapping, electric accelerations and flow drag induced loss of kinetic energy. Also, for a turbulent state, the geometry of the plasma and the type of the existing structures will have a crucial role in the behavior of the particle transport. We are in the case of anomalous transport, defined as the process in which the transport coefficients depend on the degree of disorder of the medium. Since for a turbulent state of the plasma, the electromagnetic field that acts on the particles is generated by the turbulent movement of the plasma, we will refer to this as a turbulent field.

2 Theoretical model

We consider a turbulent one-fluid plasma described in the framework of incompressible magnetohydrodynamics (constant mass density ρ), for which the dimensionless equations are:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla P + R_e^{-1} \nabla^2 \mathbf{U} + \mathbf{J} \times \mathbf{B} + \mathbf{F} \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + R_m^{-1} \nabla^2 \mathbf{B} \tag{2}$$

where $\mathbf{U} = \mathbf{U}(\mathbf{r}, t)$ is the zero mean velocity field of the plasma. The magnetic field $\mathbf{B} = \mathbf{B}(\mathbf{r}, t)$ is generated by the plasma motion and $\mathbf{J}$, given as $\mathbf{J} = \nabla \times \mathbf{B}$, is the density current. The Reynolds number $R_e = v_A L_A / \nu$ and the magnetic Reynolds number $R_m = v_A L_A / \eta$ are taken to be equal, so that the magnetic Prandtl number ($\mathrm{Pr} = R_m / R_e$) is unity. The Alfvén velocity, defined as $v_A = B / \rho \mu_0$ is taken as the characteristic turbulent velocity scale and the L_A the characteristic large scale turbulent length. When desired, we force the velocity equation by means of an known, external force $\mathbf{F} = \mathbf{F}(\mathbf{r}, t)$. The equations (1) and (2) are joined by the incompressibility condition for the fluid ($\nabla \cdot \mathbf{U} = 0$) and the magnetic field zero-divergence ($\nabla \cdot \mathbf{B} = 0$). Because of the incompressibility condition, the total pressure (hydrodynamic pressure P plus the magnetic pressure), is not an independent variable and depends on $\mathbf{U}$ and $\mathbf{B}$. In the MHD formalism the magnetic field is found directly from the MHD equations, while the electric field $\mathbf{E}(\mathbf{r}, t)$ is determined algebraically from the generalized Ohm's law:

$$\mathbf{E} = -\mathbf{U} \times \mathbf{B} + \frac{v_A L_A}{R_m} \mathbf{J} \tag{3}$$

The non-relativistic equation of motion for a heavy charged particle (the particle mass m is much larger than that of the plasma constituents) due to the plasma is:

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \tag{4}$$

$$\frac{d\mathbf{v}}{dt} = \alpha[\mathbf{E}(\mathbf{r}) + \mathbf{v} \times \mathbf{B}(\mathbf{r})] - \chi |\mathbf{v} - \mathbf{U}(\mathbf{r})| \tag{5}$$

where the $\mathbf{r} = \mathbf{r}(t)$ represents the position of the particle and $\mathbf{v} = \mathbf{v}(t)$ its velocity. The parameter $\alpha = t_A / t_p$ represents the coupling between the particle (of charge q) and the electromagnetic fields generated by the plasma and represents the ratio of the MHD time scale to the characteristic time scale of the particle, given by the inverse of the Larmor frequency ($t_p = 2\pi / \Omega$). The coupling between the plasma flow and the particle is given by $\chi = t_A / \tau$ and represents the ratio between the Alfvén time ($t_A = L_A / v_A$) and the relaxation time τ (inverse of the collision frequency).

Under the influence of a strong external magnetic field ($\mathbf{B}_0$), the plasma will develop an anisotropic direction parallel to $\mathbf{B}_0$. The influence of the external magnetic field will cause the plasma to become quasi two-dimensional, as the plasma flow will decouple in the $\mathbf{B}_0$ direction. As a simplifying approximation we consider a two-dimensional plasma for which we initialize the velocity field as $(U_x, U_y, 0)$ and the magnetic field as $(0, 0, B_z)$. In this configuration, the test particles will be constrained in a 2D plane.

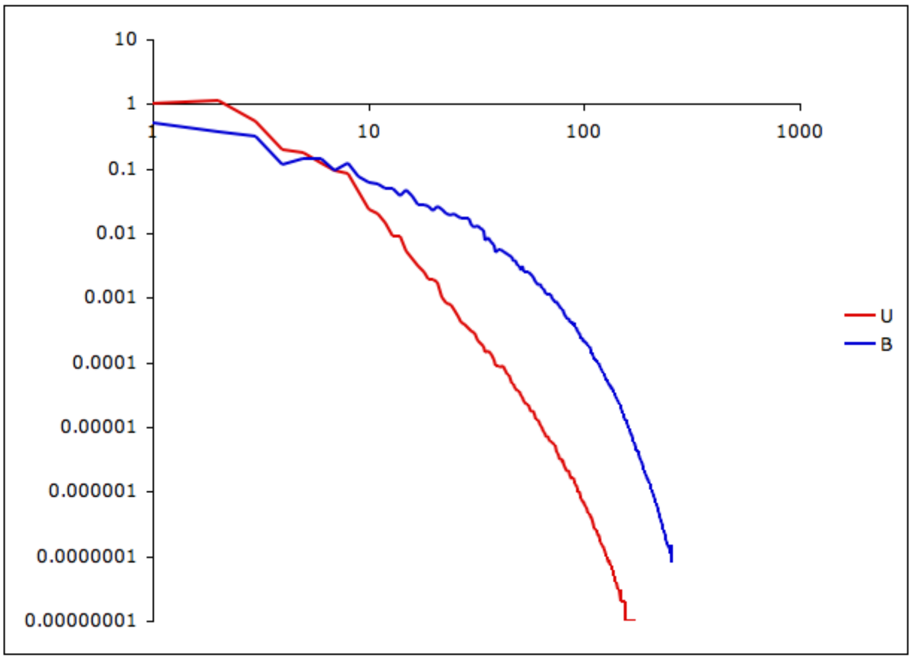

Figure 1: Velocity and magnetic spectra for the solved wave-number range

3 Numerical simulations

The MHD equations (1)-(2) are solved by a pseudo-spectral code [2] in a domain of length $\mathcal{L} = 2\pi$ with periodic boundaries conditions and a resolution of $N = 512$ modes in the X and Y direction. The time step is computed automatically by a CFL (Courant-Friedrichs-Lewy) criterion and the time advancement is based on a third order Runge-Kutta scheme. The nonlinear term is dealiased using a phase shift method. The external mechanical force $\mathbf{F}$ injects a constant level of energy ε in a shell $s_f = [7.5, 8.5]$. The choice of the boundaries for the forcing shell for two-dimensional turbulence ensures a number of 92 modes. This will ensure an isotropic forcing for our system. We evolve the MHD equations until we reach a statistically stationary state, for which the energy injected is equal to the energy lost due to dissipative effects. Throughout our simulations we check that the smallest turbulent scale (the Kolmogorov length l_K) and the largest turbulent scale (L_A) are properly solved by our simulation. Since we simulate two-dimensional turbulence, an inverse cascade is present, see Figure 1.

Once the plasma state has been generated, we use a frozen field approximation (Figure 2), and inject a number of 1000 particles. The particles are evolved using a $4th$ order symplectic solver and a cubic interpolation method is used to obtain the fields required at the particle position, [6]. We evolve the particles for a total time of about $T = 1000t_p$. In Figure 3 we plot three trajectory per coupling constant (α) to exemplify the qualitative behavior for $\chi = 0$. We see that increasing the coupling parameter between the fields and the particle will increase the trapping. Taking $\chi = 0$ is equivalent to ignoring the test particle collision effects with the plasma.

We will look at the time evolution of the mean squared displacement (MSD), defined for a direction i as: $\langle \delta r_i(t)^2 \rangle = \left\langle [r_i(t) - r_i(0)]^2 \right\rangle$. The angle brackets denote averaging over the test particle. By looking at the anomalous scaling law:

$$\left\langle (\delta r(t))^2 \right\rangle = At^{\mu} \tag{6}$$

and extracting μ we find the diffusion regimes. The time present in our figures will be

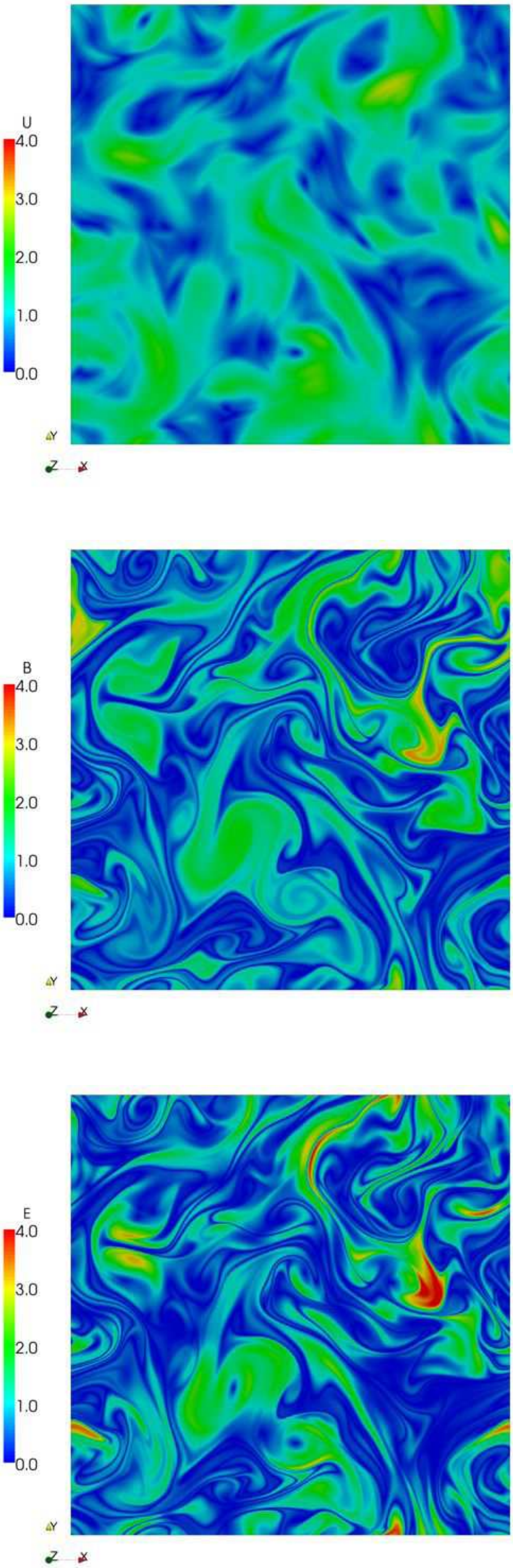

Figure 2: Plasma state. Top to bottom: the velocity, magnetic and electric fields

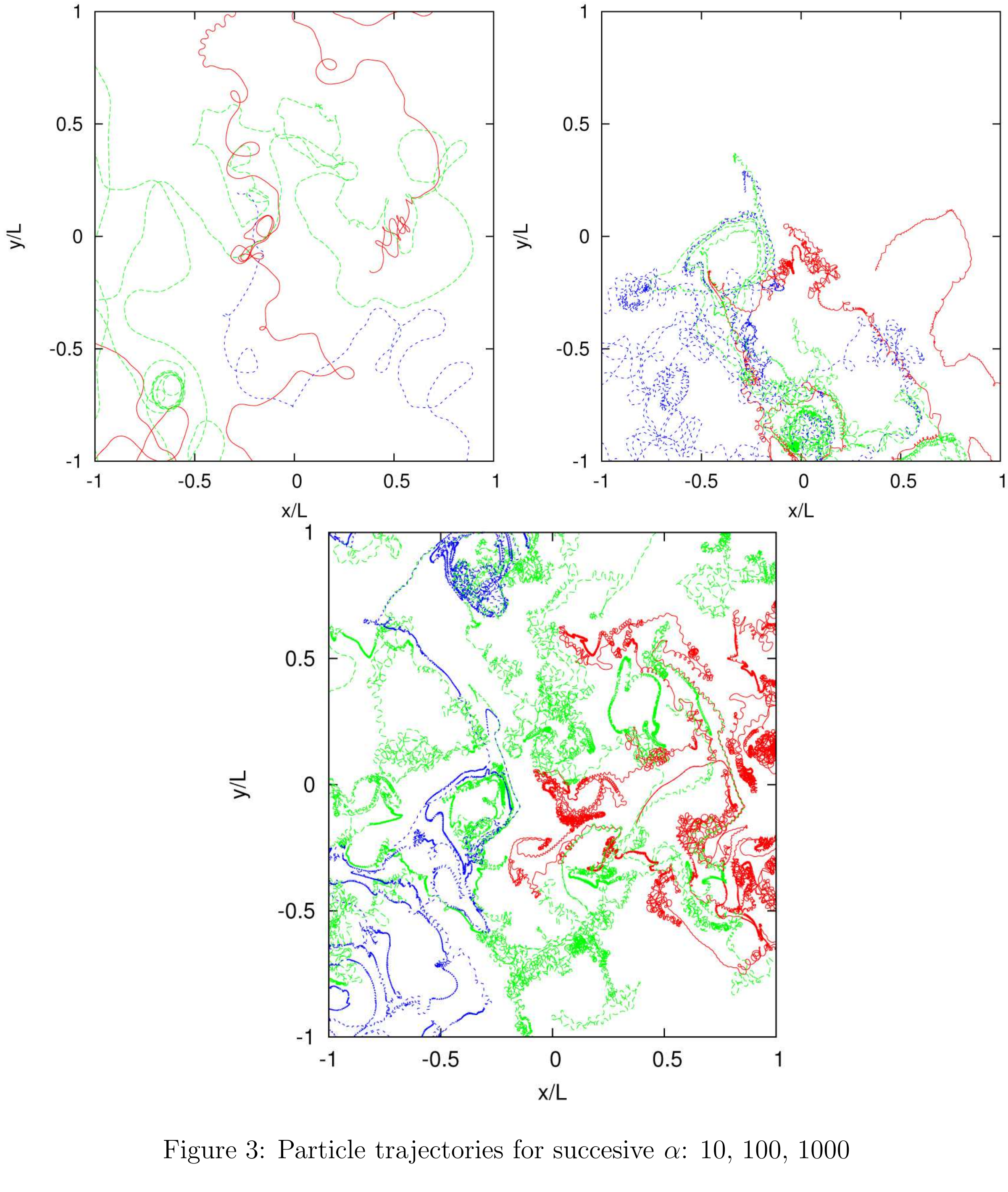

Figure 3: Particle trajectories for succesive α: 10, 100, 1000

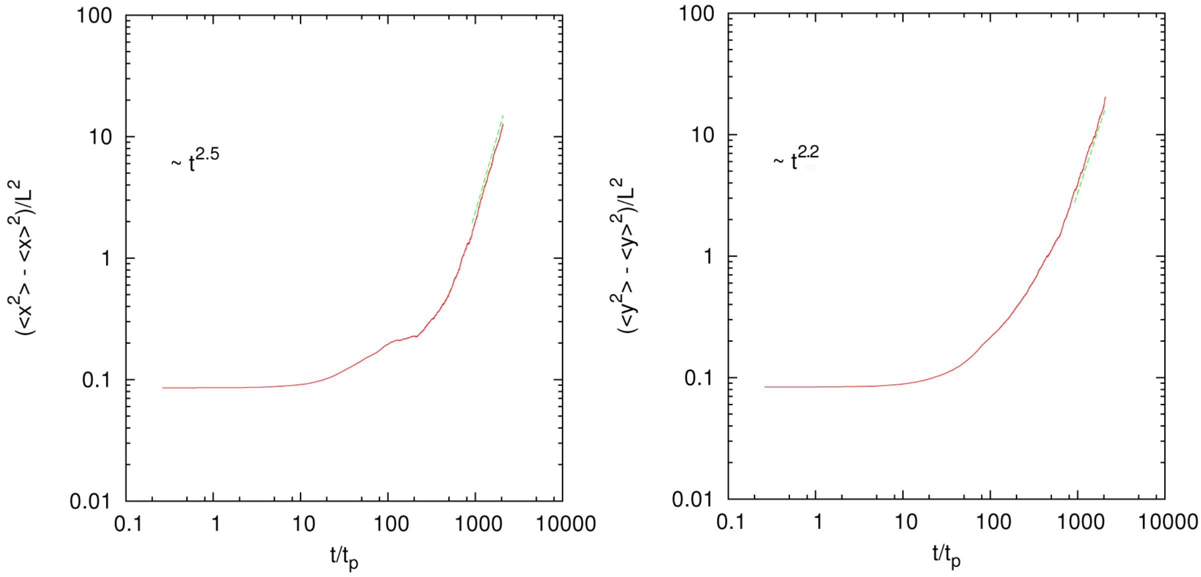

Figure 4: Spatial mean square displacement, $\alpha = 100$

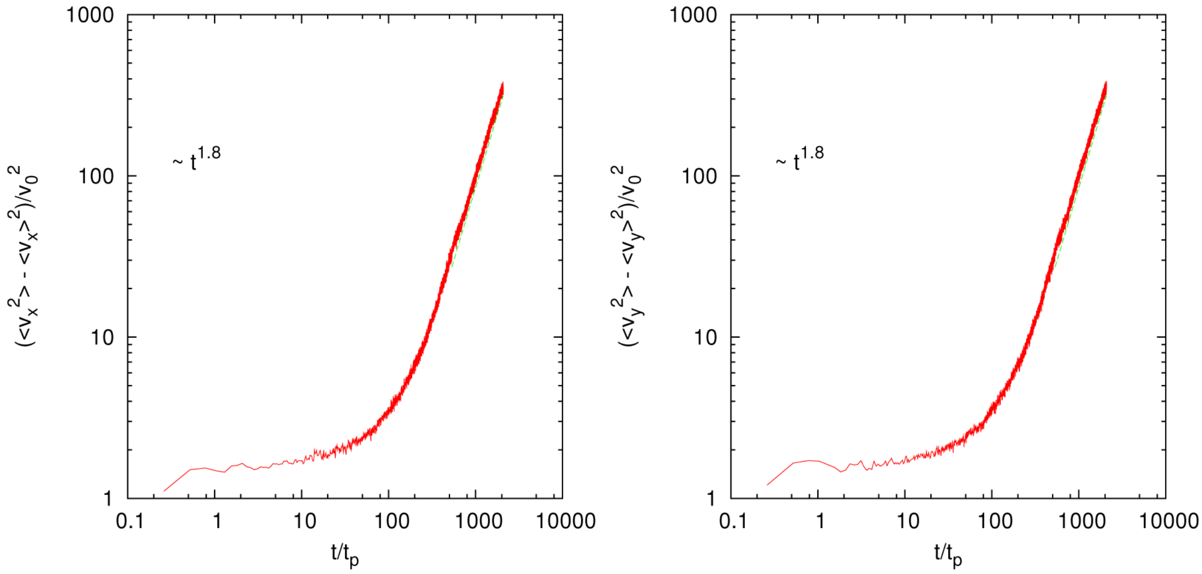

Figure 5: Velocity mean square displacement, $\alpha = 100$

measured in particle proper time $t_p = 2\pi/\Omega$). We will look at mean squared displacement evolution in time. In Figures 4 and 5 we represented the radial and poloidal spatial (Figure 4) and velocity (Figure 5) mean squared displacements.

4 Discussion

In the absence of collisions, the mean electric field (due to $\mathbf{U} \times \mathbf{B}$ contribution to $\mathbf{E}$) accelerates the particles. This gives a super-diffusive regime in the velocity space and a super-balistic regime for the position.

A large value for the coupling between the electromagnetic field and the particle (α) signifies a high trapping rate due to the magnetic field. Particles will tend to follow the magnetic field lines and the transport regime is given by the disorder (turbulence level) of the magnetic field lines. The electric field will accelerate the particles, increasing their kinetic energy and lowering the coupling α (particle velocity will increase compared to the field reference v_A). The presence of collision should decrease the level of energy gain in the unit of time and restore trapping lost due to the increase in particle velocity. We will continue our study for different collisional cases given by different values of the parameter χ.

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