Physics AUC, vol. 24, 34-55 (2014)

On a class of inconsistent deformations in BF models coupled to a dual formulation of linearized gravity in $D = 8$

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Abstract

In the context of constructing consistent cross-couplings in $D = 8$ between a topological BF model with a maximal field spectrum and a massless tensor field with the mixed symmetry $(5, 1)$ by means of the deformation of the solution to the classical master equation it is shown that all the terms from the first-order deformation depending on the ghost with maximum pure ghost number from the $(5, 1)$ sector can be eliminated for inconsistency reasons.

PACS: 11.10.Ef

1 Introduction

Topological BF field theories [1] are important in view of the fact that pure three-dimensional gravity is just a BF theory and, moreover, in higher dimensions general relativity and supergravity in Ashtekar formalism may also be formulated as topological BF theories with some extra constraints. On the other hand, tensor fields in "exotic" representations of the Lorentz group, characterized by a mixed Young symmetry type [2]–[5], held the attention lately on some important issues, like the dual formulation of field theories of spin two or higher [6]–[9], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [10], or the derivation of some exotic gravitational interactions [11, 12].

In this context the couplings between topological BF theories and certain dual formulations of linearized gravity is a topic of real interest [13]–[17]. One of the most efficient approaches to the problem of constructing interacting gauge field theories is based on the deformation of the solution to the classical master equation in the antifield-BRST setting [18, 19] by means of the local BRST cohomology [20]–[22]. The purpose of this paper is related to the construction of consistent cross-couplings in $D = 8$ between a topological BF model with a maximal field spectrum and a massless tensor field with the mixed symmetry $(5, 1)$. The free model describing a massless tensor field with the mixed symmetry $(5, 1)$ is known to be dual to linearized gravity exactly in $D = 8$. More precisely, under some standard hypotheses from field theory (such as spacetime locality), it is shown that a special class of terms from the first-order deformation involving the ghost of maximum pure ghost number belonging to the $(5, 1)$ sector can be eliminated via inconsistency arguments. This result is important in view of studying new interactions in gravity and supergravity theories whose field spectrum includes these two types of fields.

2 The free theory

The starting point is a free theory in $D = 8$, with the Lagrangian action written as the sum between the action of a topological BF model with a maximal field spectrum [two sorts of forms $(\overset{[m]}{A}, \overset{[m+1]}{B})_{m=\overline{0,3}}$] and that of a massless tensor field with the mixed symmetry $(5,1)$ $t_{\mu_1\ldots\mu_5|\alpha}$ [antisymmetric in its first 5 indices and fulfilling the identity $t_{[\mu_1\ldots\mu_5|\alpha]} \equiv 0$]

$$\begin{aligned} S^{\mathrm{L}}\left[\Phi^{\alpha_0}\right] = \int d^8x \Bigg[\sum_{m=0}^{3} \tfrac{1}{m+1} \overset{[m+1]}{B}{}^{\mu_1\ldots\mu_{m+1}} \partial_{[\mu_1} \overset{[m]}{A}{}_{\mu_2\ldots\mu_{m+1}]} \\ - \tfrac{1}{2\cdot 6!}\left(F_{\mu_1\ldots\mu_6|\alpha}F^{\mu_1\ldots\mu_6|\alpha} - 6F_{\mu_1\ldots\mu_5}F^{\mu_1\ldots\mu_5}\right)\Bigg] \\ \equiv S^{\mathrm{L,BF}}_{0,D=8}\Big[\overset{[m]}{A}{}_{\mu_1\ldots\mu_m}, \overset{[m+1]}{B}{}^{\mu_1\ldots\mu_{m+1}}\Big] + S^{\mathrm{L,t}}_{0,D=8}[t_{\mu_1\ldots\mu_5|\alpha}]. \end{aligned} \tag{1}$$

The overscript between brackets signifies the form degree [$\overset{[m]}{A}$ is a m-form and $\overset{[m+1]}{B}$ a $(m+1)$-form]. The notation $[\mu_1\mu_2\ldots\mu_n]$ means complete antisymmetry with respect to the indices between brackets, with the convention that the minimum number of terms is always used and the result is never divided by the number of terms. In this paper we work with the Minkowski metric of 'mostly plus' signature $\sigma_{\mu\nu} = \sigma^{\mu\nu} = \mathrm{diag}\,(-+\cdots+)$ and with the Levi–Civita symbol $\varepsilon^{\mu_1\cdots\mu_8}$ defined according to the convention $\varepsilon^{01\ldots7} = -\varepsilon_{01\ldots7} = -1$. The tensor $F_{\mu_1\ldots\mu_6|\alpha}$ from (1) displays the mixed symmetry $(6,1)$ and reads as

$$F_{\mu_1\ldots\mu_6|\alpha} = \partial_{[\mu_1}t_{\mu_2\ldots\mu_6]|\alpha}, \quad F_{\mu_1\ldots\mu_5} = \sigma^{\mu_6\alpha}F_{\mu_1\ldots\mu_5\mu_6|\alpha}. \tag{2}$$

The overall field spectrum, denoted by Φ^{α_0}, contains the two types of form fields from the BF sector together with the tensor field with the mixed symmetry $(5,1)$

$$\Phi^{\alpha_0} = \left\{\left(\overset{[m]}{A}{}_{\mu_1\ldots\mu_m}, \overset{[m+1]}{B}{}^{\mu_1\ldots\mu_{m+1}}\right)_{m=\overline{0,3}}, t_{\mu_1\ldots\mu_5|\alpha}\right\}. \tag{3}$$

Action (1) is invariant under a generating set of gauge symmetries of the form

$$\delta_{\Omega^{\alpha_1}}\overset{[0]}{A} = 0, \quad \delta_{\Omega^{\alpha_1}}\overset{[m]}{A}{}_{\mu_1\ldots\mu_m} = \partial_{[\mu_1}\overset{[m-1]}{\epsilon}{}_{(m,0)\mu_2\ldots\mu_m]}, \quad m=\overline{1,3}, \tag{4}$$

$$\delta_{\Omega^{\alpha_1}}\overset{[m+1]}{B}{}^{\mu_1\ldots\mu_{m+1}} = -(m+2)\,\partial_\rho\overset{[m+2]}{\xi}{}^{\rho\mu_1\ldots\mu_{m+1}}_{(m+1,0)}, \quad m=\overline{0,3}, \tag{5}$$

$$\delta_{\Omega^{\alpha_1}}t_{\mu_1\ldots\mu_5|\alpha} = \partial_{[\mu_1}\chi_{\mu_2\ldots\mu_5]|\alpha} + \partial_{[\mu_1}\theta_{\mu_2\ldots\mu_5]\alpha} + 5\partial_\alpha\theta_{\mu_1\ldots\mu_5}. \tag{6}$$

The gauge parameters were collectively denoted by Ω^{α_1}, with ϵ's and ξ's from the BF sector and χ together with θ from the $(5,1)$ sector. All these parameters are bosonic and completely antisymmetric [where applicable], excepting $\chi_{\mu_1\ldots\mu_4|\alpha}$, which displays the mixed symmetry $(4,1)$. Related to the BF gauge parameters, the overscript represents the form degree, while the other two lower indices between parentheses signify the form field to which a certain gauge parameter is associated with and respectively the reducibility level. The above gauge transformations are Abelian and off-shell, 6-order reducible. The free theory under study is a usual linear gauge theory [its field equations are linear in the fields], whose generating set of gauge transformations is 6-order reducible, such that we can define in a consistent manner its Cauchy order, which is found to be equal to 8.

According to the BRST method, we introduce the field/ghost and antifield spectra

$$\Pi^{A}\equiv\left\{\Phi^{\alpha_{0}},\{\eta^{\alpha_{l+1}}\}_{l=\overline{0,6}},\{\bar{\eta}^{\alpha_{l+1}}\}_{l=\overline{0,5}}\right\},\quad \Pi_{A}^{*}\equiv\left\{\Phi_{\alpha_{0}}^{*},\{\eta^{\alpha_{l+1}}\}_{l=\overline{0,6}},\left\{\bar{\eta}_{\alpha_{l+1}}^{*}\right\}_{l=\overline{0,5}}\right\},\tag{7}$$

where $\Phi_{\alpha_0}^*$ are the antifields corresponding to the original fields (3)

$$\Phi_{\alpha_{0}}^{*}=\left\{\left(\overset{[m]}{A}{}^{*\mu_{1}\ldots\mu_{m}},\overset{[m+1]}{B}{}^{*}_{\mu_{1}\ldots\mu_{m+1}}\right)_{m=\overline{0,3}},t^{*\mu_{1}\ldots\mu_{5}|\alpha}\right\}.\tag{8}$$

The ghosts and antifields associated with the gauge/reducibility parameters from the BF sector, $\{\eta^{\alpha_{l+1}}\}_{l=\overline{0,6}}$ and $\{\eta^{\alpha_{l+1}}\}_{l=\overline{0,6}}$, are organized as

$$\{\eta^{\alpha_{l+1}}\}_{l=\overline{0,2}}\equiv\left\{\left\{\overset{[m'-l-1]}{\eta}{}_{(m',l)}\right\}_{m'=\overline{l+1,3}},\left\{\overset{[m+l+2]}{C}{}_{(m+1,l)}\right\}_{m=\overline{0,3}}\right\}_{l=\overline{0,2}},\tag{9}$$

$$\{\eta^{\alpha_{l+1}}\}_{l=\overline{3,6}}\equiv\left\{\left\{\overset{[m''+l+2]}{C}{}_{(m''+1,l)}\right\}_{m''=\overline{0,6-l}}\right\}_{l=\overline{3,6}},\tag{10}$$

$$\{\eta_{\alpha_{l+1}}^{*}\}_{l=\overline{0,2}}\equiv\left\{\left\{\overset{[m'-l-1]}{\eta}{}^{*}_{(m',l)}\right\}_{m'=\overline{l+1,3}},\left\{\overset{[m+l+2]}{C}{}^{*}_{(m+1,l)}\right\}_{m=\overline{0,3}}\right\}_{l=\overline{0,2}},\tag{11}$$

$$\{\eta_{\alpha_{l+1}}^{*}\}_{l=\overline{3,6}}\equiv\left\{\left\{\overset{[m''+l+2]}{C}{}^{*}_{(m''+1,l)}\right\}_{m''=\overline{0,6-l}}\right\}_{l=\overline{3,6}},\tag{12}$$

while those from the $(5,1)$ sector [$\{\bar{\eta}^{\alpha_{l+1}}\}_{l=\overline{0,5}}$ and $\left\{\bar{\eta}_{\alpha_{l+1}}^{*}\right\}_{l=\overline{0,5}}$] are structured like

$$\{\bar{\eta}^{\alpha_{l+1}}\}_{l=\overline{0,3}}\equiv\left\{\mathcal{G}_{(l)\mu_{1}\ldots\mu_{4-l}|\alpha},\mathcal{C}_{(l)\mu_{1}\ldots\mu_{5-l}}\right\}_{l=\overline{0,3}},\quad\bar{\eta}^{\alpha_{5}}\equiv\mathcal{C}_{(4)\mu_{1}},\tag{13}$$

$$\left\{\bar{\eta}_{\alpha_{l+1}}^{*}\right\}_{l=\overline{0,3}}\equiv\left\{\mathcal{G}_{(l)}^{*\mu_{1}\ldots\mu_{4-l}|\alpha},\mathcal{C}_{(l)}^{*\mu_{1}\ldots\mu_{5-l}}\right\}_{l=\overline{0,3}},\quad\bar{\eta}_{\alpha_{5}}^{*}\equiv\mathcal{C}_{(4)}^{*\mu_{1}}.\tag{14}$$

The tensor fields $\mathcal{G}_{(l)\mu_1\ldots\mu_{4-l}|\alpha}$ and $\mathcal{G}_{(l)}^{*\mu_1\ldots\mu_{4-l}|\alpha}$ display the mixed symmetry $(4-l,1)$, while $\mathcal{C}_{(l)\mu_1\ldots\mu_{5-l}}$ and $\mathcal{C}_{(l)}^{*\mu_1\ldots\mu_{5-l}}$ are completely antisymmetric. In addition, $\mathcal{G}_{(3)\mu_1|\alpha}$ and $\mathcal{G}_{(3)}^{*\mu_1|\alpha}$ represent some symmetric tensors $[\mathcal{G}_{(3)[\mu_1|\alpha]}=\mathcal{G}_{(3)\mu_1|\alpha}-\mathcal{G}_{(3)\alpha|\mu_1}\equiv 0]$.

Since both the gauge generators and the reducibility functions are field-independent, it follows that the BRST differential, s, reduces to

$$s=\delta+\gamma,\tag{15}$$

where δ is the Koszul–Tate differential and γ the exterior longitudinal derivative. The Koszul–Tate differential is graded in terms of the antighost number [agh, $\mathrm{agh}(\delta)=-1$, $\mathrm{agh}(\gamma)=0$] and enforces a resolution of the algebra of smooth functions defined on the stationary surface of action (1), $C^{\infty}(\Sigma)$, $\Sigma:\delta S^{\mathrm{L}}/\delta\Phi^{\alpha_0}=0$. The exterior longitudinal derivative is in this case a true differential, graded in terms of the pure ghost number [pgh, $\mathrm{pgh}(\gamma)=1$, $\mathrm{pgh}(\delta)=0$] and correlated with the original gauge symmetries of the action via its cohomology in pure ghost number zero computed in $C^{\infty}(\Sigma)$, which is isomorphic to the algebra of physical observables for this free theory. The overall degree that grades the BRST complex is named ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = \mathrm{gh}(s) = 1$. These two degrees of the BRST generators from (7) are valued like

$$\mathrm{pgh}\left(\Phi^{\alpha_0}\right) = 0, \qquad \mathrm{agh}\left(\Phi^*_{\alpha_0}\right) = 1, \tag{16}$$

$$\mathrm{pgh}\left(\eta^{\alpha_{l+1}}\right) = l+1 = \mathrm{pgh}\left(\bar{\eta}^{\alpha_{l+1}}\right), \qquad \mathrm{agh}\left(\eta^*_{\alpha_{l+1}}\right) = l+2 = \mathrm{agh}\left(\bar{\eta}^*_{\alpha_{l+1}}\right), \tag{17}$$

$$\mathrm{pgh}(\Pi^*_A) = 0, \qquad \mathrm{agh}\left(\Pi^A\right) = 0, \tag{18}$$

while their Grassmann parities are given by

$$\varepsilon\left(\eta^{\alpha_{l+1}}\right) = (l+1) \mod 2 = \varepsilon\left(\bar{\eta}^{\alpha_{l+1}}\right), \tag{19}$$

$$\varepsilon(\Pi^*_A) = (\varepsilon(\Pi^A) + 1) \mod 2. \tag{20}$$

The actions of the differential δ on the above generators read as

$$\delta\Pi^A = 0 \Leftrightarrow (\delta\Phi^{\alpha_0} = 0, \quad \delta\eta^{\alpha_{l+1}} = 0 = \delta\bar{\eta}^{\alpha_{l+1}}, \quad l \geq 0), \tag{21}$$

$$\delta \overset{[m]}{A}{}^{*\mu_1\ldots\mu_m} = \partial_\rho \overset{[m+1]}{B}{}^{\rho\mu_1\ldots\mu_m}, \quad m = \overline{0,3}, \tag{22}$$

$$\delta \overset{[m-1]}{\eta}{}^{*\mu_1\ldots\mu_{m-1}}_{(m,0)} = -m\partial_\lambda \overset{[m]}{A}{}^{*\lambda\mu_1\ldots\mu_{m-1}}, \quad m = \overline{1,3}, \tag{23}$$

$$\delta \overset{[m-i-2]}{\eta}{}^{*\mu_1\ldots\mu_{m-i-2}}_{(m,i+1)} = (-)^i(m-i-1)\partial_\lambda \overset{[m-i-1]}{\eta}{}^{*\lambda\mu_1\ldots\mu_{m-i-2}}_{(m,i)}, \quad m = 2,3, \tag{24}$$

$$\delta \overset{[m+1]^*}{C}_{(m+1,-1)\mu_1\ldots\mu_{m+1}} = -\tfrac{1}{m+1}\partial_{[\mu_1} \overset{[m]}{A}_{\mu_2\ldots\mu_{m+1}]}, \quad m = \overline{0,3}, \tag{25}$$

$$\delta \overset{[m+l+2]^*}{C}_{(m+1,l)\mu_1\ldots\mu_{m+l+2}} = (-)^l\,\partial_{[\mu_1} \overset{[m+l+1]^*}{C}_{(m+1,l-1)\mu_2\ldots\mu_{m+l+2}]}, \quad m = \overline{0,3}, \tag{26}$$

[with $i = \overline{0, m-2}$ in (24) and respectively $l = \overline{0, 6-m}$ in (26)] and

$$\delta t^{*\mu_1\ldots\mu_5|\alpha} = -\frac{\delta S^{\mathrm{L,t}}_{0,D=8}}{\delta t_{\mu_1\ldots\mu_5|\alpha}} \equiv -\frac{1}{5!}T^{\mu_1\ldots\mu_5|\alpha}, \tag{27}$$

$$\delta\mathcal{G}^{*\mu_1\ldots\mu_4|\alpha}_{(0)} = -\partial_\lambda\left(5t^{*\lambda\mu_1\ldots\mu_4|\alpha} - t^{*\mu_1\ldots\mu_4\alpha|\lambda}\right), \ \delta\mathcal{C}^{*\mu_1\ldots\mu_5}_{(0)} = -6\partial_\alpha t^{*\mu_1\ldots\mu_5|\alpha}, \tag{28}$$

$$\delta\mathcal{G}^{*\mu_1\ldots\mu_{4-l}|\alpha}_{(l)} = (-)^{l+1}\partial_\lambda\Big((5-l)\mathcal{G}^{*\lambda\mu_1\ldots\mu_{4-l}|\alpha}_{(l-1)} + (-)^{5-l}\mathcal{G}^{*\mu_1\ldots\mu_{4-l}\alpha|\lambda}_{(l-1)}\Big), \ l = \overline{1,3}, \tag{29}$$

$$\delta\mathcal{C}^{*\mu_1\ldots\mu_{5-l}}_{(l)} = -(6-l)\partial_\alpha\Big(\mathcal{G}^{*\mu_1\ldots\mu_{5-l}|\alpha}_{(l-1)} + (-)^{6-l}\tfrac{5-l}{7-l}\mathcal{C}^{*\alpha\mu_1\ldots\mu_{5-l}}_{(l-1)}\Big), \ l = \overline{1,4}. \tag{30}$$

The tensor $T^{\mu_1\ldots\mu_5|\alpha}$ from (27) is involved in the field equations from the $(5,1)$ sector

$$\frac{\delta S^{\mathrm{L,t}}_{0,D=8}}{\delta t_{\mu_1\ldots\mu_5|\alpha}} = \frac{1}{5!}T^{\mu_1\ldots\mu_5|\alpha}, \tag{31}$$

displays the mixed symmetry $(5,1)$, and can be expressed in terms of $F^{\nu_1\ldots\nu_6|\alpha}$ from (2)

$$\begin{aligned} T^{\nu_1\ldots\nu_5|\alpha} &= \partial_\mu F^{\mu\nu_1\ldots\nu_5|\alpha} + \partial^\alpha F^{\nu_1\ldots\nu_5} - \sigma^{\alpha[\nu_1}\partial_\mu F^{\nu_2\ldots\nu_5]\mu} \\ &= \partial_\mu F^{\mu\nu_1\ldots\nu_5|\alpha} - \sigma^{\alpha[\nu_1}\partial_\mu F^{\nu_2\ldots\nu_5\mu]}. \end{aligned} \tag{32}$$

The actions of the differential γ on the above generators read as

$$\gamma\Pi^*_A = 0 \Leftrightarrow (\gamma\Phi^*_{\alpha_0} = \gamma\eta^*_{\alpha_{l+1}} = 0 = \gamma\bar{\eta}^*_{\alpha_{l+1}}, \quad l \geq 0), \tag{33}$$

$$\gamma \overset{[0]}{A} = 0, \quad \gamma \overset{[m]}{A}_{\mu_1 \ldots \mu_m} = \partial_{[\mu_1} \overset{[m-1]}{\eta}_{(m,0)\mu_2 \ldots \mu_m]}, \quad \gamma \overset{[0]}{\eta}_{(m,m-1)} = 0, \quad m = \overline{1,3}, \tag{34}$$

$$\gamma \overset{[m-i-1]}{\eta}_{(m,i)\mu_1 \ldots \mu_{m-i-1}} = \partial_{[\mu_1} \overset{[m-i]}{\eta}_{(m,i+1)\mu_2 \ldots \mu_{m-i-1}]}, \quad m = \overline{2,3}, \quad i = \overline{0, m-2}, \tag{35}$$

$$\gamma \overset{[D]}{C}{}^{\mu_1 \ldots \mu_8}_{(m+1,6-m)} = 0, \quad \gamma \overset{[m+l+1]}{C}{}^{\mu_1 \ldots \mu_{m+l+1}}_{(m+1,l-1)} = -(m+l+2)\, \partial_\rho \overset{[m+l+2]}{C}{}^{\rho\mu_1 \ldots \mu_{m+l+1}}_{(m+1,l)}, \tag{36}$$

[with $m = \overline{0,3}$ and $l = \overline{0, 6-m}$ in (36)] and

$$\gamma t_{\mu_1 \ldots \mu_5|\alpha} = \partial_{[\mu_1} \mathcal{G}_{(0)\mu_2 \ldots \mu_5]|\alpha} + \partial_{[\mu_1} \mathcal{C}_{(0)\mu_2 \ldots \mu_5]\alpha} + 5\partial_\alpha \mathcal{C}_{(0)\mu_1 \ldots \mu_5}, \tag{37}$$

$$\begin{aligned} \gamma \mathcal{G}_{(l)\mu_1 \ldots \mu_{4-l}|\alpha} &= \partial_{[\mu_1} \mathcal{G}_{(l+1)\mu_2 \ldots \mu_{4-l}]|\alpha} \\ &+ \partial_{[\mu_1} \mathcal{C}_{(l+1)\mu_2 \ldots \mu_{4-l}]\alpha} + (-)^{l+1}(4-l)\partial_\alpha \mathcal{C}_{(l+1)\mu_1 \ldots \mu_{4-l}}, \end{aligned} \tag{38}$$

$$\gamma \mathcal{C}_{(l)\mu_1 \ldots \mu_{5-l}} = \tfrac{4-l}{6-l} \partial_{[\mu_1} \mathcal{C}_{(l+1)\mu_2 \ldots \mu_{5-l}]}, \quad \gamma \mathcal{G}_{(3)\mu_1|\alpha} = \partial_{(\mu_1} \mathcal{C}_{(4)\alpha)}, \tag{39}$$

$$\gamma \mathcal{C}_{(3)\mu_1\mu_2} = \tfrac{1}{3} \partial_{[\mu_1} \mathcal{C}_{(4)\mu_2]}, \quad \gamma \mathcal{C}_{(4)\mu_1} = 0, \tag{40}$$

where $l = \overline{0,2}$ in (38) and also in the first relation from (39). Moreover, in (25) and (36) we used the supplementary conventions

$$\overset{[m+1]}{B}{}^{\mu_1 \ldots \mu_{m+1}} = \overset{[m+1]}{C}{}^{\mu_1 \ldots \mu_{m+1}}_{(m+1,-1)}, \quad m = \overline{0,3}, \tag{41}$$

$$\overset{[m+1]^*}{B}_{\mu_1 \ldots \mu_{m+1}} = \overset{[m+1]^*}{C}_{(m+1,-1)\mu_1 \ldots \mu_{m+1}}, \quad m = \overline{0,3}. \tag{42}$$

Without entering further technical details, in what follows we briefly address the general expression of the nontrivial representatives of $H(\gamma)$ [the cohomology of the exterior longitudinal derivative γ computed in the space of local functions]. An element a from $H(\gamma)$ simultaneously homogeneous with respect to the pure ghost and antighost numbers satisfies the conditions and the equation

$$\gamma a = 0, \quad \mathrm{pgh}(a) = l \geq 0, \quad \mathrm{agh}(a) = j \geq 0. \tag{43}$$

From the actions of γ on the BRST generators, given by definitions (33)–(40), it can be shown that $H(\gamma)$ in the space of local functions is generated by the antifields Π^*_A and their spacetime derivatives, by the quantities

$$\begin{aligned} \bar{F}_{\bar{\Delta}} = \Big\{ & \overset{[0]}{A} \equiv \varphi,\ \partial_{[\mu_1} \overset{[1]}{A}_{\mu_2]},\ \partial_{[\mu_1} \overset{[2]}{A}_{\mu_2\mu_3]},\ \partial_{[\mu_1} \overset{[3]}{A}_{\mu_2\mu_3\mu_4]},\ \partial_{\mu_1} \overset{[1]}{B}{}^{\mu_1},\ \partial_{\mu_1} \overset{[2]}{B}{}^{\mu_1\mu_2}, \\ & \partial_{\mu_1} \overset{[3]}{B}{}^{\mu_1\mu_2\mu_3},\ \partial_{\mu_1} \overset{[4]}{B}{}^{\mu_1 \ldots \mu_4},\ K_{\mu_1 \ldots \mu_6|\alpha\beta} \Big\} \end{aligned} \tag{44}$$

together with their spacetime derivatives, and also by the undifferentiated objects [all their spacetime derivatives are trivial in $H(\gamma)$]

$$\bar{\eta}^{\bar{\gamma}} = \left\{ \overset{[0]}{\eta}_{(1,0)},\ \overset{[0]}{\eta}_{(2,1)},\ \overset{[0]}{\eta}_{(3,2)},\ \overset{[0]}{\tilde{C}}_{(4,3)},\ \overset{[0]}{\tilde{C}}_{(3,4)},\ \overset{[0]}{\tilde{C}}_{(2,5)},\ \overset{[0]}{\tilde{C}}_{(1,6)},\ \mathcal{F}^{\mu_1 \ldots \mu_6},\ \mathcal{C}^\alpha_{(4)} \right\}. \tag{45}$$

In (44) the tensor $K_{\mu_1 \ldots \mu_6|\alpha\beta}$ defines the curvature tensor of the $(5,1)$ sector, given by

$$K_{\mu_1 \ldots \mu_6|\alpha\beta} = \partial_{[\mu_1} t_{\mu_2 \ldots \mu_6]|[\beta,\alpha]} \equiv \partial_\alpha \partial_{[\mu_1} t_{\mu_2 \ldots \mu_6]|\beta} - \partial_\beta \partial_{[\mu_1} t_{\mu_2 \ldots \mu_6]|\alpha}, \tag{46}$$

with $f_{,\mu} = \partial_\mu f$. This tensor displays the mixed symmetry $(6,2)$, so it is separately antisymmetric in its first six indices and respectively last two indices and satisfies the Bianchi identities

$$K_{[\mu_1 \ldots \mu_6|\alpha]\beta} \equiv 0, \quad \partial_{[\mu_1} K_{\mu_2 \ldots \mu_7]|\alpha\beta} \equiv 0, \quad K_{\mu_1 \ldots \mu_6|[\alpha\beta,\gamma]} \equiv 0. \tag{47}$$

We can also express (46) in terms of $F_{\mu_1\ldots\mu_6|\beta}$ as

$$K_{\mu_1\ldots\mu_6|\alpha\beta} = \partial_\alpha F_{\mu_1\ldots\mu_6|\beta} - \partial_\beta F_{\mu_1\ldots\mu_6|\alpha}. \tag{48}$$

The object $\mathcal{F}_{\mu_1\ldots\mu_6}$ from (45) is endowed with the pure ghost number equal to one and is constructed from the first-order derivatives of the ghost $\mathcal{C}_{(0)}$ belonging to the $(5,1)$ sector

$$\mathcal{F}_{\mu_1\ldots\mu_6} \equiv \partial_{[\mu_1}\mathcal{C}_{(0)\mu_2\ldots\mu_6]}. \tag{49}$$

In order to simplify the computations, it is more convenient to work with the Hodge duals of the various ghost fields associated with the BF sector [denoted by C], defined like

$$\overset{[8-p]}{\tilde{\omega}}{}^{\nu_1\ldots\nu_{8-p}} \equiv \frac{1}{p!}\varepsilon^{\nu_1\ldots\nu_{8-p}\mu_1\ldots\mu_p}\overset{[p]}{\omega}_{\mu_1\ldots\mu_p}, \quad p = \overline{0,8}. \tag{50}$$

According to (50), the objects $\left\{\overset{[0]}{\tilde{C}}_{(m+1,6-m)}\right\}_{m=\overline{0,3}}$ present in (45) are dual to the ghosts $\left\{\overset{[8]}{C}{}^{\mu_1\ldots\mu_8}_{(m+1,6-m)}\right\}_{m=\overline{0,3}}$.

In what follows we are mainly focused on the general representatives of $H(\gamma)$ of ghost number 0. Putting together the information exposed so far, we conclude that the expression of the most general, nontrivial element from $H(\gamma)$ of $\mathrm{gh}=0$ takes the form

$$a = \sum \alpha_J\left([\Pi^*_A],[\bar{F}_{\bar{\Delta}}]\right)e^J(\bar{\eta}^{\bar{\gamma}}), \tag{51}$$

with $\mathrm{agh}(\alpha_J) = J$ and $\mathrm{pgh}(e^J) = J$. By $e^J(\bar{\eta}^{\bar{\gamma}})$ we denoted the elements with pure ghost number equal to J of a basis in the space of polynomials in the objects (45). The notation $f([q])$ means that f depends on q and its spacetime derivatives up to a finite order. The quantities $\alpha_J\left([\Pi^*_A],[\bar{F}_{\bar{\Delta}}]\right)$ stand for the most general representatives of the cohomology $H(\gamma)$ in $\mathrm{pgh}=0$ [denoted by $H^0(\gamma)$]. They are called invariant 'polynomials' and are true polynomials in all the arguments excepting the undifferentiated scalar field φ, in terms of which they may be series. The cohomology $H^0(\gamma)$ is an algebra also known as the algebra of invariant 'polynomials'. A generic invariant polynomial decomposes along the antighost number into a finite number of (homogeneous) terms.

In the sequel we address the local cohomology of the Koszul–Tate differential in antighost number J, $H_J(\delta|d)$, and also the local cohomology of Koszul–Tate differential in antighost number J computed in the algebra of invariant 'polynomials', $H^{\mathrm{inv}}_J(\delta|d)$ [both in pure ghost number zero]. The model under study is a linear gauge theory of Cauchy order 8, so the general results from the literature [20]–[22] allow us to state that

$$H_J(\delta|d) = 0, \quad J > 8. \tag{52}$$

Moreover, it can be shown that if an invariant 'polynomial' α_J, with $\mathrm{agh}(\alpha_J) = J \geq 8$, is trivial in $H_J(\delta|d)$, then it can be taken to be trivial also in $H^{\mathrm{inv}}_J(\delta|d)$. This result together with (52) ensures that

$$H^{\mathrm{inv}}_J(\delta|d) = 0, \quad J > 8. \tag{53}$$

Using definitions (22)–(30), we can show that $H^{\mathrm{inv}}_J(\delta|d)$ and $H_J(\delta|d)$ are spanned by

$$H^{\mathrm{inv}}_8(\delta|d) \ : \ \{P_{\mu_1\ldots\mu_8}\}, \tag{54}$$

$$H_7^{\mathrm{inv}}(\delta|d) \quad : \quad \left\{P_{\mu_1\ldots\mu_7},\ \overset{[8]^*}{C}_{(2,5)\mu_1\ldots\mu_8}\right\}, \tag{55}$$

$$H_6^{\mathrm{inv}}(\delta|d) \quad : \quad \left\{P_{\mu_1\ldots\mu_6},\ \overset{[7]^*}{C}_{(2,4)\mu_1\ldots\mu_7},\ \overset{[8]^*}{C}_{(3,4)\mu_1\ldots\mu_8},\ \mathcal{C}^*_{(4)\alpha}\right\}, \tag{56}$$

$$H_5^{\mathrm{inv}}(\delta|d) \quad : \quad \left\{P_{\mu_1\ldots\mu_5},\ \overset{[6]^*}{C}_{(2,3)\mu_1\ldots\mu_6},\ \overset{[7]^*}{C}_{(3,3)\mu_1\ldots\mu_7},\ \overset{[8]^*}{C}_{(4,3)\mu_1\ldots\mu_8},\ \mathcal{G}'^{*\mu_1||\alpha}_{(3)}\right\}, \tag{57}$$

$$H_4^{\mathrm{inv}}(\delta|d) \quad : \quad \left\{P_{\mu_1\ldots\mu_4},\ \overset{[0]^*}{\eta}_{(3,2)},\ \overset{[5]^*}{C}_{(2,2)\mu_1\ldots\mu_5},\ \overset{[6]^*}{C}_{(3,2)\mu_1\ldots\mu_6},\ \overset{[7]^*}{C}_{(4,2)\mu_1\ldots\mu_7},\ \mathcal{G}'^{*\mu_1\mu_2||\alpha}_{(2)}\right\}, \tag{58}$$

$$\begin{aligned} H_3^{\mathrm{inv}}(\delta|d) \quad : \quad & \left\{P_{\mu_1\mu_2\mu_3},\ \overset{[0]^*}{\eta}_{(2,1)},\ \overset{[1]^{*\mu_1}}{\eta}_{(3,1)},\ \overset{[4]^*}{C}_{(2,1)\mu_1\ldots\mu_4},\ \overset{[5]^*}{C}_{(3,1)\mu_1\ldots\mu_5},\right. \\ & \left.\overset{[6]^*}{C}_{(4,1)\mu_1\ldots\mu_6},\ \mathcal{G}'^{*\mu_1\mu_2\mu_3||\alpha}_{(1)}\right\}, \end{aligned} \tag{59}$$

$$\begin{aligned} H_2^{\mathrm{inv}}(\delta|d) \quad : \quad & \left\{P_{\mu_1\mu_2},\ \overset{[0]^*}{\eta}_{(1,0)},\ \overset{[1]^{*\mu_1}}{\eta}_{(2,0)},\ \overset{[2]^{*\mu_1\mu_2}}{\eta}_{(3,2)},\ \overset{[3]^*}{C}_{(2,0)\mu_1\mu_2\mu_3},\ \overset{[4]^*}{C}_{(3,0)\mu_1\ldots\mu_4},\right. \\ & \left.\overset{[5]^*}{C}_{(4,0)\mu_1\ldots\mu_5},\ \mathcal{G}'^{*\mu_1\ldots\mu_4||\alpha}_{(0)}\right\}. \end{aligned} \tag{60}$$

The objects $\left(P_{\mu_1\ldots\mu_j}\right)_{j=\overline{2,8}}$ from (54)–(60) are invariant 'polynomials' in the antifields corresponding to the 1-form $\overset{[1]^{\mu_1}}{B}$ from the BF sector with the expression

$$\begin{aligned} P_{\mu_1\ldots\mu_j} = {} & \frac{dP}{d\varphi}\overset{[j]^*}{C}_{(1,j-2)\mu_1\ldots\mu_j} + \frac{d^jP}{d\varphi^j}\overset{[1]^*}{C}_{(1,-1)\mu_1}\cdots\overset{[1]^*}{C}_{(1,-1)\mu_j} \\ & + \sum_{l=2}^{j-1}\frac{d^lP}{d\varphi^l}\left(\sum_{\substack{1\leq i_1\leq i_2\leq\ldots\leq i_l\leq j-l+1 \\ i_1+i_2+\cdots+i_l=j}}\overset{[i_1]^*}{C}_{(1,i_1-2)[\mu_1\ldots\mu_{i_1}}\times\right. \\ & \left.\times\overset{[i_2]^*}{C}_{(1,i_2-2)\mu_{i_1+1}\ldots\mu_{i_1+i_2}}\cdots\overset{[i_l]^*}{C}_{(1,i_l-2)\mu_{i_1+i_2+\cdots+i_{l-1}+1}\ldots\mu_{i_1+i_2+\cdots+i_l}]}\right), \end{aligned} \tag{61}$$

where $P=P(\varphi)$ is an arbitrary, smooth function depending only on the undifferentiated scalar field φ. The notation (42) for $m=0$ was also employed. The 'polynomials' defined in (61) satisfy the properties

$$\delta P_{\mu_1\ldots\mu_j} = (-)^j\partial_{[\mu_1}P_{\mu_2\ldots\mu_j]},\quad j=\overline{1,8}, \tag{62}$$

$$P_{\mu_1} \equiv \frac{dP}{d\varphi}\overset{[1]^*}{C}_{(1,-1)\mu_1} \tag{63}$$

and can be written in dual notations like

$$\delta\tilde{P}^{\mu_1\ldots\mu_{8-j}} = (-)^j\partial_{\mu_{9-j}}\tilde{P}^{\mu_1\ldots\mu_{9-j}},\quad j=\overline{2,8}, \tag{64}$$

$$\tilde{P}^{\mu_1\ldots\mu_7} = \varepsilon^{\mu_1\ldots\mu_7\rho}\frac{dP}{d\varphi}\overset{[1]^*}{C}_{(1,-1)\rho}. \tag{65}$$

The representatives denoted by $\mathcal{G}'^*_{(l)}$ from $H(\delta|d)$ and $H_J^{\mathrm{inv}}(\delta|d)$, which appear starting with antighost number 5 [see (57)–(60)], are due to the $(5,1)$ sector and contain both kinds of antifields specific to this sector

$$\mathcal{G}'^{*\mu_1\ldots\mu_{4-l}||\alpha}_{(l)} \equiv \mathcal{G}^{*\mu_1\ldots\mu_{4-l}|\alpha}_{(l)} + \frac{1}{6-l}\mathcal{C}^{*\mu_1\ldots\mu_{4-l}\alpha}_{(l)},\quad l=\overline{0,3}. \tag{66}$$

They correspond [are canonically conjugated with respect to the antibracket] to the ghosts

$$\mathcal{G}'_{(l)\mu_1\ldots\mu_{4-l}||\alpha} \equiv \mathcal{G}_{(l)\mu_1\ldots\mu_{4-l}|\alpha} + (6-l)\,\mathcal{C}_{(l)\mu_1\ldots\mu_{4-l}\alpha}, \quad l = \overline{0,3}.$$

The double bar present in the expression of the primed ghosts/antifields signifies that these BRST generators are only antisymmetric with respect to the Lorentz indices placed before, but do not exhibit the mixed symmetry $(4-l,1)$ of the non-primed generators.

3 'Homogeneous' solutions

Next, we work in the context of investigating consistent cross-couplings in $D=8$ between a topological BF model with a maximal field spectrum and a massless tensor field with the mixed symmetry $(5,1)$ as a deformation problem of the solution to the classical master equation [18, 19] with the help of the local BRST cohomology [20]–[22]. We assume that the deformed solutions to the classical master equation satisfy some general hypotheses from field theory: analyticity in the coupling constant, spacetime locality, Lorentz covariance, and Poincaré invariance, combined with the preservation of the number of derivatives on each field. We strengthen the last hypothesis by requiring that the emerging cross-couplings are described at the Lagrangian level by interaction vertices containing at most one spacetime derivative of the fields at all orders in the coupling constant. Let $\bar{S}$ be the fully deformed solution to the master equation expanded according to a coupling constant g, $\bar{S} = S + gS_1 + g^2S_2 + \cdots$, so it is compelled to satisfy the equation $(\bar{S},\bar{S}) = 0$, where $(,)$ denotes the antibracket structure. In the above S, known as the zeroth order deformation, signifies the solution to the master equation for the free model under study, described by action (1) with the gauge symmetries (4)–(6), i.e. the solution to the equation $(S,S)=0$. We do not discuss here on either the antibracket structure of this model or the concrete form of S, which can be found in [13, 14], since they will not be needed in what follows. We only stress that in dual local notations, $S_1 = \int a\, d^8x$, the equation for the first-order deformation, $(S_1,S) \equiv sS_1 = 0$, is equivalently written as $sa = \partial_\mu j^\mu$, with j^μ a local current. Despite of satisfying all the working hypotheses, the non-integrated density of the first-order deformation, a, exhibits the supplementary properties of being bosonic and displaying the ghost number equal to zero. The previous equation shows that a defines a bosonic element of the local cohomology of the BRST differential s in ghost number zero, $H^0(s|d)$. Decomposing a as a sum between three components, $a = a^{\mathrm{BF}}_{D=8} + a^{\mathrm{t}}_{D=8} + a^{\mathrm{int}}_{D=8}$, where the first is responsible for the self-interactions among the BF fields, the second for those of the tensor field with the mixed symmetry $(5,1)$, and the third describes the cross-couplings between the two sectors, it follows that the first-order deformation equation becomes equivalent with three independent equations, one for each component, $sa^{\mathrm{BF}}_{D=8} = \partial_\mu w^\mu$, $sa^{\mathrm{t}}_{D=8} = \partial_\mu z^\mu$, and $sa^{\mathrm{int}}_{D=8} = \partial_\mu m^\mu$.

In what follows we investigate some particular solutions of the first-order deformation equation responsible for the cross-couplings between the BF fields and the $(5,1)$ tensor. In view of this we start from the equation

$$sa^{\mathrm{int}}_{D=8} = \partial_\mu m^\mu, \tag{67}$$

that has been shown in [14] to allow the decomposition [according to the antighost number]

$$a^{\mathrm{int}}_{D=8} = \sum_{i=0}^{8} a^{\mathrm{int}}_{i,D=8}, \quad \mathrm{agh}(a^{\mathrm{int}}_{i,D=8}) = i = \mathrm{pgh}(a^{\mathrm{int}}_{i,D=8}), \quad \varepsilon(a^{\mathrm{int}}_{i,D=8}) = 0. \tag{68}$$

In order to describe cross-couplings, every term from $a^{\rm int}_{D=8}$ has to contain al least one BRST generator from each sector and moreover to satisfy all the working hypotheses. Using decomposition (15), result (53), and expansion (68), it can be shown in a standard manner that equation (67) is equivalent to a tower of equations, ordered according to the decreasing values of the antighost number into

$$\gamma a^{\rm int}_{8,D=8} = 0, \tag{69}$$

$$\delta a^{\rm int}_{i,D=8} + \gamma a^{\rm int}_{i-1,D=8} = \partial_\mu m^\mu_{i-1}, \quad i = \overline{1,8}. \tag{70}$$

Equation (69) states that the piece of maximum antighost number from the first-order deformation responsible for cross-couplings, $a^{\rm int}_{8,D=8}$, is γ-invariant. Since we are interested in nontrivial interactions, we consider only nontrivial, bosonic elements of the cohomology $H(\gamma)$ with pgh $= 8$ and agh $= 8$. Following a standard procedure [for instance, see [14] and [15]–[17]] and taking into account formula (51), we obtain that the component of highest antighost number necessarily involves elements from the invariant local cohomology of the Koszul–Tate differential and takes the general expression

$$a^{\rm int}_{8,D=8} = \alpha_8 e^8(\bar{\eta}^{\bar{\gamma}}), \quad \alpha_8 \in H^{\rm inv}_8(\delta|d). \tag{71}$$

The other components, with lower, but strictly positive values of the antighost number, $\{a^{\rm int}_{i,D=8}\}_{i=\overline{1,7}}$, can be decomposed into

$$a^{\rm int}_{i,D=8} = a'^{\rm int}_{i,D=8} + \hat{a}^{\rm int}_{i,D=8} + \breve{a}^{\rm int}_{i,D=8}, \ i = \overline{1,8}, \quad \gamma a'^{\rm int}_{i,D=8} \neq 0, \quad \gamma \hat{a}^{\rm int}_{i,D=8} = 0 = \gamma \breve{a}^{\rm int}_{i,D=8}. \tag{72}$$

We collectively denoted by $a'^{\rm int}_{i,D=8}$ all the terms of antighost number i from the first-order deformation responsible for cross-couplings that are not γ-invariant and by $\hat{a}^{\rm int}_{i,D=8}$ and respectively $\breve{a}^{\rm int}_{i,D=8}$ the two distinct classes of possible γ-invariant terms. By abuse of terminology in what follows we call both these classes of γ-invariant elements 'homogeneous' solutions. The 'homogeneous' solutions $\hat{a}^{\rm int}_{i,D=8}$ ensure the consistency of $a^{\rm int}_{i,D=8}$ in antighost number $(i-1)$ [more precisely, are required by the existence of $a^{\rm int}_{i-1,D=8}$ as solution to the equation $\delta a^{\rm int}_{i,D=8} + \gamma a^{\rm int}_{i-1,D=8} = \partial_\mu m^\mu_{i-1}$ for $i = \overline{1,7}$]. The γ-closed pieces denoted by $\breve{a}^{\rm int}_{i,D=8}$ contain only the so-called "potentially independently consistent" 'homogeneous' solutions with antighost number i, introduced by having an imposed expression, of the type

$$\breve{a}^{\rm int}_{i,D=8} = \alpha_i e^i(\bar{\eta}^{\bar{\gamma}}), \quad \alpha_i \in H^{\rm inv}_i(\delta|d), \quad i = \overline{2,7}, \tag{73}$$

$$\breve{a}^{\rm int}_{1,D=8} = \alpha_1 \left([\Phi^*_{\alpha_0}], [\bar{F}_{\bar{\Delta}}]\right) e^1\left(\bar{\eta}^{\bar{\gamma}}\right), \quad {\rm agh}(\alpha_1) = 1, \tag{74}$$

where $\Phi^*_{\alpha_0}$ are the antifields corresponding to the original fields (8). The attribute "potentially independently consistent" accredited to the γ-closed contributions of the above form is due to the fact that independent $(\breve{a}^{\rm int}_{i,D=8})_{i=\overline{1,7}}$'s may generate in principle independent components of the first-order deformation describing cross-couplings, $a^{\rm int}_{D=8}$. We remark that expression (71) qualifies $a^{\rm int}_{8,D=8}$ also as a "potentially independently consistent" 'homogeneous' solution in antighost number equal to 8.

The declared purpose of this work is to construct all the "potentially independently consistent" 'homogeneous' solutions [in antighost numbers $i = \overline{1,8}$] and prove that certain classes are not consistent at the level of the first-order deformation and therefore can be safely eliminated. In view of this, we need to couple the representatives of $H^{\rm inv}_i(\delta|d)$ given in (54)–(60) to the basis elements $e^i(\bar{\eta}^{\bar{\gamma}})$ constructed from (45) [taking into account that each term has to effectively mix the two sectors and satisfy all the working hypotheses]. We recall that the pure ghost number of the γ-closed objects (45) are valued like

$${\rm pgh}(\overset{[0]}{\eta}_{(1,0)}) = {\rm pgh}(\mathcal{F}^{\mu_1\cdots\mu_6}) = 1, \quad {\rm pgh}(\overset{[0]}{\eta}_{(2,1)}) = 2, \tag{75}$$

$$\begin{aligned}
\mathrm{pgh}(\overset{[0]}{\eta}_{(3,2)}) &= 3, \quad \mathrm{pgh}(\overset{[0]}{\tilde{C}}_{(4,3)}) = 4, \quad \mathrm{pgh}(\overset{[0]}{\tilde{C}}_{(3,4)}) = 5 = \mathrm{pgh}(\mathcal{C}^{\alpha}_{(4)}), \qquad (76)\\
\mathrm{pgh}(\overset{[0]}{\tilde{C}}_{(2,5)}) &= 6, \quad \mathrm{pgh}(\overset{[0]}{\tilde{C}}_{(1,6)}) = 7, \qquad (77)
\end{aligned}$$

while their Grassmann parities read as

$$\varepsilon(\overset{[0]}{\eta}_{(2,1)}) = \varepsilon(\overset{[0]}{\tilde{C}}_{(2,5)}) = \varepsilon(\overset{[0]}{\tilde{C}}_{(4,3)}) = 0, \tag{78}$$

$$\varepsilon(\overset{[0]}{\eta}_{(1,0)}) = \varepsilon(\overset{[0]}{\eta}_{(3,2)}) = \varepsilon(\overset{[0]}{\tilde{C}}_{(3,4)}) = \varepsilon(\overset{[0]}{\tilde{C}}_{(1,6)}) = \varepsilon(\mathcal{F}^{\mu_1\ldots\mu_6}) = \varepsilon(\mathcal{C}^{\nu_1}_{(4)}) = 1. \tag{79}$$

In antighost number 8, we start from the general expression (71) and observe that formula (54) restricts $H^{\mathrm{inv}}_{8}(\delta|d)$ to involve only generators from the BF sector. Thus, in order to construct cross-couplings from $a^{\mathrm{int}}_{8,D=8}$ one should retain from the basis elements $e^{8}(\bar{\eta}^{\bar{\gamma}})$ only those objects containing at least one ghost from the $(5,1)$ sector, namely $\mathcal{C}^{\alpha}_{(4)}$ or $\mathcal{F}^{\nu_1\ldots\nu_6}$ [the latter defined in (49)]. Moreover, the hypothesis on the maximum derivative order of the interacting Lagrangian density to be equal to one limits the basis elements to contain at most one object of the type $\mathcal{F}^{\nu_1\ldots\nu_6}$. In view of these observations, we remain with the following eligible basis elements in pure ghost number equal to 8:

$$e^{8}_{\mathrm{eligible}}(\bar{\eta}^{\bar{\gamma}}) = \left\{e^{2}(\mathrm{BF})\mathcal{C}^{\alpha}_{(4)}\mathcal{F}^{\nu_1\ldots\nu_6},\ e^{3}(\mathrm{BF})\mathcal{C}^{\alpha}_{(4)},\ e^{7}(\mathrm{BF})\mathcal{F}^{\nu_1\ldots\nu_6}\right\}. \tag{80}$$

The notation $e^{i}(\mathrm{BF})$ from the above formula signifies the elements with pure ghost number i [where i takes here the values 2, 3, and respectively 7] of a basis in the space of polynomials in the objects from (45) pertaining only to the BF sector [the first seven ghosts]. Since all the elements $e^{i}(\mathrm{BF})$ are 0-forms, it follows that none of the terms from the right-hand side of (80) can be coupled in a covariant manner in $D=8$ to the generic representative from $H^{\mathrm{inv}}_{8}(\delta|d)$, given in (54), such that we are compelled to set

$$a^{\mathrm{int}}_{8,D=8} = 0. \tag{81}$$

This is a very important result since it shows that expansion (68) of the cross-coupling first-order deformation can be safely replaced by one that stops in antighost number 7.

In antighost number 7, we start from the "potentially independently consistent" 'homogeneous' solution (73) particularized to $i=7$. With the help of result (55) we notice that the general representatives of $H^{\mathrm{inv}}_{7}(\delta|d)$ entirely belong to the BF sector. Reprising exactly the same arguments as before, we determine the eligible basis elements as

$$e^{7}_{\mathrm{eligible}}(\bar{\eta}^{\bar{\gamma}}) = \left\{e^{1}(\mathrm{BF})\mathcal{C}^{\alpha}_{(4)}\mathcal{F}^{\nu_1\ldots\nu_6},\ e^{2}(\mathrm{BF})\mathcal{C}^{\alpha}_{(4)},\ e^{6}(\mathrm{BF})\mathcal{F}^{\nu_1\ldots\nu_6}\right\}, \tag{82}$$

$$e^{1}(\mathrm{BF}) = \{\overset{[0]}{\eta}_{(1,0)}\}, \quad e^{2}(\mathrm{BF}) = \{\overset{[0]}{\eta}_{(2,1)}\}. \tag{83}$$

Analyzing the structure of (55), it follows by covariance arguments that only the first two elements from (82) may be 'glued' to the former representative in $H^{\mathrm{inv}}_{7}(\delta|d)$

$$\breve{a}^{\mathrm{int}}_{7,D=8} = -P_{1\mu_1\ldots\mu_7}\mathcal{C}^{[\mu_1}_{(4)}\mathcal{F}^{\mu_2\ldots\mu_7]}\overset{[0]}{\eta}_{(1,0)} + \varepsilon^{\mu_1\ldots\mu_8}P_{2[\mu_1\ldots\mu_7}\overset{[0]}{\eta}_{(2,1)}\mathcal{C}_{(4)\mu_8]}. \tag{84}$$

In the above $P_{1\mu_1\ldots\mu_7}$ and $P_{2\mu_1\ldots\mu_7}$ read as in (61) for $j=7$, but they are constructed by means of two distinct smooth functions of φ, denoted by P_1 and respectively P_2.

Related to the "potentially independently consistent" 'homogeneous' solution (73) in $\mathrm{agh} = 6$, we remark from (56) that $H_6^{\mathrm{inv}}(\delta|d)$ contains for the first time a representative specific to the $(5,1)$ sector, namely the antifield $\mathcal{C}^{*}_{(4)\mu_1}$. Accordingly, the eligible basis in pure ghost number 6 will contain also elements strictly belonging to the BF sector

$$e^{6}_{\mathrm{eligible}}(\bar{\eta}^{\bar{\gamma}}) = \Big\{ \mathcal{C}^{\alpha}_{(4)} \mathcal{F}^{\nu_1 \ldots \nu_6},\ e^{1}(\mathrm{BF}) \mathcal{C}^{\alpha}_{(4)},\ e^{5}(\mathrm{BF}) \mathcal{F}^{\nu_1 \ldots \nu_6},\ e^{6}(\mathrm{BF}) \Big\}. \tag{85}$$

On the one hand, the cross-coupling requirement restricts the first three representatives from (56) to be connected only to the first three classes of elements from (85) and the last antifield from (56) to be linked just to the last three elements from (85). On the other hand, the derivative-order assumption forbids the coupling of $\mathcal{C}^{*}_{(4)\mu_1}$ to the elements form (85) depending on $\mathcal{F}^{\nu_1 \ldots \nu_6}$. [Indeed, using the actions of δ and γ on the BRST generators, it can be shown that, if consistent, such a term would generate in $a^{\mathrm{int}}_{1,D=8}$, via equations (70), a term that is simultaneously linear in $t^{*}_{\mu_1 \ldots \mu_5|\alpha}$ and $\mathcal{F}^{\nu_1 \ldots \nu_6}$ (and also depends on the undifferentiated BF fields in a way that is not important here). Further assuming that equation (70) is also consistent in $\mathrm{agh} = 0$ (for $i = 1$), the previously mentioned term would induce in the Lagrangian density $a^{\mathrm{int}}_{0,D=8}$ an interaction vertex generically written as $(\partial t)(\mathrm{BF\ fields})\partial t$, which clearly exceeds the maximum derivative order imposed (one).] Finally mixing in the Lorentz covariance, we remain with the following classes of possible terms in $\breve{a}^{\mathrm{int}}_{6,D=8}$:

$$\begin{aligned} \breve{a}^{\mathrm{int}}_{6,D=8} = \Big\{ & P_{\mu_1 \ldots \mu_6} \leftrightarrow e^{5}(\mathrm{BF}) \mathcal{F}^{\nu_1 \ldots \nu_6},\ \overset{[7]^{*}}{C}_{(2,4)\mu_1 \ldots \mu_7} \leftrightarrow \mathcal{C}^{\alpha}_{(4)} \mathcal{F}^{\nu_1 \ldots \nu_6}, \\ & \overset{[7]^{*}}{C}_{(2,4)\mu_1 \ldots \mu_7} \leftrightarrow e^{1}(\mathrm{BF}) \mathcal{C}^{\alpha}_{(4)},\ \mathcal{C}^{*}_{(4)\mu_1} \leftrightarrow e^{1}(\mathrm{BF}) \mathcal{C}^{\alpha}_{(4)} \Big\}. \end{aligned} \tag{86}$$

The former expression in (83) together with

$$e^{5}(\mathrm{BF}) = \Big\{ \overset{[0]}{\tilde{C}}_{(3,4)},\ \overset{[0]}{\eta}_{(1,0)} \overset{[0]}{\tilde{C}}_{(4,3)},\ \overset{[0]}{\eta}_{(2,1)} \overset{[0]}{\eta}_{(3,2)},\ \overset{[0]}{\eta}_{(1,0)} \Big(\overset{[0]}{\eta}_{(2,1)} \Big)^{2} \Big\}, \tag{87}$$

plus the Poincaré invariance requirement yield in the end

$$\begin{aligned} \breve{a}^{\mathrm{int}}_{6,D=8} = {} & \frac{5}{6} \Big(-8! V_{\mu_1 \ldots \mu_6} \overset{[0]}{\tilde{C}}_{(3,4)} + \frac{8!}{4} V_{1\mu_1 \ldots \mu_6} \overset{[0]}{\eta}_{(1,0)} \overset{[0]}{\tilde{C}}_{(4,3)} - U_{3\mu_1 \ldots \mu_6} \overset{[0]}{\eta}_{(2,1)} \overset{[0]}{\eta}_{(3,2)} \\ & - \frac{1}{2} U_{4\mu_1 \ldots \mu_6} \overset{[0]}{\eta}_{(1,0)} \overset{[0]}{\eta}_{(2,1)} \overset{[0]}{\eta}_{(2,1)} \Big) \mathcal{F}^{\mu_1 \ldots \mu_6} + q \varepsilon^{\mu_1 \ldots \mu_8} \overset{[7]^{*}}{C}_{(2,4)[\mu_1 \ldots \mu_7} \mathcal{C}_{(4)\mu_8]} \overset{[0]}{\eta}_{(1,0)} \\ & - q_1 \overset{[7]^{*}}{C}_{(2,4)\mu_1 \ldots \mu_7} \mathcal{C}^{[\mu_1}_{(4)} \mathcal{F}^{\mu_2 \ldots \mu_7]} + \frac{7!}{20} q_1' \mathcal{C}^{*\alpha}_{(4)} \mathcal{C}_{(4)\alpha} \overset{[0]}{\eta}_{(1,0)}. \end{aligned} \tag{88}$$

The objects $V_{\mu_1 \ldots \mu_6}$, $V_{1\mu_1 \ldots \mu_6}$, $U_{3\mu_1 \ldots \mu_6}$, and $U_{4\mu_1 \ldots \mu_6}$ read as in (61) with $j = 6$ and are constructed with the aid of the smooth functions $V(\varphi)$, $V_1(\varphi)$, $U_3(\varphi)$, and respectively $U_4(\varphi)$. By q, q_1, and q_1' we denoted three arbitrary real constants.

Regarding (73) in $i = 5$, we start from the eligible basis elements

$$e^{5}_{\mathrm{eligible}}(\bar{\eta}^{\bar{\gamma}}) = \Big\{ \mathcal{C}^{\alpha}_{(4)},\ e^{4}(\mathrm{BF}) \mathcal{F}^{\nu_1 \ldots \nu_6},\ e^{5}(\mathrm{BF}) \Big\} \tag{89}$$

and also from the general representatives of $H_5^{\mathrm{inv}}(\delta|d)$, established in (57). Invoking the cross-coupling and derivative-order hypotheses we are led to the observations that the first four representatives from (57) may be coupled only to the first two elements in (89) and the antifield $\mathcal{G}'^{*}_{(3)\mu_1||\beta}$ may be related exclusively to the last object from (89). Moreover, noticing that the components of the antifield $\mathcal{G}'^{*}_{(3)\mu_1||\beta}$ [defined by formula (66) for $j=5$] can be combined into a symmetric tensor and respectively a 0-form

$$\mathcal{G}'^{*}_{(3)\mu_1||\beta} \rightarrow \left\{ \mathcal{G}'^{*}_{(3)[\mu_1||\beta]} \sim \mathcal{C}^{*}_{(3)\mu_1\beta},\ \mathcal{G}'^{*}_{(3)} \equiv \mathcal{G}'^{*}_{(3)\mu_1||\beta}\sigma^{\mu_1\beta} = \mathcal{G}^{*}_{(3)} \right\} \tag{90}$$

and taking into account the Lorentz covariance, we are led to three classes of terms

$$\breve{a}^{\mathrm{int}}_{5,D=8} = \left\{ \overset{[6]}{C}{}^{*}_{(2,3)\mu_1\ldots\mu_6} \leftrightarrow e^4(\mathrm{BF})\mathcal{F}^{\nu_1\ldots\nu_6},\ \overset{[7]}{C}{}^{*}_{(3,3)\mu_1\ldots\mu_7} \leftrightarrow \mathcal{C}^{\alpha}_{(4)},\ \mathcal{G}^{*}_{(3)} \leftrightarrow e^5(\mathrm{BF}) \right\}. \tag{91}$$

By means of expression (87) and

$$e^4(\mathrm{BF}) = \left\{ \overset{[0]}{\tilde{C}}_{(4,3)},\ \overset{[0]}{\eta}_{(1,0)}\overset{[0]}{\eta}_{(3,2)},\ \left(\overset{[0]}{\eta}_{(2,1)}\right)^2 \right\} \tag{92}$$

and requiring the Poincaré invariance of the first-order deformation, we eventually obtain

$$\begin{aligned}
\breve{a}^{\mathrm{int}}_{5,D=8} = {} & \frac{5}{6}\overset{[6]}{C}{}^{*}_{(2,3)\mu_1\ldots\mu_6}\left(\frac{8!}{2}v_1\overset{[0]}{\tilde{C}}_{(4,3)} + \bar{q}\overset{[0]}{\eta}_{(1,0)}\overset{[0]}{\eta}_{(3,2)} - u_4\overset{[0]}{\eta}_{(2,1)}\overset{[0]}{\eta}_{(2,1)}\right)\mathcal{F}^{\mu_1\ldots\mu_6} \\
& - 3q_2\varepsilon^{\mu_1\ldots\mu_8}\overset{[7]}{C}{}^{*}_{(3,3)[\mu_1\ldots\mu_7}\mathcal{C}_{(4)\mu_8]} + 5!\mathcal{G}^{*}_{(3)}\left(-\frac{8!}{2}v\overset{[0]}{\tilde{C}}_{(3,4)} + 7!v'_1\overset{[0]}{\eta}_{(1,0)}\overset{[0]}{\tilde{C}}_{(4,3)}\right. \\
& \left. - \frac{1}{2}u_3\overset{[0]}{\eta}_{(2,1)}\overset{[0]}{\eta}_{(3,2)} - \frac{1}{4}u'_4\overset{[0]}{\eta}_{(1,0)}\overset{[0]}{\eta}_{(2,1)}\overset{[0]}{\eta}_{(2,1)}\right),
\end{aligned} \tag{93}$$

with v, v_1, v'_1, u_3, u_4, u'_4, $\bar{q}$, and q_2 some arbitrary real constants.

We pass to constructing the "potentially independently consistent" 'homogeneous' solutions in antighost number 4, of the form (73) with $i=4$. This is the first time when the basis elements cannot contain the ghost $\mathcal{C}^{\alpha}_{(4)}$ from the $(5,1)$ sector since its pure ghost number is equal to 5; otherwise they are spanned by two types of objects, namely

$$e^4_{\mathrm{eligible}}(\bar{\eta}^{\bar{\gamma}}) = \left\{ e^3(\mathrm{BF})\mathcal{F}^{\nu_1\ldots\nu_6},\ e^4(\mathrm{BF}) \right\}. \tag{94}$$

Inspecting the structure of $H_4^{\mathrm{inv}}(\delta|d)$ [see formula (58)], it follows that its first five representatives may be correlated only to the former class of elements from (94), while the antifield $\mathcal{G}'^{*}_{(2)\mu_1\mu_2||\beta}$ solely to the latter class. In addition, the manipulation of the components of the antifield $\mathcal{G}'^{*}_{(2)\mu_1\mu_2||\beta}$, defined in (66) for $j=4$, yields two tensors

$$\mathcal{G}'^{*}_{(2)\mu_1\mu_2||\beta} \rightarrow \left\{ \mathcal{G}'^{*}_{(2)[\mu_1\mu_2||\beta]} \sim \mathcal{C}^{*}_{(2)\mu_1\mu_2\beta},\ \mathcal{G}'^{*}_{(2)\mu_1} \equiv \mathcal{G}'^{*}_{(2)\mu_1\mu_2||\beta}\sigma^{\mu_2\beta} = \mathcal{G}^{*}_{(2)\mu_1} \right\}. \tag{95}$$

Invoking the Lorentz covariance, we find that $\breve{a}^{\mathrm{int}}_{4,D=8}$ reduces to a single object

$$\breve{a}^{\mathrm{int}}_{4,D=8} = \left\{ \overset{[6]}{C}{}^{*}_{(3,2)\mu_1\ldots\mu_6} \leftrightarrow e^3(\mathrm{BF})\mathcal{F}^{\nu_1\ldots\nu_6} \right\}, \tag{96}$$

$$e^3(\mathrm{BF}) = \left\{ \overset{[0]}{\eta}_{(3,2)},\ \overset{[0]}{\eta}_{(1,0)}\overset{[0]}{\eta}_{(2,1)} \right\}. \tag{97}$$

With the help of results (96) and (97) and under the assumption of Poincaré invariance we reach the final expression of the "potentially independently consistent" 'homogeneous' solution in antighost number 4

$$\breve{a}^{\mathrm{int}}_{4,D=8} = \frac{5}{2}\overset{[6]}{C}{}^{*}_{(3,2)\mu_1\ldots\mu_6}\left(u'_3\overset{[0]}{\eta}_{(3,2)} + u''_4\overset{[0]}{\eta}_{(1,0)}\overset{[0]}{\eta}_{(2,1)}\right)\mathcal{F}^{\mu_1\ldots\mu_6}, \tag{98}$$

with u_3' and u_4'' some arbitrary real constants.

In antighost number 3 we start from the eligible basis elements with pgh = 3

$$e^3_{\text{eligible}}(\bar{\eta}^{\bar{\gamma}}) = \left\{e^2(\text{BF})\mathcal{F}^{\nu_1\ldots\nu_6},\ e^3(\text{BF})\right\}, \tag{99}$$

together with the representatives (59) of $H_3^{\text{inv}}(\delta|d)$. Using exactly the same arguments like before, we deduce that $\breve{a}^{\text{int}}_{3,D=8}$ includes only one class of terms

$$\breve{a}^{\text{int}}_{3,D=8} = \left\{\overset{[6]^*}{C}_{(4,1)\mu_1\ldots\mu_6} \leftrightarrow e^2(\text{BF})\mathcal{F}^{\nu_1\ldots\nu_6}\right\}. \tag{100}$$

Inserting the concrete expression of $e^2(\text{BF})$ from (83) in (100) we arrive at

$$\breve{a}^{\text{int}}_{3,D=8} = \frac{10}{3}u_3''\overset{[6]^*}{C}_{(4,1)\mu_1\ldots\mu_6}\overset{[0]}{\eta}_{(2,1)}\mathcal{F}^{\mu_1\ldots\mu_6}, \tag{101}$$

with u_3'' an arbitrary real constant.

At antighost number 2, the corresponding eligible basis elements reduce to

$$e^2_{\text{eligible}}(\bar{\eta}^{\bar{\gamma}}) = \left\{e^1(\text{BF})\mathcal{F}^{\nu_1\ldots\nu_6},\ e^2(\text{BF})\right\}, \tag{102}$$

while the general representatives spanning $H_2^{\text{inv}}(\delta|d)$ can be found in (60). In principle, the BF-sector antifields from (60) may be related only to the former element in (102), while $\mathcal{G}'^{*}_{(0)\mu_1\ldots\mu_4||\beta}$ solely to the latter, but covariance arguments provide two possibilities

$$\breve{a}^{\text{int}}_{2,D=8} = \left\{\overset{[2]^*}{\eta}_{(3,0)\mu_1\mu_2} \leftrightarrow e^1(\text{BF})\mathcal{F}^{\nu_1\ldots\nu_6},\ P_{\mu_1\mu_2} \leftrightarrow e^1(\text{BF})\mathcal{F}^{\nu_1\ldots\nu_6}\right\}. \tag{103}$$

Taking into account the expression of $e^1(\text{BF})$ present in (83) we conclude that

$$\breve{a}^{\text{int}}_{2,D=8} = -\frac{5}{3}v_1''\overset{[6]^*}{\tilde{\eta}}_{(3,0)\mu_1\ldots\mu_6}\overset{[0]}{\eta}_{(1,0)}\mathcal{F}^{\mu_1\ldots\mu_6} + \varepsilon^{\mu_1\ldots\mu_8}P_{3[\mu_1\mu_2}\mathcal{F}_{\mu_3\ldots\mu_8]}, \tag{104}$$

where v_1'' is an arbitrary real constant and $P_{3\mu_1\mu_2}$ follows from (61) with $j = 2$ and $P \to P_3$, while $\overset{[6]^*}{\tilde{\eta}}_{(3,0)}$ denotes the Hodge dual of the antifield $\overset{[2]^*}{\eta}_{(3,0)}$, defined according to (50).

At this point we are left only with the "potentially independently consistent" 'homogeneous' solution in antighost number equal to 1, of the form (74). Since the antighost number of all the antifields corresponding to the original fields, $\Phi^*_{\alpha_0}$, and of their spacetime derivatives is equal to one, it follows that $\breve{a}^{\text{int}}_{1,D=8}$ will be written as a monomial of degree one in the antifields (8) and their spacetime derivatives. Moreover, using the same line employed in [15]–[17], it can be shown that the entire dependence of $\breve{a}^{\text{int}}_{1,D=8}$ on the quantities from (44) pertaining to the BF sector and on their derivatives, excepting that on the undifferentiated scalar field, can be eliminated based on the observation that these objects are δ-exact and thus, even consistent, they would generate only trivial first-order cross-couplings. At the same time, the dependence of $\breve{a}^{\text{int}}_{1,D=8}$ on the curvature tensor and its spacetime derivatives, $[K]$, is forbidden since it would break the derivative-order hypothesis [if the corresponding $a^{\text{int}}_{0,D=8}$ exists, then it will contain at least two spacetime derivatives]. Putting together all these considerations, we can write that

$$\breve{a}^{\text{int}}_{1,D=8} = \alpha_1\Big([\Phi^*_{\alpha_0}], \varphi\Big)e^1(\bar{\eta}^{\bar{\gamma}}), \tag{105}$$

with

$$e^1(\bar{\eta}^{\bar{\gamma}}) = \left\{ \overset{[0]}{\eta}_{(1,0)},\ \mathcal{F}^{\nu_1 \ldots \nu_6} \right\}, \tag{106}$$

where the invariant polynomial α_1 is linear in the antifields from (8) together with their spacetime derivatives. The antifields $[t^*]$ cannot be satisfactorily coupled to $\mathcal{F}^{\nu_1 \ldots \nu_6}$ [via a function of φ] since, if consistent, such terms would generate minimum two spacetime derivatives in the associated Lagrangian density. Consequently, we are left with two generic cross-coupling terms

$$\breve{a}^{\mathrm{int}}_{1,D=8} = \left\{ [\Phi^*_{\mathrm{BF}}] f(\varphi) \leftrightarrow \mathcal{F}^{\nu_1 \ldots \nu_6},\ [t^*] f(\varphi) \leftrightarrow \overset{[0]}{\eta}_{(1,0)} \right\}. \tag{107}$$

Invoking the actions of the operator δ on the BF antifields Φ^*_{BF}, which are linear in the first-order derivatives of BF fields [see formulas (22) and (25) and also notation (42)], accompanied by the result $\partial \mathcal{F} \sim \gamma(\partial t)$ together with the actions $\delta t^* \sim \partial\partial t$ and $\partial_\lambda \overset{[0]}{\eta}_{(1,0)} = \gamma \overset{[1]}{A}_\lambda$, it follows that the Lagrangian density $\tilde{a}^{\mathrm{int}}_{0,D=8}$ as solution to the equation $\delta \breve{a}^{\mathrm{int}}_{1,D=8} + \gamma \tilde{a}^{\mathrm{int}}_{0,D=8} = \partial_\mu \tilde{m}^\mu_0$ satisfies the derivative-order hypothesis if and only if the dependence on $[\Phi^*_{\mathrm{BF}}]$ and respectively on $[t^*]$ of (107) is linear in the undifferentiated antifields

$$\breve{a}^{\mathrm{int}}_{1,D=8} = \left\{ \Phi^*_{\mathrm{BF}} f(\varphi) \leftrightarrow \mathcal{F}^{\nu_1 \ldots \nu_6},\ t^*_{\nu_1 \ldots \nu_5 | \beta} f(\varphi) \leftrightarrow \overset{[0]}{\eta}_{(1,0)} \right\}. \tag{108}$$

Lorentz covariance finally selects only two possible terms, both belonging to the former class displayed in (108)

$$\breve{a}^{\mathrm{int}}_{1,D=8} = \left(P_4(\varphi) \overset{[6]^*}{\tilde{A}}_{(2)\mu_1 \ldots \mu_6} + P_5(\varphi) \overset{[6]^*}{\tilde{B}}_{(2)\mu_1 \ldots \mu_6} \right) \mathcal{F}^{\mu_1 \ldots \mu_6}, \tag{109}$$

where $\overset{[6]^*}{\tilde{A}}_{(2)}$ and $\overset{[6]^*}{\tilde{B}}_{(2)}$ represent the Hodge duals of the antifields $\overset{[2]^*}{A}$ and respectively $\overset{[2]^*}{B}$ and P_4 together with P_5 denote two arbitrary, smooth functions of φ.

So far, we managed to construct all "potentially independently consistent" 'homogeneous' solutions with the antighost number ranging from 8 to 1 and found that only those with $\mathrm{agh} = i$ and $i = \overline{1,7}$ are non-vanishing. Their expressions are listed in formulas (84), (88), (93), (98), (101), (104), and (109) [according to the decreasing values of i]. The various pieces composing these solutions can be classified into two broad distinct classes, according to the generic form of their corresponding cross-couplings, of course assuming they are consistent at the level of the first-order deformation:

I. terms linear in either $\mathcal{F}$ or a single antifield from the $(5,1)$ sector and with no other dependence on this sector [the four components from (88) involving the functions V, V_1, U_3, and respectively U_4, the seven pieces from (93) depending on the constants v_1, $\bar{q}$, u_4, v, v'_1, u_3, and respectively u'_4, together with all the terms from (98), (101), (104), and (109)];

II. terms containing the ghost $\mathcal{C}_{(4)}$ and possibly other ghosts or antifields from the $(5,1)$ sector [both components of (84), the three pieces from (88) proportional with the constants q, q_1, and respectively q'_1, and also the term from (93) involving q_2].

In other words, if consistent at order one in the coupling constant, then these two classes of 'homogeneous' solutions are independently consistent. In the next section we will show that all the terms from class II are inconsistent and can be removed from $a^{\mathrm{int}}_{D=8}$.

4 Elimination of the terms containing $\mathcal{C}_{(4)}$

Analyzing the structure of the 'homogeneous' solutions from class II [containing the ghost $\mathcal{C}_{(4)}$ from the $(5,1)$ sector] discussed in the previous section, we observe that they can be again classified into three subclasses according to their potential of generating independent cross-couplings: II.1. the term involving the function P_1 from (84) as well as the pieces depending on the constants q_1 and respectively q_1' from (88), II.2. the component built with the help of the function P_2 from (84) together with the term linear in the constant q_2 and present in (93), and II.3. the piece proportional with q appearing in (88). Due to the fact that the representatives of the first two subclasses display the same maximum value of the antighost number, namely 7, we will treat them together and approach separately only the third subclass.

In order to show that the first two subclasses cause inconsistencies at the level of the cross-coupling first-order deformation we prove that there is no density $a_{D=8}^{II.1+2}$ complying with all the working hypotheses and endowed with the properties

$$a_{D=8}^{II.1+2}=\sum_{i=0}^{7}a_{i,D=8}^{II.1+2},\ \varepsilon(a_{i,D=8}^{II.1+2})=0,\ \mathrm{agh}(a_{i,D=8}^{II.1+2})=i=\mathrm{pgh}(a_{i,D=8}^{II.1+2}),\tag{110}$$

$$a_{7,D=8}^{II.1+2}=-P_{1\mu_1\ldots\mu_7}\mathcal{C}_{(4)}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}\overset{[0]}{\eta}_{(1,0)}+\varepsilon^{\mu_1\ldots\mu_8}P_{2[\mu_1\ldots\mu_7}\overset{[0]}{\eta}_{(2,1)}\mathcal{C}_{(4)\mu_8]},\tag{111}$$

$$a_{6,D=8}^{II.1+2}=-q_1\overset{[7]^*}{C}_{(2,4)\mu_1\ldots\mu_7}\mathcal{C}_{(4)}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}+\frac{7!}{20}q_1'\mathcal{C}_{(4)}^{*\alpha}\mathcal{C}_{(4)\alpha}\overset{[0]}{\eta}_{(1,0)}+\cdots,\tag{112}$$

$$a_{5,D=8}^{II.1+2}=-3q_2\varepsilon^{\mu_1\ldots\mu_8}\overset{[7]^*}{C}_{(3,3)[\mu_1\ldots\mu_7}\mathcal{C}_{(4)\mu_8]}+\cdots,\tag{113}$$

that satisfies the equation specific to the first-order deformation of the solution to the classical master equation, $sa_{D=8}^{II.1+2}=\partial_\mu m'^\mu$, equivalent to the tower of equations

$$\gamma a_{7,D=8}^{II.1+2}=0,\tag{114}$$

$$\delta a_{i,D=8}^{II.1+2}+\gamma a_{i-1,D=8}^{II.1+2}=\partial_\mu m_{i-1}'^{\mu},\quad i=\overline{1,7}.\tag{115}$$

We notice that at this stage $a_{D=8}^{II.1+2}$ is parameterized by two arbitrary, real, smooth functions of φ (P_1 and P_2) and three arbitrary, real constants (q_1, q_2, and q_1').

Equation (114) is satisfied by construction [see (84)]. Inserting (111) in equation (115) for $i=7$, using the actions of δ and γ on the BRST generators [formulas (21)–(30) and (33)–(40)], and taking into account condition (112), we obtain the partial expression of $a_{6,D=8}^{II.1+2}$

$$\begin{aligned}a_{6,D=8}^{II.1+2}=&-P_{1[\mu_1\ldots\mu_6}\Big(\frac{1}{2}\mathcal{G}'_{(3)\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}\overset{[0]}{\eta}_{(1,0)}+\overset{[1]}{A}_{\mu_7]}\mathcal{C}_{(4)}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}\Big)\\&+\frac{1}{5}\delta_a^{[\beta}F^{\mu_1\ldots\mu_6]|}{}_{[\beta}P_{1\mu_1\ldots\mu_6]}\mathcal{C}_{(4)}^{\alpha}\overset{[0]}{\eta}_{(1,0)}+\varepsilon^{\mu_1\ldots\mu_8}P_{2[\mu_1\ldots\mu_6}\Big(-\overset{[1]}{\eta}_{(2,0)\mu_7}\mathcal{C}_{(4)\mu_8]}+3\overset{[0]}{\eta}_{(2,1)}\mathcal{C}_{(3)\mu_7\mu_8]}\Big)\\&-q_1\overset{[7]^*}{C}_{(2,4)\mu_1\ldots\mu_7}\mathcal{C}_{(4)}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}+\frac{7!}{20}q_1'\mathcal{C}_{(4)}^{*\alpha}\mathcal{C}_{(4)\alpha}\overset{[0]}{\eta}_{(1,0)}+\hat{a}_{6,D=8}^{II.1+2}.\end{aligned}\tag{116}$$

In the above $P_{1\mu_1\ldots\mu_6}$ and $P_{2\mu_1\ldots\mu_6}$ are of the form (61) with $j=6$ and $P(\varphi)\to P_1(\varphi)$, respectively $P(\varphi)\to P_2(\varphi)$. The 'homogeneous' solution $\hat{a}_{6,D=8}^{II.1+2}$ [$\gamma\hat{a}_{6,D=8}^{II.1+2}=0$] has the same meaning with $\hat{a}_{i,D=8}^{\mathrm{int}}$ from (72), namely, it is a nontrivial, bosonic element of $H(\gamma)$ of both pure ghost number and antighost number equal to 6 that ensures the consistency of $a^{II.1+2}_{D=8}$ in antighost number 5. Its concrete expression will be determined during the next step.

Related to equation (115) for $i=6$, initially we act with δ on (116) and infer the expression of the 'homogeneous' solution of antighost number 6, $\hat{a}^{II.1+2}_{6,D=8}$

$$\begin{aligned}\hat{a}^{II.1+2}_{6,D=8} = & -2\Big(P_{1[\mu_1\ldots\mu_5}\overset{[2]^*}{B}_{\mu_6\mu_7]} + P_{1[\mu_1\ldots\mu_4}\overset{[3]^*}{C}_{(2,0)\mu_5\mu_6\mu_7]} + P_{1[\mu_1\mu_2\mu_3}\overset{[4]^*}{C}_{(2,1)\mu_4\ldots\mu_7]} \\ & + P_{1[\mu_1\mu_2}\overset{[5]^*}{C}_{(2,2)\mu_3\ldots\mu_7]} + P_{1[\mu_1}\overset{[6]^*}{C}_{(2,3)\mu_2\ldots\mu_7]} + P_1\overset{[7]^*}{C}_{(2,4)\mu_1\ldots\mu_7}\Big)\mathcal{C}_{(4)}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]} \\ & + \frac{7!}{5}\Big(t^{*\mu_1\ldots\mu_5|}{}_{\alpha}P_{1\mu_1\ldots\mu_5} + \mathcal{G}'^{*\mu_1\ldots\mu_4||}_{(0)}{}_{\alpha}P_{1\mu_1\ldots\mu_4} - \mathcal{G}'^{*\mu_1\mu_2\mu_3||}_{(1)}{}_{\alpha}P_{1\mu_1\mu_2\mu_3} \\ & - \mathcal{G}'^{*\mu_1\mu_2||}_{(2)}{}_{\alpha}P_{1\mu_1\mu_2} + \mathcal{G}'^{*\mu_1||}_{(3)}{}_{\alpha}P_{1\mu_1} + \frac{1}{2}\mathcal{C}^{*}_{(4)\alpha}P_1\Big)\mathcal{C}^{\alpha}_{(4)}\overset{[0]}{\eta}_{(1,0)},\end{aligned} \tag{117}$$

which substituted in (116) provides the complete expression of $a^{II.1+2}_{6,D=8}$. Then, we employ this final expression, compute $\delta a^{II.1+2}_{6,D=8}$, take into account condition (113), and by means of equation (115) for $i=6$ we partially identify the piece of antighost number 5 of the first-order deformation $a^{II.1+2}_{D=8}$

$$\begin{aligned}a^{II.1+2}_{5,D=8} = & \frac{1}{2}P_{1[\mu_1\ldots\mu_5}\mathcal{G}'_{(2)\mu_6\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}\overset{[0]}{\eta}_{(1,0)} + \frac{1}{2}\Big[P_{1[\mu_1\ldots\mu_5}\overset{[1]}{A}_{\mu_6} \\ & + 2\Big(P_{1[\mu_1\ldots\mu_4}\overset{[2]^*}{B}_{\mu_5\mu_6} + P_{1[\mu_1\mu_2\mu_3}\overset{[3]^*}{C}_{(2,0)\mu_4\mu_5\mu_6} + P_{1[\mu_1\mu_2}\overset{[4]^*}{C}_{(2,1)\mu_3\ldots\mu_6} + P_{1[\mu_1}\overset{[5]^*}{C}_{(2,2)\mu_2\ldots\mu_6]} \\ & + P_1\overset{[6]^*}{C}_{(2,3)[\mu_1\ldots\mu_6}\Big)\Big]\mathcal{G}'_{(3)\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]} + \frac{1}{10}\Big[\delta^{[\beta}_{a}F^{\mu_1\ldots\mu_5\nu_1]||}{}_{[\beta}P_{1\mu_1\ldots\mu_5} + 7!\Big(t^{*\mu_1\ldots\mu_4\nu_1|}{}_{\alpha}P_{1[\mu_1\ldots\mu_4} \\ & + \mathcal{G}'^{*\mu_1\mu_2\mu_3\nu_1||}_{(0)}{}_{\alpha}P_{1[\mu_1\mu_2\mu_3} - \mathcal{G}'^{*\mu_1\mu_2\nu_1||}_{(1)}{}_{\alpha}P_{1[\mu_1\mu_2} - \mathcal{G}'^{*\mu_1\nu_1||}_{(2)}{}_{\alpha}P_{1[\mu_1]}\Big)\Big]\mathcal{G}'_{(3)\nu_1||}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)} \\ & + \frac{7!}{10}\mathcal{G}'^{*\nu_1||}_{(3)}{}_{\alpha}P_1\mathcal{G}'_{(3)\nu_1||}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)} - \frac{1}{5}\Big\{\delta^{[\beta}_{a}F^{\mu_1\ldots\mu_6]||}{}_{[\beta}\Big[P_{1\mu_1\ldots\mu_5}\overset{[1]}{A}_{\mu_6]} + 2\Big(P_{1\mu_1\ldots\mu_4}\overset{[2]^*}{B}_{\mu_5\mu_6]} \\ & + P_{1\mu_1\mu_2\mu_3}\overset{[3]^*}{C}_{(2,0)\mu_4\mu_5\mu_6]} + P_{1\mu_1\mu_2}\overset{[4]^*}{C}_{(2,1)\mu_3\ldots\mu_6]} + P_{1\mu_1}\overset{[5]^*}{C}_{(2,2)\mu_2\ldots\mu_6]} + P_1\overset{[6]^*}{C}_{(2,3)\mu_1\ldots\mu_6]}\Big)\Big] \\ & + 7!\Big(t^{*\mu_1\ldots\mu_5|}{}_{\alpha}P_{1[\mu_1\ldots\mu_4}\overset{[1]}{A}_{\mu_5]} + \mathcal{G}'^{*\mu_1\ldots\mu_4||}_{(0)}{}_{\alpha}P_{1[\mu_1\mu_2\mu_3}\overset{[1]}{A}_{\mu_4]} - \mathcal{G}'^{*\mu_1\mu_2\mu_3||}_{(1)}{}_{\alpha}P_{1[\mu_1\mu_2}\overset{[1]}{A}_{\mu_3]} \\ & - \mathcal{G}'^{*\mu_1\mu_2||}_{(2)}{}_{\alpha}P_{1[\mu_1}\overset{[1]}{A}_{\mu_2]} + \mathcal{G}'^{*\mu_1||}_{(3)}{}_{\alpha}P_1\overset{[1]}{A}_{\mu_1}\Big)\Big\}\mathcal{C}^{\alpha}_{(4)} - \varepsilon^{\mu_1\ldots\mu_8}P_{2[\mu_1\ldots\mu_5}\Big(\overset{[2]}{A}_{\mu_6\mu_7}\mathcal{C}_{(4)\mu_8]} \\ & + 3\overset{[1]}{\eta}_{(2,0)\mu_6}\mathcal{C}_{(3)\mu_7\mu_8]} + 6\overset{[0]}{\eta}_{(2,1)}\mathcal{C}_{(2)\mu_6\mu_7\mu_8]}\Big) - q_1\overset{[6]^*}{C}_{(2,3)[\mu_1\ldots\mu_6}\Big(\frac{1}{5}F^{[\mu_1\ldots\mu_6|}{}_{\alpha]}\mathcal{C}^{\alpha]}_{(4)} \\ & - \frac{1}{2}\mathcal{G}'_{(3)\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}\Big) + \frac{7!}{10}q'_1\mathcal{G}'^{*\mu_1||}_{(3)}{}_{\alpha}\Big(\frac{1}{2}\mathcal{G}'_{(3)\mu_1||}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)} - \overset{[1]}{A}_{\mu_1}\mathcal{C}^{\alpha}_{(4)}\Big) \\ & - 3q_2\varepsilon^{\mu_1\ldots\mu_8}\overset{[7]^*}{C}_{(3,3)[\mu_1\ldots\mu_7}\mathcal{C}_{(4)\mu_8]} + \hat{a}^{II.1+2}_{5,D=8},\end{aligned} \tag{118}$$

where we used formula (61) for various values of j together with (63) in which we replaced the function P by P_1 or P_2 and included the γ-closed component $\hat{a}^{II.1+2}_{5,D=8}$ necessary at the consistency of $a^{II.1+2}_{D=8}$ in antighost number 4.

Regarding equation (115) for $i=5$, first we act with δ on (118) and identify the concrete expression of $\hat{a}^{II.1+2}_{5,D=8}$

$$\hat{a}^{II.1+2}_{5,D=8} = -\frac{2}{5}7!\Big[t^{*\mu_1\ldots\mu_5|}{}_{\alpha}\Big(P_{1[\mu_1\mu_2\mu_3}\overset{[2]^*}{B}_{\mu_4\mu_5]} + P_{1[\mu_1\mu_2}\overset{[3]^*}{C}_{(2,0)\mu_3\mu_4\mu_5]} + P_{1[\mu_1}\overset{[4]^*}{C}_{(2,1)\mu_2\ldots\mu_5]}$$

$$+ P_1 \overset{[5]^*}{C}_{(2,2)\mu_1\ldots\mu_5}\Big) + \mathcal{G}'^{*\mu_1\ldots\mu_4||}_{(0)\quad\alpha}\Big(P_{1[\mu_1\mu_2}\overset{[2]^*}{B}_{\mu_3\mu_4]} + P_{1[\mu_1}\overset{[3]^*}{C}_{(2,0)\mu_2\mu_3\mu_4]} + P_1\overset{[4]^*}{C}_{(2,1)\mu_1\ldots\mu_4}\Big)$$
$$- \mathcal{G}'^{*\mu_1\mu_2\mu_3||}_{(1)\quad\alpha}\Big(P_{1[\mu_1}\overset{[2]^*}{B}_{\mu_2\mu_3]} + P_1\overset{[3]^*}{C}_{(2,0)\mu_1\mu_2\mu_3}\Big) - \mathcal{G}'^{*\mu_1\mu_2||}_{(2)\quad\alpha}P_1\overset{[2]^*}{B}_{\mu_1\mu_2}\Big]\mathcal{C}^{\alpha}_{(4)}$$
$$- 3\varepsilon^{\mu_1\ldots\mu_8}\Big(P_{2[\mu_1\ldots\mu_4}\overset{[3]^*}{B}_{\mu_5\mu_6\mu_7} + P_{2[\mu_1\mu_2\mu_3}\overset{[4]^*}{C}_{(3,0)\mu_4\ldots\mu_7} + P_{2[\mu_1\mu_2}\overset{[5]^*}{C}_{(3,1)\mu_3\ldots\mu_7}$$
$$+ P_{2[\mu_1}\overset{[6]^*}{C}_{(3,2)\mu_2\ldots\mu_7} + P_2\overset{[7]^*}{C}_{(3,3)[\mu_1\ldots\mu_7}\Big)\mathcal{C}_{(4)\mu_8]} - \frac{7!}{5}q_1\Big(t^{*\mu_1\ldots\mu_5|}{}_{\alpha}\overset{[5]^*}{C}_{(2,2)\mu_1\ldots\mu_5}$$
$$+ \mathcal{G}'^{*\mu_1\ldots\mu_4||}_{(0)\quad\alpha}\overset{[4]^*}{C}_{(2,1)\mu_1\ldots\mu_4} - \mathcal{G}'^{*\mu_1\mu_2\mu_3||}_{(1)\quad\alpha}\overset{[3]^*}{C}_{(2,0)\mu_1\mu_2\mu_3} - \mathcal{G}'^{*\mu_1\mu_2||}_{(2)\quad\alpha}\overset{[2]^*}{B}_{\mu_1\mu_2}\Big)\mathcal{C}_{(4)}^{\ \alpha}. \tag{119}$$

Second, by inserting (119) in (118) we get the full expression of $a^{II.1+2}_{5,D=8}$. Computing now $\delta a^{II.1+2}_{5,D=8}$ and comparing the emerging result with equation (115) for $i=5$, we observe that the existence of $a^{II.1+2}_{4,D=8}$ requires the following condition

$$\frac{7!}{20}q_1'\mathcal{G}'^{*\mu_1\mu_2||}_{(2)\quad\alpha}\Big(\gamma\overset{[1]}{A}_{[\mu_1}\Big)\mathcal{G}'_{(3)\mu_2]||}{}^{\alpha} - \frac{7!}{5}\Big[q_1'\mathcal{G}'^{*\mu_1\mu_2||}_{(2)\quad\alpha}\Big(\partial_{\mu_1}\overset{[1]}{A}_{\mu_2}\Big) + q_1\Big(\partial_{\mu_1}\mathcal{G}'^{*\mu_1\mu_2||}_{(2)\quad\alpha}\Big)\overset{[1]}{A}_{\mu_2}\Big]\mathcal{C}_{(4)}^{\ \alpha}$$
$$= \partial_\mu m''^{\mu}(q_1,q_1') - \gamma a^{II.1+2}_{4,D=8}(q_1,q_1'). \tag{120}$$

The last equation is satisfied if and only if the constants q_1' and q_1 are related by

$$q_1' = q_1, \tag{121}$$

which furnishes the solution $a^{II.1+2}_{4,D=8}(q_1,q_1')$ to equation (120) in the form

$$a^{II.1+2}_{4,D=8}(q_1,q_1'=q_1) \equiv a''^{II.1+2}_{4,D=8}(q_1) = -\frac{7!}{20}q_1\mathcal{G}'^{*\mu_1\mu_2||}_{(2)\quad\alpha}\overset{[1]}{A}_{[\mu_1}\mathcal{G}'_{(3)\mu_2]||}{}^{\alpha}. \tag{122}$$

Substituting (119) and (121) in (118), we determine the final expression of $a^{II.1+2}_{5,D=8}$. Third, we use this expression, compute $\delta a^{II.1+2}_{5,D=8}$, take into account relations (121)–(122), and consequently find the partial solution $a^{II.1+2}_{4,D=8}$ to equation (115) for $i=5$

$$a^{II.1+2}_{4,D=8} = \frac{1}{2}P_{1[\mu_1\ldots\mu_4}\mathcal{G}'_{(1)\mu_5\mu_6\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}\overset{[0]}{\eta}_{(1,0)} + \frac{1}{2}\Big[P_{1[\mu_1\ldots\mu_4}\overset{[1]}{A}_{\mu_5} + 2\Big(P_{1[\mu_1\mu_2\mu_3}\overset{[2]^*}{B}_{\mu_4\mu_5}$$
$$+ P_{1[\mu_1\mu_2}\overset{[3]^*}{C}_{(2,0)\mu_3\mu_4\mu_5} + P_{1[\mu_1}\overset{[4]^*}{C}_{(2,1)\mu_2\ldots\mu_5} + P_1\overset{[5]^*}{C}_{(2,2)\mu_1\ldots\mu_5}\Big)\Big]\mathcal{G}'_{(2)\mu_6\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}$$
$$- \frac{1}{10}\Big[\delta^{[\beta}_{a}F^{\mu_1\ldots\mu_4\nu_1\nu_2]|}{}_{[\beta}P_{1\mu_1\ldots\mu_4} + 7!\Big(t^{*\mu_1\mu_2\mu_3\nu_1\nu_2|}{}_{\alpha}P_{1[\mu_1\mu_2\mu_3} + \mathcal{G}'^{*\mu_1\mu_2\nu_1\nu_2||}_{(0)\quad\alpha}P_{1[\mu_1\mu_2}$$
$$- \mathcal{G}'^{*\mu_1\nu_1\nu_2||}_{(1)\quad\alpha}P_{1[\mu_1}\Big)\Big]\mathcal{G}'_{(2)\nu_1\nu_2||}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)} + \frac{7!}{10}\mathcal{G}'^{*\nu_1\nu_2||}_{(2)\quad\alpha}P_1\mathcal{G}'_{(2)\nu_1\nu_2||}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)}$$
$$+ \frac{1}{10}\Big\{\delta^{[\beta}_{a}F^{\mu_1\ldots\mu_5\nu_1]|}{}_{[\beta}\Big[P_{1\mu_1\ldots\mu_4}\overset{[1]}{A}_{\mu_5} + 2\Big(P_{1\mu_1\mu_2\mu_3}\overset{[2]^*}{B}_{\mu_4\mu_5} + P_{1\mu_1\mu_2}\overset{[3]^*}{C}_{(2,0)\mu_3\mu_4\mu_5}$$
$$+ P_{1\mu_1}\overset{[4]^*}{C}_{(2,1)\mu_2\ldots\mu_5} + P_1\overset{[5]^*}{C}_{(2,2)\mu_1\ldots\mu_5}\Big)\Big] + 7!\Big\{t^{*\mu_1\ldots\mu_4\nu_1|}{}_{\alpha}\Big[P_{1[\mu_1\mu_2\mu_3}\overset{[1]}{A}_{\mu_4}$$
$$+ 2\Big(P_{1[\mu_1\mu_2}\overset{[2]^*}{B}_{\mu_3\mu_4} + P_{1[\mu_1}\overset{[3]^*}{C}_{(2,0)\mu_2\mu_3\mu_4} + P_1\overset{[3]^*}{C}_{(2,0)\mu_1\mu_2\mu_3}\Big)\Big]$$
$$+ \mathcal{G}'^{*\mu_1\mu_2\mu_3\nu_1||}_{(0)\quad\alpha}\Big[P_{1[\mu_1\mu_2}\overset{[1]}{A}_{\mu_3} + 2\Big(P_{1[\mu_1}\overset{[2]^*}{B}_{\mu_2\mu_3} + P_1\overset{[3]^*}{C}_{(2,0)[\mu_1\mu_2\mu_3}\Big)\Big]$$

$$
\begin{aligned}
&- \mathcal{G}'^{*\mu_1\mu_2\nu_1||}_{(1)}{}_{\alpha}\Big(P_{1[\mu_1}\overset{[1]}{A}_{\mu_2} + 2P_1\overset{[2]^*}{B}_{[\mu_1\mu_2}\Big) - \mathcal{G}'^{*\mu_1\nu_1||}_{(2)}{}_{\alpha}P_1\overset{[1]}{A}_{[\mu_1}\Big\}\Big\}\mathcal{G}'_{(3)\nu_1]||}{}^{\alpha} \\
&+ \varepsilon^{\mu_1\ldots\mu_8}\Big\{-3\Big[P_{2[\mu_1\ldots\mu_4}\overset{[2]}{A}_{\mu_5\mu_6} + 3\Big(P_{2[\mu_1\mu_2\mu_3}\overset{[3]^*}{B}_{\mu_4\mu_5\mu_6} + P_{2[\mu_1\mu_2}\overset{[4]^*}{C}_{(3,0)\mu_3\ldots\mu_6} \\
&+ P_{2[\mu_1}\overset{[5]^*}{C}_{(3,1)\mu_2\ldots\mu_6} + P_2\overset{[6]^*}{C}_{(3,2)[\mu_1\ldots\mu_6}\Big)\Big]\mathcal{C}_{(3)\mu_7\mu_8]} + 6P_{2[\mu_1\ldots\mu_4}\overset{[1]}{\eta}_{(2,0)\mu_5}\mathcal{C}_{(2)\mu_6\mu_7\mu_8]} \\
&- 10P_{2[\mu_1\ldots\mu_4}\mathcal{C}_{(1)\mu_5\ldots\mu_8]}\overset{[0]}{\eta}_{(2,1)}\Big\} - \frac{q_1}{2}\overset{[5]^*}{C}_{(2,2)[\mu_1\ldots\mu_5}\mathcal{G}'_{(2)\mu_6\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]} \\
&+ \frac{7!}{20}q_1\mathcal{G}'^{*\mu_1\mu_2||}_{(2)}{}_{\alpha}\mathcal{G}'_{(2)\mu_1\mu_2]|}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)} + \frac{q_1}{10}\Big[\delta^{[\beta}_{a}F^{\mu_1\ldots\mu_6]|}{}_{[\beta}\overset{[5]^*}{C}_{(2,2)\mu_1\ldots\mu_5} \\
&+ 7!\Big(t^{*\mu_2\ldots\mu_6|}{}_{\alpha}\overset{[4]^*}{C}_{(2,1)[\mu_2\ldots\mu_5} + \mathcal{G}'^{*\mu_3\ldots\mu_6||}_{(0)}{}_{\alpha}\overset{[3]^*}{C}_{(2,0)[\mu_3\ldots\mu_5} - \mathcal{G}'^{*\mu_4\ldots\mu_6||}_{(1)}{}_{\alpha}\overset{[2]^*}{B}_{[\mu_4\mu_5} \\
&- \frac{1}{2}\mathcal{G}'^{*\mu_5\mu_6||}_{(2)}{}_{\alpha}\overset{[1]}{A}_{[\mu_5}\Big)\Big]\mathcal{G}'_{(3)\mu_6]||}{}^{\alpha} - 9q_2\varepsilon^{\mu_1\ldots\mu_8}\overset{[6]^*}{C}_{(3,2)[\mu_1\ldots\mu_6}\mathcal{C}_{(3)\mu_7\mu_8]} + \hat{a}^{II.1+2}_{4,D=8}.
\end{aligned}
\tag{123}
$$

We remark that at this point $a^{II.1+2}_{D=8}$ is parameterized again by the functions P_1 and P_2, but only by two arbitrary, real constants [q_1 and q_2] instead of the initial three ones.

Next, we approach equation (115) for $i = 4$. In this respect we act with δ on (123) and on the one hand we observe that we can safely take

$$
\hat{a}^{II.1+2}_{4,D=8} = 0 \tag{124}
$$

in (123), which completes the form $a^{II.1+2}_{4,D=8}$. On the other hand, we identify the partial solution to (115) for $i = 4$

$$
\begin{aligned}
a^{II.1+2}_{3,D=8} &= -\frac{1}{2}P_{1[\mu_1\mu_2\mu_3}\mathcal{G}'_{(0)\mu_4\ldots\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]}\overset{[0]}{\eta}_{(1,0)} - \frac{1}{2}\Big[P_{1[\mu_1\mu_2\mu_3}\overset{[1]}{A}_{\mu_4} + 2\Big(P_{1[\mu_1\mu_2}\overset{[2]^*}{B}_{\mu_3\mu_4} \\
&+ P_{1[\mu_1}\overset{[3]^*}{C}_{(2,0)\mu_2\mu_3\mu_4} + P_1\overset{[4]^*}{C}_{(2,1)[\mu_1\ldots\mu_4}\Big)\Big]\mathcal{G}'_{(1)\mu_5\mu_6\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]} - \frac{1}{10}\Big[\delta^{[\beta}_{a}F^{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3]|}{}_{[\beta}P_{1\mu_1\mu_2\mu_3} \\
&+ 7!\Big(t^{*\mu_1\mu_2\nu_1\nu_2\nu_3|}{}_{\alpha}P_{1[\mu_1\mu_2} + \mathcal{G}'^{*\mu_1\nu_1\nu_2\nu_3||}_{(0)}{}_{\alpha}P_{1[\mu_1}\Big)\Big]\mathcal{G}'_{(1)\nu_1\nu_2\nu_3]||}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)} \\
&+ \frac{7!}{10}\mathcal{G}'^{*\nu_1\nu_2\nu_3||}_{(1)}{}_{\alpha}P_1\mathcal{G}'_{(1)\nu_1\nu_2\nu_3]||}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)} + \frac{1}{10}\Big\{\delta^{[\beta}_{a}F^{\mu_1\ldots\mu_4\nu_1\nu_2]|}{}_{[\beta}\Big[P_{1\mu_1\mu_2\mu_3}\overset{[1]}{A}_{\mu_4} + 2\Big(P_{1\mu_1\mu_2}\overset{[2]^*}{B}_{\mu_3\mu_4} \\
&+ P_{1\mu_1}\overset{[3]^*}{C}_{(2,0)\mu_2\mu_3\mu_4} + P_1\overset{[4]^*}{C}_{(2,1)\mu_1\ldots\mu_4}\Big)\Big] + 7!\Big\{t^{*\mu_1\mu_2\mu_3\nu_1\nu_2|}{}_{\alpha}\Big[P_{1[\mu_1\mu_2}\overset{[1]}{A}_{\mu_3} + 2\Big(P_{1[\mu_1}\overset{[2]^*}{B}_{\mu_2\mu_3} \\
&+ P_1\overset{[3]^*}{C}_{(2,0)[\mu_1\mu_2\mu_3)}\Big)\Big] + \mathcal{G}'^{*\mu_1\mu_2\nu_1\nu_2||}_{(0)}{}_{\alpha}\Big(P_{1[\mu_1}\overset{[1]}{A}_{\mu_2} + 2P_1\overset{[2]^*}{B}_{[\mu_1\mu_2)}\Big) \\
&- \mathcal{G}'^{*\mu_1\nu_1\nu_2||}_{(1)}{}_{\alpha}P_1\overset{[1]}{A}_{[\mu_1}\Big\}\Big\}\mathcal{G}'_{(2)\nu_1\nu_2]||}{}^{\alpha} + \varepsilon^{\mu_1\ldots\mu_8}\Big\{6\Big[P_{2[\mu_1\mu_2\mu_3}\overset{[2]}{A}_{\mu_4\mu_5} + 3\Big(P_{2[\mu_1\mu_2}\overset{[3]^*}{B}_{\mu_3\mu_4\mu_5} \\
&+ P_{2[\mu_1}\overset{[4]^*}{C}_{(3,0)\mu_2\ldots\mu_5} + P_2\overset{[5]^*}{C}_{(3,1)[\mu_1\ldots\mu_5}\Big)\Big]\mathcal{C}_{(2)\mu_6\mu_7\mu_8]} + 10P_{2[\mu_1\mu_2\mu_3}\overset{[1]}{\eta}_{(2,0)\mu_4}\mathcal{C}_{(1)\mu_5\ldots\mu_8]} \\
&+ 15P_{2[\mu_1\mu_2\mu_3}\overset{[0]}{\eta}_{(2,1)}\mathcal{C}_{(0)\mu_4\ldots\mu_8]}\Big\} - \frac{q_1}{2}\overset{[4]^*}{C}_{(2,1)[\mu_1\ldots\mu_4}\mathcal{G}'_{(1)\mu_5\mu_6\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]} \\
&+ \frac{7!}{20}q_1\mathcal{G}'^{*\mu_1\mu_2\mu_3||}_{(1)}{}_{\alpha}\mathcal{G}'_{(1)\mu_1\mu_2\mu_3]|}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)} + \frac{q_1}{10}\Big[\delta^{[\beta}_{a}F^{\mu_1\ldots\mu_4\nu_1\nu_2]|}{}_{[\beta}\overset{[4]^*}{C}_{(2,1)\mu_1\ldots\mu_4} \\
&+ 7!\Big(t^{*\mu_1\mu_2\mu_3\nu_1\nu_2|}{}_{\alpha}\overset{[3]^*}{C}_{(2,0)[\mu_1\mu_2\mu_3} + \mathcal{G}'^{*\mu_1\mu_2\nu_1\nu_2||}_{(0)}{}_{\alpha}\overset{[2]^*}{B}_{[\mu_1\mu_2} - \frac{1}{2}\mathcal{G}'^{*\mu_1\nu_1\nu_2||}_{(1)}{}_{\alpha}\overset{[1]}{A}_{[\mu_1}\Big)\Big]\mathcal{G}'_{(2)\nu_1\nu_2]||}{}^{\alpha}
\end{aligned}
$$

$$+ 18q_2\varepsilon^{\mu_1\ldots\mu_8}\overset{[5]^*}{C}_{(3,1)[\mu_1\ldots\mu_5}\mathcal{C}_{(2)\mu_6\mu_7\mu_8]} + \hat{a}^{II.1+2}_{3,D=8}. \tag{125}$$

In the next step we employ (125) in equation (115) for $i = 3$ and, after some tedious computations, we find three types of results. Firstly, it follows that we can take

$$\hat{a}^{II.1+2}_{3,D=8} = 0 \tag{126}$$

in (125) without affecting the generality of our approach. With this finding in mind, secondly we show that the existence of $a^{II.1+2}_{2,D=8}$ as solution to (115) for $i = 3$ implies the following condition

$$\begin{aligned}
&- 105P_{1\mu_1\mu_2}\mathcal{F}^{\mu_1\lambda_1\ldots\lambda_4}\mathcal{F}^{\mu_2}{}_{\lambda_1\ldots\lambda_4}\overset{[0]}{\eta}_{(1,0)} - 15\varepsilon^{\mu_1\ldots\mu_8}P_{2[\mu_1\mu_2}\mathcal{F}_{\mu_3\ldots\mu_8]}\overset{[0]}{\eta}_{(2,1)} \\
&= \partial_\mu m''^\mu(P_1, P_2) - \gamma a^{II.1+2}_{2,D=8}(P_1) - \gamma a^{II.1+2}_{2,D=8}(P_2).
\end{aligned} \tag{127}$$

Since the two terms from the left-hand side of (127) are independent, they must be separately written as γ-exact modulo d objects. Each of them is a nontrivial element of $H(\gamma)$ in pure ghost number 3 that does not reduce to a divergence, so the existence of $a^{II.1+2}_{2,D=8}$ requires the vanishing of both terms, which takes place if and only if

$$P_{k\mu_1\mu_2} \equiv \frac{dP_k}{d\varphi}\overset{[2]^*}{C}_{(1,0)\mu_1\mu_2} + \frac{d^2P_k}{d\varphi^2}\overset{[1]^*}{B}_{\mu_1}\overset{[1]^*}{B}_{\mu_2} = 0, \quad k = 1, 2. \tag{128}$$

From (128) we obtain that (127) is satisfied if and only if the functions $P_1(\varphi)$ and $P_2(\varphi)$ reduce to some arbitrary real constants

$$P_1(\varphi) = p_1, \qquad P_2(\varphi) = p_2, \qquad p_1, p_2 \in \mathbb{R}. \tag{129}$$

Inserting (129) in (111), we notice that the component of highest antighost number (7) from $a^{II.1+2}_{D=8}$ [see expansion (110)] vanishes

$$a^{II.1+2}_{7,D=8} = 0. \tag{130}$$

Consequently, $a^{II.1+2}_{D=8}$ exhibit non-vanishing terms of maximum antighost number 6 and is parameterized now by four arbitrary, real constants (q_1, q_2, p_1, and p_2). Moreover, replacing (129) as well as (121) in the concrete expressions of the components of $a^{II.1+2}_{D=8}$ determined so far [formulas (116)–(119) and (123)–(126)], we find that all the terms depending on the constants p_1 and respectively p_2 exhibit exactly the same structure like those involving q_1 and respectively q_2. In order to avoid term duplication and introduction of unnecessary constants we make the notations

$$q_1 + 2p_1 \equiv q_1, \quad q_2 + p_2 \equiv q_2 \tag{131}$$

and parameterize $a^{II.1+2}_{D=8}$ in terms of two distinct real constants only. Introducing (126), (129), and (131) in (125), the full expression of $a^{II.1+2}_{3,D=8}$ becomes

$$\begin{aligned}
a^{II.1+2}_{3,D=8} = &-\frac{q_1}{2}\overset{[4]^*}{C}_{(2,1)[\mu_1\ldots\mu_4}\mathcal{G}'_{(1)\mu_5\mu_6\mu_7]||}{}^{[\mu_1}\mathcal{F}^{\mu_2\ldots\mu_7]} + \frac{7!}{20}q_1\mathcal{G}'^{*\mu_1\mu_2\mu_3||}_{(1)}{}_{\alpha}\mathcal{G}'_{(1)\mu_1\mu_2\mu_3||}{}^{\alpha}\overset{[0]}{\eta}_{(1,0)} \\
&+ \frac{q_1}{10}\Big[\delta^{[\beta}_{\alpha}F^{\mu_1\ldots\mu_4\nu_1\nu_2]|}{}_{[\beta}\overset{[4]^*}{C}_{(2,1)\mu_1\ldots\mu_4} + 7!\Big(t^{*\mu_1\mu_2\mu_3\nu_1\nu_2|}{}_{\alpha}\overset{[3]^*}{C}_{(2,0)[\mu_1\mu_2\mu_3} + \mathcal{G}'^{*\mu_1\mu_2\nu_1\nu_2||}_{(0)}{}_{\alpha}\overset{[2]^*}{B}_{[\mu_1\mu_2} \\
&- \frac{1}{2}\mathcal{G}'^{*\mu_1\nu_1\nu_2||}_{(1)}{}_{\alpha}\overset{[1]}{A}_{[\mu_1}\Big)\Big]\mathcal{G}'_{(2)\nu_1\nu_2]||}{}^{\alpha} + 18q_2\varepsilon^{\mu_1\ldots\mu_8}\overset{[5]^*}{C}_{(3,1)[\mu_1\ldots\mu_5}\mathcal{C}_{(2)\mu_6\mu_7\mu_8]}.
\end{aligned} \tag{132}$$

Thirdly, acting with δ on (132) we deduce the partial solution to (115) for $i = 3$

$$
\begin{aligned}
a^{II.1+2}_{2,D=8} &= -\frac{q_1}{2} \overset{[3]^*}{C}_{(2,0)[\mu_1\mu_2\mu_3} \mathcal{G}'_{(0)\mu_4\ldots\mu_7]||}{}^{[\mu_1} \mathcal{F}^{\mu_2\ldots\mu_7]} + \frac{7!}{20} q_1 \mathcal{G}'^{*\mu_1\ldots\mu_4||}_{(0)}{}_{\alpha} \mathcal{G}'_{(0)\mu_1\ldots\mu_4||}{}^{\alpha} \overset{[0]}{\eta}_{(1,0)} \\
&- \frac{q_1}{10} \Big[\delta^{[\beta}_{a} F^{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3]||}{}_{[\beta} \overset{[3]^*}{C}_{(2,1)\mu_1\mu_2\mu_3} + 7! \Big(t^{*\mu_1\mu_2\nu_1\nu_2\nu_3|}{}_{\alpha} \overset{[2]^*}{B}_{[\mu_1\mu_2} \\
&+ \frac{1}{2} \mathcal{G}'^{*\mu_1\nu_1\nu_2\nu_3||}_{(0)}{}_{\alpha} \overset{[1]}{A}_{[\mu_1} \Big) \Big] \mathcal{G}'_{(1)\nu_1\nu_2\nu_3]||}{}^{\alpha} + 30 q_2 \varepsilon^{\mu_1\ldots\mu_8} \overset{[4]^*}{C}_{(3,0)[\mu_1\ldots\mu_4} \mathcal{C}_{(1)\mu_5\ldots\mu_8]} + \hat{a}^{II.1+2}_{2,D=8}.
\end{aligned} \tag{133}
$$

Now, we analyze equation (115) for $i = 2$. To this aim, we evaluate $\delta a^{II.1+2}_{2,D=8}$. From the resulting expression on the one hand we conclude that we can set

$$\hat{a}^{II.1+2}_{2,D=8} = 0 \tag{134}$$

in (133) and on the other hand we obtain that the existence of $a^{II.1+2}_{1,D=8}$ as solution to equation (115) for $i = 2$ induces the condition

$$-105 q_1 \overset{[2]^*}{B}_{\mu_1\mu_2} \mathcal{F}^{\mu_1\lambda_1\ldots\lambda_5} \mathcal{F}^{\mu_2}{}_{\lambda_1\ldots\lambda_5} \overset{[0]}{\eta}_{(1,0)} = \partial_\mu m''^{\mu}_1(q_1) - \gamma a^{II.1+2}_{1,D=8}(q_1). \tag{135}$$

Due to the fact that the object from the left-hand side of the above condition is a nontrivial element of $H(\gamma)$ in pure ghost number 2 which cannot be written as a divergence, it results that (135) is fulfilled if and only if the constant q_1 is vanishing

$$q_1 = 0. \tag{136}$$

Substituting (136) in all the components of $a^{II.1+2}_{D=8}$ computed so far, we find that all the terms depending on q_1 vanish, so expansion (110) ends at antighost number 5 [we already established at a previous step that the non-vanishing component of maximum antighost number was 6 instead of 7] and, more important, that $a^{II.1+2}_{D=8}$ is being parameterized by a single arbitrary, real constant, q_2. Replacing (134) in (133), implementing (136) in the resulting relation, and further computing $\delta a^{II.1+2}_{2,D=8}$, we partially identify the piece of antighost number 1 from $a^{II.1+2}_{D=8}$ as solution to equation (115) for $i = 2$

$$a^{II.1+2}_{1,D=8} = -45 q_2 \varepsilon^{\mu_1\ldots\mu_8} \overset{[3]^*}{B}_{[\mu_1\mu_2\mu_3} \mathcal{C}_{(0)\mu_4\ldots\mu_8]} + \hat{a}^{II.1+2}_{1,D=8}. \tag{137}$$

Finally, we apply δ on (137) and investigate the consistency of $a^{II.1+2}_{D=8}$ in antighost number 0 [the existence of solutions $a^{II.1+2}_{0,D=8}$ to equation (115) for $i = 1$]. Firstly we infer that we can take

$$\hat{a}^{II.1+2}_{1,D=8} = 0 \tag{138}$$

in (137) and secondly we find that the existence of $a^{II.1+2}_{0,D=8}$ imposes the condition

$$15 q_2 \varepsilon^{\mu_1\ldots\mu_8} \overset{[2]}{A}_{[\mu_1\mu_2} \mathcal{F}_{\mu_3\ldots\mu_8]} = \partial_\mu m''^{\mu}_0 - \gamma a^{II.1+2}_{0,D=8}. \tag{139}$$

We analyze (139) taking into account the fact that the 2-form gauge field $\overset{[2]}{A}$ is not γ-exact [there are no BRST generators of strictly negative pure ghost number], which further restricts the quantity $\mathcal{F}_{\mu_1\ldots\mu_6} \equiv \partial_{[\mu_1} \mathcal{C}_{(0)\mu_2\ldots\mu_6]}$ to be γ-exact. This is indeed the case since

$$\mathcal{F}_{\mu_1\ldots\mu_6} = \gamma \Big[-\frac{1}{\Box} \Big(\frac{1}{4} \partial^\alpha F_{\mu_1\ldots\mu_6|\alpha} \Big) \Big], \tag{140}$$

where $1/\Box$ denotes the inverse of the d'Alembert operator $\Box$. Due to the fact that the right-hand side of the last formula is a non-local object, it follows that, even if there exist solutions $a_{0,D=8}^{II.1+2}$ to (139), they would be non-local, which breaks the locality hypothesis imposed on the deformations of the solution to the master equation. Therefore, we remove these contributions by setting

$$q_2 = 0. \tag{141}$$

The last result plays a key role since it eliminates all the remaining pieces from (110) [indeed, we established earlier that $a_{D=8}^{II.1+2}$ is parameterized only by q_2, so the vanishing of this constant sets $a_{D=8}^{II.1+2}$ to zero].

All the results presented until now allow us to conclude that the first two subclasses of "potentially independently consistent" 'homogeneous' solutions depending on the ghost $\mathcal{C}_{(4)}$ from the $(5,1)$ sector can be safely eliminated from the first-order deformation responsible for the cross-couplings in $D = 8$ between the BF fields and the $(5,1)$ tensor field due to inconsistency reasons.

At this stage we only need to show that the third subclass of 'homogeneous' solutions from class II also provokes inconsistencies at the level of the first-order deformation. In this respect we prove that there is no density $a_{D=8}^{II.3}$ in agreement with all the working hypotheses and displaying the properties

$$a_{D=8}^{II.3} = \sum_{i=0}^{6} a_{i,D=8}^{II.3},\ \varepsilon(a_{i,D=8}^{II.3}) = 0,\ \mathrm{agh}(a_{i,D=8}^{II.3}) = i = \mathrm{pgh}(a_{i,D=8}^{II.3}), \tag{142}$$

$$a_{6,D=8}^{II.3} = q\varepsilon^{\mu_1\ldots\mu_8}\overset{[7]^*}{C}_{(2,4)[\mu_1\ldots\mu_7}\mathcal{C}_{(4)\mu_8]}\overset{[0]}{\eta}_{(1,0)}, \tag{143}$$

that fulfils the equation governing the first-order deformation of the solution to the classical master equation, $sa_{D=8}^{II.3} = \partial_\mu m'^\mu$, equivalent to the tower of equations

$$\gamma a_{6,D=8}^{II.3} = 0, \tag{144}$$

$$\delta a_{i,D=8}^{II.3} + \gamma a_{i-1,D=8}^{II.3} = \partial_\mu m'^\mu_{i-1}, \quad i = \overline{1,6}. \tag{145}$$

Clearly, equation (144) is satisfied by construction [see (88)]. Acting with δ on (143) we partially infer the solution to equation (145) for $i = 6$ under the form

$$a_{5,D=8}^{II.3} = q\varepsilon^{\mu_1\ldots\mu_8}\overset{[6]^*}{C}_{(2,3)[\mu_1\ldots\mu_6}\Big(-3\mathcal{C}_{(3)\mu_7\mu_8]}\overset{[0]}{\eta}_{(1,0)} - \overset{[1]}{A}_{\mu_7}\mathcal{C}_{(4)\mu_8]}\Big) + \hat{a}_{5,D=8}^{II.3}, \tag{146}$$

where $\hat{a}_{5,D=8}^{II.3}$ collects the γ-invariant contributions to $a_{D=8}^{II.3}$ in antighost number 5 that ensure the consistency of $a_{5,D=8}^{II.3}$ in antighost number 4 [the existence of solutions $a_{4,D=8}^{II.3}$ to equation (145) for $i = 5$]. Computing the action of δ on (146), on the one hand we deduce the expression of $\hat{a}_{5,D=8}^{II.3}$

$$\hat{a}_{5,D=8}^{II.3} = 2q\varepsilon^{\mu_1\ldots\mu_8}\Big(-\overset{[5]^*}{C}_{(2,2)[\mu_1\ldots\mu_5}\overset{[2]^*}{B}_{\mu_6\mu_7} - \overset{[4]^*}{C}_{(2,1)[\mu_1\ldots\mu_4}\overset{[3]^*}{C}_{(2,0)\mu_5\mu_6\mu_7}\Big)\mathcal{C}_{(4)\mu_8]} \tag{147}$$

and on the other hand we find that the existence of $a_{4,D=8}^{II.3}$ as a solution to equation (145) for $i = 5$ imposes the condition

$$2q\varepsilon^{\mu_1\ldots\mu_8}\Big(\partial_{[\mu_1}\overset{[3]^*}{C}_{(2,0)\mu_2\mu_3\mu_4}\Big)\overset{[3]^*}{C}_{(2,0)\mu_5\mu_6\mu_7}\mathcal{C}_{(4)\mu_8]} = \partial_\mu m''^\mu_4 - \gamma a'^{II.3}_{4,D=8}. \tag{148}$$

The antifield $\overset{[3]^{*}}{C}_{(2,0)\mu_1\mu_2\mu_3}$ is bosonic and meanwhile bears an odd number of Lorentz indices, so we cannot transfer the derivative acting on it to $\mathcal{C}_{(4)\mu_8}$ [up to a total divergence] in order to provide a γ-exact term $[\partial_{[\mu_1}\mathcal{C}_{(4)\mu_2]} = \gamma(3\mathcal{C}_{(3)\mu_1\mu_2})]$. In consequence, (148) takes place if and only if its left-hand side vanishes

$$q = 0, \tag{149}$$

which further yields $a^{II.3}_{D=8} = 0$ and concludes both the proof and the paper since it confirms that neither the third subclass of 'homogeneous' solutions from class II can contribute to the first-order deformation for the model under study.

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