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Cohomological properties of the massless tensor field with the mixed symmetry $(k, 1)$. II. Results on the local cohomology of the Koszul–Tate differential

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Abstract

In this latter part of a paper dedicated to the free massless tensor field with the mixed symmetry $(k, 1)$ $(k \geq 4)$ we focus on the main cohomological properties of the Koszul–Tate differential and on the computation of the local BRST cohomology in maximum form degree.

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1 Introduction

The aim of this paper is to continue the main properties of the local BRST cohomology for the free theory describing a massless tensor field with the mixed symmetry $(k, 1)$ for $k \geq 4$ started in [1]. Actually, here we will evaluate the local cohomology of the Koszul–Tate differential and its invariant version and finally expose the key features of the local cohomology of the BRST differential in maximum form degree. In the sequel we extensively employ the conventions, notations, and results from [1].

2 Cohomologies $H(\delta|d)$ and $H^{\text{inv}}(\delta|d)$

The results included in the former part of this section are valid for any field theory with nontrivial gauge symmetries that satisfies the usual regularity conditions [2] (normal gauge theory). These results involve also the local cohomology of the Koszul–Tate differential evaluated in the algebra of local forms explicitly depending on x^μ, to be denoted by Λ_x. The form coefficients are in this situation elements of the BRST algebra depending on the spacetime coordinates, $\mathcal{A}_x$. The structure $\mathcal{A}_x$ is generated starting with the standard BRST algebra $\mathcal{A}$ enhanced with the definitions

$$\delta x^\mu = 0, \quad \gamma x^\mu = 0, \quad \cdots, \quad s x^\mu = 0, \quad \text{agh}(x^\mu) = \text{pgh}(x^\mu) = \text{gh}(x^\mu) = 0 \qquad (1)$$

and in which the space of field histories I is replaced by $\mathcal{M} \times I$, the stationary surface (of field equations) Σ by $\mathcal{M} \times \Sigma$, etc. In the context of these general statements the

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notations $\mathcal{A}$, $\mathcal{A}_x$, Λ, and Λ_x together with the operators s, δ, d, and so on refer to an arbitrary gauge field theory. The presence of eventual additional relations from (1), denoted by '$\cdots$', takes into account the fact that it is possible that the BRST differential s does not reduce to $\delta + \gamma$, but decomposes according to a (possibly infinite) number of operators (odd derivations), $s = \delta + \gamma + \sum_{j \in \mathbb{N}, j \geq 1} \overset{(j)}{s}$, the antighost number of each $\overset{(j)}{s}$ being equal to j, such that the nilpotency of s is achieved, $s^2 = 0$. (In the case where at least the operator $\overset{(1)}{s}$ is nontrivial, then γ is no longer a true differential, but just a differential modulo δ, $\gamma^2 = -(\delta \overset{(1)}{s} + \overset{(1)}{s} \delta)$, even if it still anticommutes with δ, $\gamma\delta + \delta\gamma = 0$.)

The essential difference between the algebras Λ and Λ_x consists in the different structures of the cohomology of the exterior spacetime differential computed in each of these algebras in strictly positive form degrees, known as the algebraic Poincaré Lemma [3]–[6]. Concerning the local cohomology of the Koszul–Tate differential in this general setting, the following statements can be shown to hold.

Theorem 1 *The local cohomology of the Koszul–Tate differential computed either in Λ or in Λ_x at strictly positive values of both antighost number (j) and pure ghost number is trivial*

$$H_j(\delta|d) = 0, \quad j > 0, \quad \mathrm{pgh} > 0. \tag{2}$$

Due to relation (2) from now on it is understood that the evaluation of the local cohomology of the Koszul–Tate differential in strictly positive antighost numbers is carried out only with respect to the subalgebras of local forms from Λ or Λ_x that are ghost independent.

Proposition 2 *The local cohomology of the Koszul–Tate differential in form degree zero and in strictly positive values of the antighost number computed in the BRST algebra of local functions depending or not on x^μ, but independent of ghosts, is trivial*

$$H_j^0(\delta|d) = 0, \quad j > 0. \tag{3}$$

Theorem 3 *The following isomorphism holds*

$$H_j^p(\delta|d) \simeq H_{j-1}^{p-1}(\delta|d), \quad p \geq 1, \quad j > 1, \tag{4}$$

where the local cohomology algebra $H(\delta|d)$ is computed in the algebra of local forms depending on x^μ and ghost-independent.

If we eliminate the dependence on x^μ, then Theorem 3 is still valid, but we have to factorize each side of relation (4), where possible, to the space of constant forms of appropriate degree. A significant corollary of Proposition 2 and of the last theorem is expressed by the triviality of the local cohomology $H(\delta|d)$ in antighost number strictly greater than the form degree, $H_j^p(\delta|d) = 0$ for $j > p$. Without entering further details, we mention that by terminology abuse the local cohomology of the Koszul–Tate differential at maximum form degree and strictly positive antighost number, $H_j^D(\delta|d)$ with $j > 0$, computed in the algebra of local forms dependent on x^μ and ghost-independent is also known as characteristic cohomology. For gauge field theories endowed with a well-defined Cauchy order the next theorem, essential at the evaluation of the local BRST cohomology, holds.

Theorem 4 *The characteristic cohomology for a gauge field theory with the Cauchy order equal to q is trivial in antighost numbers strictly greater than q*

$$H_j^D(\delta|d) = 0, \quad j > q. \tag{5}$$

From now on we focus on the free massless tensor with the mixed symmetry $(k,1)$. Theorem 4 together with the value $(k+1)$ of the Cauchy order for this model yields the following corollary.

Corollary 5 *The characteristic cohomology of the free theory describing a massless tensor field with the mixed symmetry $(k,1)$ is trivial in antighost numbers strictly greater than $(k+1)$*

$$H_j^D(\delta|d) = 0, \quad j > k+1. \tag{6}$$

Another ingredient necessitated by the computation of the local BRST cohomology for this model is represented by the local cohomology of the Koszul–Tate differential computed in the algebra of invariant polynomials, denoted by $H^{\mathrm{inv}}(\delta|d)$. The elements of the algebra of invariant polynomials, $H^0(\gamma) \equiv (\mathrm{Ker}(\gamma))^0$, do not depend on the ghosts; a generic representative of this algebra with a given form degree reads as in formula (44) from [1]. The cohomology $H^{\mathrm{inv}}(\delta|d)$ is defined like the set of equivalence classes of δ-closed modulo d elements $\alpha \in H^0(\gamma)$, $\delta\alpha = d\beta$, with β also an invariant polynomial, two such elements pertaining to the same equivalence class if and only if their difference is trivial. By trivial elements in this context we understand invariant polynomials that are δ-exact modulo d-exact, $\bar{\alpha} = \delta\bar{\beta} + d\bar{\gamma}$, where it is essential that both $\bar{\beta}$ and $\bar{\gamma}$ be invariant polynomials. The cohomology $H^{\mathrm{inv}}(\delta|d)$ is supercommutative and $\mathbb{N}$-graded in terms of the antighost number, $H^{\mathrm{inv}}(\delta|d) = \bigoplus_{j\in\mathbb{N}} H_j^{\mathrm{inv}}(\delta|d)$, where the space $H_j^{\mathrm{inv}}(\delta|d)$ is called invariant local cohomology of the Koszul–Tate differential in antighost number j. In addition, the invariant local cohomology of the Koszul–Tate differential at each fixed value j of agh decomposes along the form degree, $H_j^{\mathrm{inv}}(\delta|d) = \bigoplus_{p=0}^{D} H_j^{\mathrm{inv}p}(\delta|d)$, with $H_j^{\mathrm{inv}p}(\delta|d)$ the invariant local cohomology of the Koszul–Tate differential in antighost number j and form degree p. In maximum form degree and strictly positive antighost number the space $H_j^{\mathrm{inv}D}(\delta|d)$ is named, again by abuse of terminology, invariant characteristic cohomology in antighost number j. In the sequel we show that the result of Corollary 5 remains valid at the level of invariant characteristic cohomology. In view of this, we need the following lemma.

Lemma 6 *Let α be an invariant polynomial corresponding to the free theory that describes a massless tensor field with the mixed symmetry $(k,1)$, which is δ-exact, $\alpha = \delta\beta$. Then, β can be taken to be also an invariant polynomial.*

The proof of this lemma is standard [7]–[11] etc. and will not be given here, but the statement of this lemma is extremely useful in the context of the next theorem, which represents the key element in establishing the validity of (6) at the level of invariant characteristic cohomology as well as for providing some general properties of the local BRST cohomology in maximum form degree.

Theorem 7 *Let $\overset{[p]}{\alpha}_j$ be an invariant polynomial corresponding to the free theory that describes a massless tensor field with the mixed symmetry $(k,1)$, with $\mathrm{agh}(\overset{[p]}{\alpha}_j) = j \geq k+1$ and $\deg(\overset{[p]}{\alpha}_j) = p \leq D$, which is trivial in $H_j^p(\delta|d)$*

$$\overset{[p]}{\alpha}_j = \delta \overset{[p]}{\zeta}_{j+1} + d \overset{[p-1]}{\zeta}_j, \quad j \geq k+1, \quad k \geq 4, \quad 0 \leq p \leq D, \tag{7}$$

$$\mathrm{agh}(\overset{[p]}{\zeta}_{j+1}) = j+1, \quad \mathrm{agh}(\overset{[p-1]}{\zeta}_{j}) = j, \quad \deg(\overset{[p]}{\zeta}_{j+1}) = p, \quad \deg(\overset{[p-1]}{\zeta}_{j}) = p-1. \qquad (8)$$

Then, $\overset{[p]}{\alpha}_j$ is trivial also in $H_j^{\mathrm{inv}p}(\delta|d)$, so both $\overset{[p]}{\zeta}_{j+1}$ and $\overset{[p-1]}{\zeta}_{j}$ can be taken to be invariant polynomials.

Proof. We start from relation (7). Assuming $p < D$, we act with d, use its nilpotency ($d^2 = 0$) together with its anticommutation with δ, and obtain

$$d\overset{[p]}{\alpha}_j = \delta(-d\overset{[p]}{\zeta}_{j+1}). \qquad (9)$$

Due to the fact that $\overset{[p]}{\alpha}_j$ is an invariant polynomial, $d\overset{[p]}{\alpha}_j$ is also an invariant polynomial. According to formula (9), we have an invariant polynomial that is also δ-exact. Lemma 6 ensures that $d\overset{[p]}{\alpha}_j = -\delta\overset{[p+1]}{\alpha}_{j+1}$, where $\overset{[p+1]}{\alpha}_{j+1}$ is also invariant. From (9) and the previous equation we deduce that $\delta(\overset{[p+1]}{\alpha}_{j+1} - d\overset{[p]}{\zeta}_{j+1}) = 0$. Because of the acyclicity of δ in strictly positive antighost numbers, it follows that the invariant polynomial $\overset{[p+1]}{\alpha}_{j+1}$ is trivial in the space $H_{j+1}^{p+1}(\delta|d)$

$$\overset{[p+1]}{\alpha}_{j+1} = \delta\overset{[p+1]}{\zeta}_{j+2} + d\overset{[p]}{\zeta}_{j+1}. \qquad (10)$$

Applying the same procedure, after $(D-p-1)$ steps we infer the equations

$$\overset{[p+2]}{\alpha}_{j+2} = \delta\overset{[p+2]}{\zeta}_{j+3} + d\overset{[p+1]}{\zeta}_{j+2}, \qquad (11)$$

$$\vdots$$

$$\overset{[D]}{\alpha}_{j-p+D} = \delta\overset{[D]}{\zeta}_{j-p+D+1} + d\overset{[D-1]}{\zeta}_{j-p+D}, \qquad (12)$$

where all the elements denoted by α are invariant polynomials.

We start again with relation (7). Supposing $j > k+1$ and $p > 0$, we act with the operator δ and account for its nilpotency and its anticommutation with d, which imply

$$\delta\overset{[p]}{\alpha}_j = d(-\delta\overset{[p-1]}{\zeta}_{j}). \qquad (13)$$

As $\overset{[p]}{\alpha}_j$ is an invariant polynomial, we obtain that $\delta\overset{[p]}{\alpha}_j$ is also an invariant polynomial with the antighost number $j-1 \geq k+1 > 0$. According to relation (13), an invariant polynomial with agh > 0 is d-exact, such that Theorem 1 from [1] ensures that $\delta\overset{[p]}{\alpha}_j = d(-\overset{[p-1]}{\alpha}_{j-1})$, with $\overset{[p-1]}{\alpha}_{j-1}$ an invariant polynomial. Formula (13) and the last result imply the equation $d(\overset{[p-1]}{\alpha}_{j-1} - \delta\overset{[p-1]}{\zeta}_{j}) = 0$. The triviality of the cohomology of d in the algebra of local forms with agh > 0 and pgh $= 0$ grants that

$$\overset{[p-1]}{\alpha}_{j-1} = \delta\overset{[p-1]}{\zeta}_{j} + d\overset{[p-2]}{\zeta}_{j-1}. \qquad (14)$$

If $j = k+1$ in (7), then we cannot lower the antighost number like in (14). Starting with (14) and resuming the procedure between formulas (13) and (14), we notice that there appear two distinct situations, namely:

– if $j - p < k + 1$, then this procedure ends after $(j - k - 1)$ steps $(j \geq k + 1)$, at $\mathrm{agh} = k + 1$

$$\overset{[p-j+k+1]}{\alpha}_{k+1} = \delta \overset{[p-j+k+1]}{\zeta}_{k+2} + d \overset{[p-j+k]}{\zeta}_{k+1}; \tag{15}$$

– if $j - p \geq k + 1$, then this algorithm stops after p steps, at $\deg = 0$

$$\overset{[0]}{\alpha}_{j-p} = \delta \overset{[0]}{\zeta}_{j-p+1}. \tag{16}$$

In both situations the left-hand sides are invariant polynomials.

By means of results (10)–(12) and (14)–(16) it follows that equation (7) leads to a chain of similar equations, that can be organized according to the decreasing values of both agh and deg as

$$\overset{[D]}{\alpha}_{j-p+D} = \delta \overset{[D]}{\zeta}_{j-p+D+1} + d \overset{[D-1]}{\zeta}_{j-p+D}, \tag{17}$$

$$\vdots$$

$$\overset{[p+1]}{\alpha}_{j+1} = \delta \overset{[p+1]}{\zeta}_{j+2} + d \overset{[p]}{\zeta}_{j+1}, \tag{18}$$

$$\overset{[p]}{\alpha}_{j} = \delta \overset{[p]}{\zeta}_{j+1} + d \overset{[p-1]}{\zeta}_{j}, \tag{19}$$

$$\overset{[p-1]}{\alpha}_{j-1} = \delta \overset{[p-1]}{\zeta}_{j} + d \overset{[p-2]}{\zeta}_{j-1}, \tag{20}$$

$$\vdots$$

$$\overset{[p-j+k+1]}{\alpha}_{k+1} = \delta \overset{[p-j+k+1]}{\zeta}_{k+2} + d \overset{[p-j+k]}{\zeta}_{k+1} \quad \text{or} \quad \overset{[0]}{\alpha}_{j-p} = \delta \overset{[0]}{\zeta}_{j-p+1}, \tag{21}$$

in the sense that all the quantities denoted by α are invariant polynomials with $\mathrm{agh} \geq k+1$. We notice the limit case $j = k + 1$ and $p = D$, where there appear no more equations excepting the original one, (7), such that neither the superior chain (17)–(18) nor the inferior one (20)–(21) is present. If $p = D$, then the superior chain is absent; if $p = 0$ or $j = k + 1$, then the inferior one is removed.

In what follows we show that if at least one object denoted by ζ is an invariant polynomial, then all the other quantities denoted by ζ can be taken to be also invariant polynomials. Such objects may be involved in a single equation, $\overset{[D]}{\zeta}_{j-p+D+1}$ in (17) or respectively $\overset{[p-j+k]}{\zeta}_{k+1}$ in (21) if $j-p < k+1$, or in two equations otherwise. Thus, in the first case we assume that $\overset{[D]}{\zeta}_{j-p+D+1}$ is an invariant polynomial, such that $\delta \overset{[D]}{\zeta}_{j-p+D+1}$ is also invariant. Consequently, from (17) we get

$$d \overset{[D-1]}{\zeta}_{j-p+D} = \overset{[D]}{\alpha}_{j-p+D} - \delta \overset{[D]}{\zeta}_{j-p+D+1}. \tag{22}$$

Since the right-hand side is an invariant polynomial with $\mathrm{agh} \geq k+1 > 0$, the last equation shows that this is d-exact, so Theorem 1 from [1] induces that we can take $\overset{[D-1]}{\zeta}_{j-p+D}$ to be an invariant polynomial. Based on the last result, we approach the next equation and repeat this procedure until we exhaust also the last equation from the above chain. In the next case, if $j - p < k + 1$, then the last equation of the previous chain becomes the first equation from (21). Assuming that $\overset{[p-j+k]}{\zeta}_{k+1}$ is an invariant polynomial, then $d \overset{[p-j+k]}{\zeta}_{k+1}$

will also be an invariant polynomial with agh $= k+1$, such that the first equation from (21), written as

$$\delta \overset{[p-j+k+1]}{\zeta}_{k+2} = \overset{[p-j+k+1]}{\alpha}_{k+1} - d \overset{[p-j+k]}{\zeta}_{k+1}, \tag{23}$$

expresses the fact that an invariant polynomial is δ-exact, so Lemma 6 further implies that $\overset{[p-j+k+1]}{\zeta}_{k+2}$ can also be taken to be invariant. Relying on the last result we manipulate along the same line the rising equations from the chain until the first one and deduce the same conclusion, namely, that all the objects denoted by ζ can be taken to be invariant polynomials. Next, we investigate the last possibility, where an invariant polynomial $\overset{[A-1]}{\zeta}_{B}$ is involved in two equations

$$\overset{[A]}{\alpha}_{B} = \delta \overset{[A]}{\zeta}_{B+1} + d \overset{[A-1]}{\zeta}_{B}, \tag{24}$$

$$\overset{[A-1]}{\alpha}_{B-1} = \delta \overset{[A-1]}{\zeta}_{B} + d \overset{[A-2]}{\zeta}_{B-1}. \tag{25}$$

Related to (24), we reprise the same reasoning as in the above starting with $\overset{[p-j+k]}{\zeta}_{k+1}$ and formula (23). Concerning (24), we resume the arguments carried out previously beginning with $\overset{[D]}{\zeta}_{j-p+D+1}$ and relation (22). In the end we conclude that both $\overset{[A]}{\zeta}_{B+1}$ and $\overset{[A-2]}{\zeta}_{B-1}$ can be taken to be invariant polynomials. Further, each of these objects are involved in two other equations from the chain, so a similar reasoning can be repeated until we exhaust all the quantities of interest, such that all elements denoted by ζ can indeed be taken to be invariant polynomials.

The structure of the equation chain (17)–(21) accompanied by the above results shows that it is enough to prove the theorem in form degree D and in antighost numbers $j \geq k+1$ (in other words, only with respect to the first equation, of the type (17))

$$\overset{[D]}{\alpha}_{j} = \delta \overset{[D]}{\zeta}_{j+1} + d \overset{[D-1]}{\zeta}_{j}, \quad j \geq k+1, \quad k \geq 4. \tag{26}$$

More precisely, due to the previous argument related to equation (22), it is enough to prove that $\overset{[D]}{\zeta}_{j+1}$ is an invariant polynomial. In view of this, we remark that the theorem is true in form degree D and in antighost numbers $j \geq D+k+1$. Indeed, supposing that the first equation, (17), takes the form

$$\overset{[D]}{\alpha}_{j} = \delta \overset{[D]}{\zeta}_{j+1} + d \overset{[D-1]}{\zeta}_{j}, \quad j \geq D+k+1, \tag{27}$$

then we can descend to deg $= 0$, the last equation of the chain being of the latter type from (21)

$$\overset{[0]}{\alpha}_{j-D} = \delta \overset{[0]}{\zeta}_{j-D+1}, \quad j-D \geq k+1. \tag{28}$$

The last equation expresses that an invariant polynomial is δ-exact, so Lemma 6 guarantees that $\overset{[0]}{\zeta}_{j-D+1}$ can be taken to be invariant, while the arguments from the previous paragraph ensure that $\overset{[D]}{\zeta}_{j+1}$ can also be taken to be an invariant polynomial. Under these considerations, we only need to prove the theorem in form degree D and in antighost numbers $k+1 \leq j \leq D+k$, so we have to show that if the invariant polynomial $\overset{[D]}{\alpha}_{j}$ is δ-exact modulo d,

$$\overset{[D]}{\alpha}_{j} = \delta \overset{[D]}{\zeta}_{j+1} + d \overset{[D-1]}{\zeta}_{j}, \quad k+1 \leq j \leq D+k, \tag{29}$$

then we can take $\overset{[D]}{\zeta}_{j+1}$ to be an invariant polynomial.

For the sake of simplicity we pass (29) in dual language and show in what follows that if α_j is an invariant polynomial (see formula (44) from [1]) satisfying

$$\alpha_j = \delta\zeta_{j+1} + \partial_\mu m^\mu, \quad \mathrm{agh}(\alpha_j) = \mathrm{agh}(\zeta_{j+1}) - 1 = \mathrm{agh}(m^\mu) = j, \quad k+1 \leq j \leq D+k, \tag{30}$$

then ζ_{j+1} can also be taken as an element of the algebra of invariant polynomials (see formula (44) from [1]). The current m^μ and the object ζ_{j+1} are at this stage elements of the algebra $\mathcal{A}$ with pgh $= 0$ that are ghost-independent, but not necessarily invariant polynomials, in the sense that they depend only on the original field t, on the antifields Φ^*_A, and their derivatives up to a finite order, but we still do not know whether the dependence on $[t]$ is implemented or not through the curvature tensor $[K]$. Since by assumption the antighost number of ζ_{j+1} is $j+1 > k+1$, while the maximum value of the degree agh with respect to all the BRST generators is equal to $k+1$ (see Table 1 from [1]), it follows that in principle ζ_{j+1} may explicitly involve all the antifields (see formula (13) from [1]) and their derivatives up to a finite order. In order to simplify the procedure we choose to replace the antifields denoted by $\overset{(m)}{\eta}{}^*$ and $\overset{(m)}{C}{}^*$ (with $m = \overline{1, k-1}$) with the generators $\overset{(m)}{C'}{}^*$ (defined by transformation (25) from [1]), so from now on we will work with

$$\Phi^*_A \equiv \left\{ t^{*\mu_1 \ldots \mu_k|\alpha}, \left\{ \overset{(m)}{C'}{}^{*\mu_1 \ldots \mu_{k-m}||\alpha} \right\}_{m=\overline{1,k-1}}, \overset{(k)}{\eta}{}^{*\mu} \right\}. \tag{31}$$

Under these considerations, if we denote the right Euler–Lagrange derivatives of ζ_{j+1} by

$$\frac{\delta^{\mathrm{R}}\zeta_{j+1}}{\delta \overset{(k)}{\eta}{}^*_\alpha} \equiv g^\alpha_{j-k}, \qquad \frac{\delta^{\mathrm{R}}\zeta_{j+1}}{\delta \overset{(m)}{C'}{}^*_{\mu_1 \ldots \mu_{k-m}||\alpha}} \equiv g^{\mu_1 \ldots \mu_{k-m}||\alpha}_{j-m}, \quad m = \overline{1, k-1}, \tag{32}$$

$$\frac{\delta^{\mathrm{R}}\zeta_{j+1}}{\delta t^*_{\mu_1 \ldots \mu_k|\alpha}} \equiv g^{\mu_1 \ldots \mu_k|\alpha}_j, \qquad \frac{\delta^{\mathrm{R}}\zeta_{j+1}}{\delta t_{\mu_1 \ldots \mu_k|\alpha}} \equiv f^{\mu_1 \ldots \mu_k|\alpha}_{j+1}, \tag{33}$$

where the subscript of the objects denoted by g or f signifies their antighost number, then we notice that all these quantities display strictly positive values of agh on behalf of the assumption $j \geq k+1$. We maintain the significance of the double bar explained in the former part of this paper [1] and highlight that only the quantities $g^{\mu_1 \ldots \mu_k|\alpha}_j$ and $f^{\mu_1 \ldots \mu_k|\alpha}_{j+1}$ are endowed with the mixed symmetry $(k, 1)$. We employ the actions of δ on the BRST generators mentioned in the previous part (see formulas (19) and (28)–(29) in [1]) and evaluate the Euler–Lagrange derivatives of both sides of equation (30) (taking into account the right action of the Koszul–Tate differential on ζ_{j+1} and the vanishing of all Euler–Lagrange derivatives of the divergence), which produce the following equations

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta \overset{(k)}{\eta}{}^*_\alpha} = (-)^{k+1}\delta g^\alpha_{j-k}, \quad \frac{\delta^{\mathrm{R}}\alpha_j}{\delta \overset{(k-1)}{C'}{}^*_{\mu_1||\alpha}} = (-)^k \left(\delta g^{\mu_1||\alpha}_{j-k+1} - 2\partial^{\mu_1} g^\alpha_{j-k}\right), \tag{34}$$

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta \overset{(m)}{C'}{}^*_{\mu_1 \ldots \mu_{k-m}||\alpha}} = (-)^{m+1}\left(\delta g^{\mu_1 \ldots \mu_{k-m}||\alpha}_{j-m} - \partial^{[\mu_1} g^{\mu_2 \ldots \mu_{k-m}]||\alpha}_{j-m-1}\right), \quad m = \overline{1, k-2}, \tag{35}$$

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta t^*_{\mu_1 \ldots \mu_k|\alpha}} = -\left(\delta g^{\mu_1 \ldots \mu_k|\alpha}_j - \partial^{[\mu_1} g^{\mu_2 \ldots \mu_k]||\alpha}_{j-1}\right), \tag{36}$$

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta t_{\mu_1\ldots\mu_k|}{}^{\alpha}} = \delta f_{j+1}^{\mu_1\ldots\mu_k|}{}_{\alpha} + \tfrac{1}{k!}\partial_{\mu_{k+1}}\partial^{\beta}\left(\delta_{\alpha}^{[\mu_1}\delta_{\beta}^{\mu_2}g_j^{\mu_3\ldots\mu_{k+1}\mu]|}{}_{\mu}\right). \tag{37}$$

In deducing equation (37) we took into account the action of δ on the antifield t^* (see the second definition in formula (19) from [1]), the expression of the tensor involved in the Euler–Lagrange derivatives of the Lagrangian action associated with the massless tensor field $(k,1)$ (see relations (6) and (7) from [1]), and also the fact that this tensor can be equivalently written as

$$T^{\mu_1\ldots\mu_k|\alpha} = \partial_{\mu_{k+1}}\partial_{\beta}\Phi^{\mu_1\ldots\mu_{k+1}|\alpha\beta}, \tag{38}$$

where the quantity denoted by Φ exhibits the mixed symmetry $(k+1,2)$ and reads as

$$\Phi^{\mu_1\ldots\mu_{k+1}|}{}_{\alpha\beta} = -\delta_{\alpha}^{[\mu_1}\delta_{\beta}^{\mu_2}t^{\mu_3\ldots\mu_{k+1}\rho]|}{}_{\rho}. \tag{39}$$

We emphasize that relations (38)–(39) have a special meaning: they implement the Noether identities at the level of the free theory of a massless tensor field with the mixed symmetry $(k,1)$, i.e. are the general solutions to the equations

$$\partial_{\mu_1}T^{\mu_1\ldots\mu_k|\alpha} = 0, \quad \partial_{\alpha}T^{\mu_1\ldots\mu_k|\alpha} = 0. \tag{40}$$

If we adopt the notation

$$G_j^{\mu_1\ldots\mu_{k+1}|}{}_{\alpha\beta} \equiv \tfrac{1}{k!}\delta_{\alpha}^{[\mu_1}\delta_{\beta}^{\mu_2}g_j^{\mu_3\ldots\mu_{k+1}\mu]|}{}_{\mu}, \tag{41}$$

then the contravariant tensor $G_j^{\mu_1\ldots\mu_{k+1}|\alpha\beta}$ displays the mixed symmetry $(k+1,2)$ of the curvature tensor precisely due to the mixed symmetry $(k,1)$ of $g_j^{\mu_1\ldots\mu_k|\alpha}$ from (33). Consequently, (37) can be written as

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta t_{\mu_1\ldots\mu_k|\alpha}} = \delta f_{j+1}^{\mu_1\ldots\mu_k|\alpha} + \partial_{\mu_{k+1}}\partial_{\beta}G_j^{\mu_1\ldots\mu_{k+1}|\alpha\beta}. \tag{42}$$

Let us analyze now relations (34)–(36) and (42). Because α_j is by hypothesis an invariant polynomial, it follows that its Euler-Lagrange derivatives are also invariant. Moreover, we assumed that $j \geq k+1$, so $j-k>0$. The former equation in (34) shows that an invariant polynomial is δ-exact, so Lemma 6 guarantees that we can take g_{j-k}^{α} to be an invariant polynomial. Based on the last result, from the latter equation in (34) we deduce immediately that $g_{j-k+1}^{\mu_1||\alpha}$ can also be taken to be an invariant polynomial. Extending the same argument to the remaining equations we conclude that all the objects denoted by g or f are invariant polynomials and therefore (34)–(36) and (42) can be written as

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta \overset{(k)}{\eta}{}^{*}_{\alpha}} = (-)^{k+1}\delta\bar{g}_{j-k}^{\alpha}, \quad \frac{\delta^{\mathrm{R}}\alpha_j}{\delta \overset{(k-1)}{C'}{}^{*}_{\mu_1||\alpha}} = (-)^{k}\left(\delta\bar{g}_{j-k+1}^{\mu_1||\alpha} - 2\partial^{\mu_1}\bar{g}_{j-k}^{\alpha}\right), \tag{43}$$

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta \overset{(m)}{C'}{}^{*}_{\mu_1\ldots\mu_{k-m}||\alpha}} = (-)^{m+1}\left(\delta\bar{g}_{j-m}^{\mu_1\ldots\mu_{k-m}||\alpha} - \partial^{[\mu_1}\bar{g}_{j-m-1}^{\mu_2\ldots\mu_{k-m}]||\alpha}\right), \;\; m = \overline{1,k-2}, \tag{44}$$

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta t^{*}_{\mu_1\ldots\mu_k|\alpha}} = -\left(\delta\bar{g}_j^{\mu_1\ldots\mu_k|\alpha} - \partial^{[\mu_1}\bar{g}_{j-1}^{\mu_2\ldots\mu_k]||\alpha}\right), \tag{45}$$

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta t_{\mu_1\ldots\mu_k|\alpha}} = \delta\bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha} + \partial_{\mu_{k+1}}\partial_{\beta}\bar{G}_j^{\mu_1\ldots\mu_{k+1}|\alpha\beta}, \tag{46}$$

where all the bar quantities are invariant polynomials and

$$\bar{G}_j^{\mu_1\ldots\mu_{k+1}|}{}_{\alpha\beta} = \tfrac{1}{k!}\delta_\alpha^{[\mu_1}\delta_\beta^{\mu_2}\bar{g}_j^{\mu_3\ldots\mu_{k+1}\mu]|}{}_\mu. \tag{47}$$

We reconstruct the invariant polynomial α_j from its Euler–Lagrange derivatives by means of the homotopy formula

$$\alpha_j = \int_0^1 d\tau \left(\frac{\delta^{\mathrm{R}}\alpha_j}{\delta \overset{(k)}{\eta}{}^*_\alpha}(\tau)\overset{(k)}{\eta}{}^{*}_{\alpha} + \sum_{m=1}^{k-1} \frac{\delta^{\mathrm{R}}\alpha_j}{\delta \overset{(m)}{C'}{}^*_{\mu_1\ldots\mu_{k-m}||\alpha}}(\tau)\overset{(m)}{C'}{}^*_{\mu_1\ldots\mu_{k-m}||\alpha} \right.$$
$$\left. + \frac{\delta^{\mathrm{R}}\alpha_j}{\delta t^*_{\mu_1\ldots\mu_k|\alpha}}(\tau)t^*_{\mu_1\ldots\mu_k|\alpha} + \frac{\delta^{\mathrm{R}}\alpha_j}{\delta t_{\mu_1\ldots\mu_k|\alpha}}(\tau)t_{\mu_1\ldots\mu_k|\alpha} \right) + \partial_\mu l^\mu, \tag{48}$$

where, assuming that $f([y])$, we used the notation $f(\tau) \equiv f(\tau[y])$. Replacing relations (43)–(46) in (48) and recalling the actions of δ on the involved BRST generators, after some computations we arrive at

$$\alpha_j = \delta \left[\int_0^1 d\tau \left(\bar{g}^\alpha_{j-k}(\tau)\overset{(k)}{\eta}{}^*_\alpha + \sum_{m=1}^{k-1} \bar{g}_{j-m}^{\mu_1\ldots\mu_{k-m}||\alpha}(\tau)\overset{(m)}{C'}{}^*_{\mu_1\ldots\mu_{k-m}||\alpha} \right.\right.$$
$$\left.\left. + \bar{g}_j^{\mu_1\ldots\mu_k|\alpha}(\tau)t^*_{\mu_1\ldots\mu_k|\alpha} + \bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha}(\tau)t_{\mu_1\ldots\mu_k|\alpha} \right)\right] + \partial_\mu s^\mu. \tag{49}$$

We remark that all the terms on which the operator δ acts excepting the last one are true invariant polynomials. Comparing (49) with (30), it is clear that in order to complete the proof it remains to be shown that the last term on which δ acts can also be represented as an invariant polynomial. In view of this, we proceed as follows.

We start from relation (46) and recall that the Euler–Lagrange derivatives of an invariant polynomial α_j, $\delta^{\mathrm{R}}\alpha_j/\delta t_{\mu_1\ldots\mu_k|\alpha}$, are also invariant polynomials. This means that α_j depends on $[t]$ only through the components of the curvature tensor $K_{\mu_1\ldots\mu_{k+1}|\alpha\beta}$ (see formula (9) from [1]) and their derivatives up to a finite order, so

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta t_{\mu_1\ldots\mu_k|\alpha}} = (-)^{k+1}2(k+1)\partial_{\mu_{k+1}}\partial_\beta \frac{\delta^{\mathrm{R}}\alpha_j}{\delta K_{\mu_1\ldots\mu_{k+1}|\alpha\beta}}, \tag{50}$$

where the mixed symmetry $(k+1,2)$ of the curvature tensor implies the same symmetry for the Euler–Lagrange derivatives of α_j with respect to this tensor. By means of the notation

$$(-)^{k+1}2(k+1)\frac{\delta^{\mathrm{R}}\alpha_j}{\delta K_{\mu_1\ldots\mu_{k+1}|\alpha\beta}} \equiv G'^{\mu_1\ldots\mu_{k+1}|\alpha\beta}_j, \tag{51}$$

formula (50) takes the equivalent form

$$\frac{\delta^{\mathrm{R}}\alpha_j}{\delta t_{\mu_1\ldots\mu_k|\alpha}} = \partial_{\mu_{k+1}}\partial_\beta G'^{\mu_1\ldots\mu_{k+1}|\alpha\beta}_j, \tag{52}$$

where, since its left-hand side is an invariant polynomial, its right-hand side will inherit the same property, but we still do not know whether G'_j is invariant. Substituting (52) into equation (46) and denoting the difference $G'_j - \bar{G}_j$ by Δ_j, the latter becomes

$$\delta \bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha} = \partial_{\mu_{k+1}}\partial_\beta \Delta_j^{\mu_1\ldots\mu_{k+1}|\alpha\beta} \Leftrightarrow \delta \bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha} = \partial_{\mu_{k+1}}\Delta_j^{\mu_1\ldots\mu_{k+1}|\alpha\beta,}{}_\beta, \tag{53}$$

where the tensor Δ_j displays the mixed symmetry $(k+1,2)$ of the curvature tensor. Both $\bar{f}_{j+1}$ and the right-hand side are invariant polynomials, but this property cannot be passed for now to Δ_j. Equation (53) indicates that for every fixed value of the index α the quantity $\bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha}$ defines an invariant polynomial with deg $= D-k$ and agh $= j+1$, which is trivial in the cohomology $H_{j+1}^{D-k}(\delta|d)$. Applying successively Theorem 3 we find the isomorphism $H_{j+1}^{D-k}(\delta|d) \simeq H_{j+k+1}^{D}(\delta|d)$, so by Corollary 5 we have that $H_{j+k+1}^{D}(\delta|d)$=0, so we deduce that $H_{j+1}^{D-k}(\delta|d)=0$. The triviality of the last space together with (53) for each distinct value of α yields that $\bar{f}_{j+1}$ is trivial in $H_{j+1}^{D-k}(\delta|d)$

$$\bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha} = \delta R_{j+2}^{\mu_1\ldots\mu_k||\alpha} + \partial_{\mu_{k+1}} U_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha}, \tag{54}$$

where R_{j+2} and U_{j+1} are not invariant polynomials for now. Related to their symmetry properties, we only know that they are antisymmetric with respect to the Lorentz indices preceding the double bar. In the sequel the proof of the theorem proceeds in an inductive manner according to the antighost number, namely we presume it is true in deg $= D$ and in antighost numbers $(j+2)$ and respectively $(j+k+1)$ and show that is also holds in deg $= D$ and agh $= j$.

The isomorphism $H_{j+1}^{D-k}(\delta|d) \simeq H_{j+k+1}^{D}$ accompanied by the induction hypothesis in deg $= D$ and agh $= j+k+1$ implies that R_{j+2} and U_{j+1} from (54) can be taken to be some invariant polynomials, to be redenoted by $\bar{R}_{j+2}$ and $\bar{U}_{j+1}$

$$\bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha} = \delta \bar{R}_{j+2}^{\mu_1\ldots\mu_k||\alpha} + \partial_{\mu_{k+1}} \bar{U}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha}. \tag{55}$$

We modify the Lorentz indices from (55) according to further purposes and in the last term we bring the last index before the double bar to the front by successive permutations

$$\bar{f}_{j+1}^{\mu_1\ldots\mu_k|\mu_{k+1}} = \delta \bar{R}_{j+2}^{\mu_1\ldots\mu_k||\mu_{k+1}} + (-)^k \partial_\alpha \bar{U}_{j+1}^{\alpha\mu_1\ldots\mu_k||\mu_{k+1}}. \tag{56}$$

We have shown in the above that $\bar{f}_{j+1}$ possesses the mixed symmetry $(k,1)$ and hence satisfies the identity $\bar{f}_{j+1}^{[\mu_1\ldots\mu_k|\mu_{k+1}]} \equiv 0$. Taking the antisymmetric of the last equation with respect to the indices $\{\mu_1,\ldots,\mu_{k+1}\}$ and making use of the above mentioned identity, we obtain that

$$0 = \delta \bar{R}_{j+2}^{[\mu_1\ldots\mu_k||\mu_{k+1}]} + (-)^k \partial_\alpha \bar{U}_{j+1}^{\alpha[\mu_1\ldots\mu_k||\mu_{k+1}]}. \tag{57}$$

Acting with δ on (57), from its nilpotency combined with its commutation with the spacetime derivatives it follows that

$$\partial_\alpha \big(\delta \bar{U}_{j+1}^{\alpha[\mu_1\ldots\mu_k||\mu_{k+1}]}\big) = 0. \tag{58}$$

In this way, for each set of fixed values of the indices $\{\mu_1,\ldots,\mu_{k+1}\}$ the last equation defines the components of a d-closed $(D-1)$-form with pgh $= 0$ and agh $= j > 0$. The triviality of the cohomology of d (in deg $= D-1$, pgh $= 0$, and agh > 0) further induces that

$$\delta \bar{U}_{j+1}^{\alpha[\mu_1\ldots\mu_k||\mu_{k+1}]} = \partial_\beta V_j^{\mu_1\ldots\mu_{k+1}||\alpha\beta}, \tag{59}$$

where V_j is endowed with pgh $= 0$ and agh $= j > 0$, is separately antisymmetric in its first $(k+1)$ indices and in its last two ones, but does not exhibit in general the mixed symmetry $(k+1,2)$. (The detailed analysis of equation (58) discloses that since $\bar{U}_{j+1}$ is an invariant polynomial of antighost number $j+1 > k+1 > 1$, then $\delta\bar{U}_{j+1}$ is also invariant and with agh $= j > 0$, such that for any fixed values of the indices $\{\mu_1,\ldots,\mu_{k+1}\}$ we obtain a d-closed invariant polynomial with deg $= D-1$ and strictly positive agh. Theorem 1 from [1] further guarantees that we can take V_j from (59) to be also invariant. Nevertheless, in this context the invariant property of V_j is not essential.) Equation (59) signifies that for any fixed values of the indices $\{\mu_1, \ldots, \mu_{k+1}\}$ the quantity $\bar{U}_{j+1}^{\alpha[\mu_1 \ldots \mu_k||\mu_{k+1}]}$ defines an element of the local cohomology $H_{j+1}^{D-1}(\delta|d)$, so, by means of Theorem 3, we deduce the isomorphism $H_{j+1}^{D-1}(\delta|d) \simeq H_{j+2}^{D}(\delta|d)$. By hypothesis $j+2 > k+1$, and hence Corollary 5 leads to $H_{j+2}^{D}(\delta|d) = 0$, which yields $H_{j+1}^{D-1}(\delta|d) = 0$, meaning that $\bar{U}_{j+1}^{\alpha[\mu_1 \ldots \mu_k||\mu_{k+1}]}$ is trivial

$$\bar{U}_{j+1}^{\alpha[\mu_1 \ldots \mu_k||\mu_{k+1}]} = \delta W_{j+2}^{\mu_1 \ldots \mu_{k+1}||\alpha} + \partial_\beta S_{j+1}^{\mu_1 \ldots \mu_{k+1}||\alpha\beta}, \tag{60}$$

with S_{j+1} separately antisymmetric in its first $(k+1)$ and respectively last two Lorentz indices. Because all the components of $\bar{U}_{j+1}$ are invariant polynomials, it follows that its antisymmetric over the last $(k+1)$ indices is also invariant. Equation (60) defines, for each set of fixed indices $\{\mu_1, \ldots, \mu_{k+1}\}$, an invariant polynomial with $\deg = D-1$ and $\mathrm{agh} = j+1$, which is δ-exact modulo d, such that the isomorphism $H_{j+1}^{D-1}(\delta|d) \simeq H_{j+2}^{D}(\delta|d)$ together with the induction hypothesis in $\deg = D$ and $\mathrm{agh} = j+2$ imply that both W_{j+2} and S_{j+1} become invariant polynomials (to be respectively redenoted by $\bar{W}_{j+2}$ and $\bar{S}_{j+1}$)

$$\bar{U}_{j+1}^{\alpha[\mu_1 \ldots \mu_k||\mu_{k+1}]} = \delta \bar{W}_{j+2}^{\mu_1 \ldots \mu_{k+1}||\alpha} + \partial_\beta \bar{S}_{j+1}^{\mu_1 \ldots \mu_{k+1}||\alpha\beta}. \tag{61}$$

At this stage we reconstruct the tensor $\bar{U}_{j+1}^{\mu_1 \ldots \mu_{k+1}||\alpha}$ from its components antisymmetric with respect to the last $(k+1)$ indices with the help of formula

$$\begin{aligned} \bar{U}_{j+1}^{\mu_1 \ldots \mu_{k+1}||\alpha} = \tfrac{1}{k+1} \Bigg(& \bar{U}_{j+1}^{\mu_1[\mu_2 \ldots \mu_{k+1}||\alpha]} + \sum_{i=2}^{k+1} (-)^{k(i-1)} \bar{U}_{j+1}^{\mu_i[\mu_{i+1} \ldots \mu_{k+1}\mu_1 \ldots \mu_{i-1}||\alpha]} \\ & + (-)^k k \bar{U}_{j+1}^{\alpha[\mu_1 \ldots \mu_k||\mu_{k+1}]} \Bigg), \end{aligned} \tag{62}$$

where all the $(k+1)$ terms from the first line of (62) generates a quantity that is antisymmetric with respect to the indices $\{\mu_1, \ldots, \mu_{k+1}\}$. Inserting (61) in each term from the right-hand side of (62), adapting the indices appropriately, and conveniently grouping the resulting expressions, we get

$$\begin{aligned} \bar{U}_{j+1}^{\mu_1 \ldots \mu_{k+1}||\alpha} = & \delta \left[\tfrac{1}{k+1} \left(\bar{W}_{j+2}^{[\alpha\mu_1 \ldots \mu_k||\mu_{k+1}]} + (-)^k (k+1) \bar{W}_{j+2}^{\mu_1 \ldots \mu_{k+1}||\alpha} \right) \right] \\ & + \partial_{\mu_{k+2}} \left[\tfrac{1}{k+1} \left(\bar{S}_{j+1}^{\alpha[\mu_1 \ldots \mu_k||\mu_{k+1}]\mu_{k+2}} + (-)^k k \bar{S}_{j+1}^{\mu_1 \ldots \mu_{k+1}||\alpha\mu_{k+2}} \right) \right]. \end{aligned} \tag{63}$$

Obviously, the operator δ acts now on an invariant polynomial since all the components of $\bar{W}_{j+2}$ are so, which, moreover, is completely antisymmetric with respect to the indices $\{\mu_1, \ldots, \mu_{k+1}\}$. A similar observation holds with respect to the object on which the spacetime derivatives $\partial_{\mu_{k+2}}$ act. By means of the notation

$$\tfrac{1}{k+1} \left(\bar{W}_{j+2}^{[\alpha\mu_1 \ldots \mu_k||\mu_{k+1}]} + (-)^k (k+1) \bar{W}_{j+2}^{\mu_1 \ldots \mu_{k+1}||\alpha} \right) \equiv \tilde{W}_{j+2}^{\mu_1 \ldots \mu_{k+1}||\alpha}, \tag{64}$$

where $\tilde{W}_{j+2}$ is an invariant polynomial, antisymmetric with respect to its Lorentz indices placed before the double bar, result (63) becomes

$$\bar{U}_{j+1}^{\mu_1 \ldots \mu_{k+1}||\alpha} = \delta \tilde{W}_{j+2}^{\mu_1 \ldots \mu_{k+1}||\alpha} + \partial_{\mu_{k+2}} \left[\tfrac{1}{k+1} \left(\bar{S}_{j+1}^{\alpha[\mu_1 \ldots \mu_k||\mu_{k+1}]\mu_{k+2}} + (-)^k k \bar{S}_{j+1}^{\mu_1 \ldots \mu_{k+1}||\alpha\mu_{k+2}} \right) \right]. \tag{65}$$

Acting now with $\partial_{\mu_{k+1}}$ on (65) leads to

$$\partial_{\mu_{k+1}}\bar{U}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha} = \delta\big(\partial_{\mu_{k+1}}\tilde{W}_{j+2}^{\mu_1\ldots\mu_{k+1}||\alpha}\big) + \tfrac{1}{k+1}\partial_{\mu_{k+1}}\partial_\beta\big(\bar{S}_{j+1}^{[\alpha\mu_1\ldots\mu_k||\mu_{k+1}]\beta} + (-)^k(k+1)\bar{S}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha\beta}\big). \tag{66}$$

We can add the terms

$$-\tfrac{1}{k+1}\partial_{\mu_{k+1}}\partial_\beta\bar{S}_{j+1}^{[\beta\mu_1\ldots\mu_k||\mu_{k+1}]\alpha} \equiv 0 \tag{67}$$

to the right-hand side of (66) since they identically vanish due on the one hand to the symmetry with respect to the pair of indices $\{\mu_{k+1},\beta\}$ of the second-order derivative operator and on the other hand to the antisymmetry with respect to the same index pair of the quantity $\bar{S}_{j+1}$, such that formula (66) can be equivalently written as

$$\partial_{\mu_{k+1}}\bar{U}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha} = \delta\big(\partial_{\mu_{k+1}}\tilde{W}_{j+2}^{\mu_1\ldots\mu_{k+1}||\alpha}\big) + \tfrac{1}{k+1}\partial_{\mu_{k+1}}\partial_\beta\big(\bar{S}_{j+1}^{[\alpha\mu_1\ldots\mu_k||\mu_{k+1}]\beta} - \bar{S}_{j+1}^{[\beta\mu_1\ldots\mu_k||\mu_{k+1}]\alpha} + (-)^k(k+1)\bar{S}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha\beta}\big). \tag{68}$$

We notice that the operator $\partial_{\mu_{k+1}}\partial_\beta$ acts now on an object that is obviously an invariant polynomial, and, essentially, became separately antisymmetric in $\{\mu_1,\ldots,\mu_{k+1}\}$ and respectively in $\{\alpha,\beta\}$

$$\tilde{S}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha\beta} \equiv \tfrac{1}{k+1}\big(\bar{S}_{j+1}^{[\alpha\mu_1\ldots\mu_k||\mu_{k+1}]\beta} - \bar{S}_{j+1}^{[\beta\mu_1\ldots\mu_k||\mu_{k+1}]\alpha} + (-)^k(k+1)\bar{S}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha\beta}\big). \tag{69}$$

In this way we showed that

$$\partial_{\mu_{k+1}}\bar{U}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha} = \delta\big(\partial_{\mu_{k+1}}\tilde{W}_{j+2}^{\mu_1\ldots\mu_{k+1}||\alpha}\big) + \partial_{\mu_{k+1}}\partial_\beta\tilde{S}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha\beta}. \tag{70}$$

Substituting (70) in formula (55) leads to

$$\bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha} = \delta\big(\bar{R}_{j+2}^{\mu_1\ldots\mu_k||\alpha} + \partial_{\mu_{k+1}}\tilde{W}_{j+2}^{\mu_1\ldots\mu_{k+1}||\alpha}\big) + \partial_{\mu_{k+1}}\partial_\beta\tilde{S}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha\beta}. \tag{71}$$

Acting now with δ on the last result and using the nilpotency of this operator, we have that

$$\delta\bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha} = \delta\big(\partial_{\mu_{k+1}}\partial_\beta\tilde{S}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha\beta}\big). \tag{72}$$

Based on the previous result we can arrange the last term in the right-hand side of formula (49) like

$$\delta\big(\bar{f}_{j+1}^{\mu_1\ldots\mu_k|\alpha}(\tau)t_{\mu_1\ldots\mu_k|\alpha}\big) = \tfrac{1}{2(k+1)}(-)^{k+1}\delta\big[\tilde{S}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha\beta}(\tau)K_{\mu_1\ldots\mu_{k+1}|\alpha\beta}\big] + \partial_\mu\rho^\mu, \tag{73}$$

where $K_{\mu_1\ldots\mu_{k+1}|\alpha\beta}$ is the curvature tensor, so it is γ-invariant, such that the product $\tilde{S}_{j+1}K$ finally defines an invariant polynomial. Replacing relation (73) into the homotopy formula (49) for α_j we derive in the end that

$$\alpha_j = \delta\Bigg[\int_0^1 d\tau\Bigg(\bar{g}_{j-k}^{\alpha}(\tau)\overset{(k)}{\eta}{}^*_{\alpha} + \sum_{m=1}^{k-1}\bar{g}_{j-m}^{\mu_1\ldots\mu_{k-m}||\alpha}(\tau)\overset{(m)}{C'}{}^*_{\mu_1\ldots\mu_{k-m}||\alpha} + \bar{g}_j^{\mu_1\ldots\mu_k|\alpha}(\tau)t^*_{\mu_1\ldots\mu_k|\alpha} + \tfrac{1}{2(k+1)}(-)^{k+1}\tilde{S}_{j+1}^{\mu_1\ldots\mu_{k+1}||\alpha\beta}(\tau)K_{\mu_1\ldots\mu_{k+1}|\alpha\beta}\Bigg)\Bigg] + \partial_\mu\psi^\mu. \tag{74}$$

At this stage all the terms present in the argument of the integrand are invariant polynomials. A prominent role in deducing the last result is played by formula (72), where the double divergence acts on the tensor $\tilde{S}_{j+1}$, which is precisely an invariant polynomial simultaneously antisymmetric in its first $(k+1)$ and respectively last two indices (even if it does not necessarily exhibit the mixed symmetry $(k+1,2)$). Indeed, on the one hand the double antisymmetry of $\tilde{S}_{j+1}$ allows us to transfer the two derivatives on the field t and generate in (73) (up to some total derivatives) precisely the curvature tensor, which is an invariant polynomial, and on the other hand the property of $\tilde{S}_{j+1}$ of being an invariant polynomial instates the same feature at the level of $\tilde{S}_{j+1}K$.

Coming back to equation (30) compared with (74) and recalling the arguments developed so far we conclude that the induction hypothesis in form degree D and in antighost numbers $j+2$ and $j+k+1$ leads to the same property in form degree D and in antighost number j ($j=\overline{k+1,D+k}$). This completes the proof since we have shown previously that the theorem holds in $\deg = D$ and $\mathrm{agh} \geq D+k+1$. ■

The last theorem allows us to transfer the result expressed by (6) to the level of the invariant characteristic cohomology.

Corollary 8 *The invariant characteristic cohomology of the free theory that describes a massless tensor field with the mixed symmetry* $(k,1)$ *is trivial in antighost numbers strictly greater than* $(k+1)$

$$H_j^{\mathrm{inv}D}(\delta|d)=0, \quad j>k+1. \tag{75}$$

Proof. The proof is quite straightforward. Indeed, let $\overset{[D]}{\alpha}_j$ be an element of an arbitrary class from the invariant characteristic cohomology $H_j^{\mathrm{inv}D}(\delta|d)$. This means it is an invariant polynomial with $\deg = D$ and $\mathrm{agh} = j > k+1$ that fulfils the equation $\delta\overset{[D]}{\alpha}_j = d\overset{[D-1]}{\zeta}_{j-1}$, with $\overset{[D-1]}{\zeta}_{j-1}$ also an invariant polynomial. The last equation shows that $\overset{[D]}{\alpha}_j$ meanwhile pertains to an equivalence class of elements from $H_j^D(\delta|d)$ with $j>k+1$. But, in agreement with Corollary 5 (formula (6)), we have that $H_j^D(\delta|d)=0$ for $j>k+1$, such that $\overset{[D]}{\alpha}_j$ is automatically trivial in the considered class from $H_j^D(\delta|d)=0$, $\overset{[D]}{\alpha}_j = \delta\overset{[D]}{\zeta}_{j-1} + d\overset{[D-1]}{\zeta}_j$. Nevertheless, $\overset{[D]}{\alpha}_j$ is by assumption an invariant polynomial of antighost number $j>k+1$, such that Theorem 7 ensures that we can take both $\overset{[D]}{\zeta}_{j-1}$ and $\overset{[D-1]}{\zeta}_j$ from the previous relation to be also invariant polynomials. In other words, $\overset{[D]}{\alpha}_j$ is in fact trivial in the (arbitrary) chosen class from $H_j^{\mathrm{inv}D}(\delta|d)$, which proves the corollary. ■

In terms of the antifield spectrum organized like in (31) and recalling the actions of the Koszul–Tate differential on the BRST generators (see formulas (19) and (28)–(29) from [1]), it can be shown that the spaces associated with both the characteristic and respectively invariant characteristic cohomology, $H_j^D(\delta|d)$ and $H_j^{\mathrm{inv}D}(\delta|d)$, for $j=\overline{2,k+1}$, are linearly generated by the complete sets of nontrivial representatives given in Table 1. We notice that there is no nontrivial representative of either $\left(H_j^D(\delta|d)\right)_{j=\overline{2,k+1}}$ or $\left(H_j^{\mathrm{inv}D}(\delta|d)\right)_{j=\overline{2,k+1}}$ that depends on the curvature tensor or its derivatives. The same observation holds with respect to a more than linear dependence on the antifields or their derivatives. Unlike these characteristic cohomology spaces, which are finite-dimensional, the cohomology $H_1^D(\delta|d)$ (in pure ghost number zero), correlated with usual global symmetries and conservation laws, is infinite-dimensional since the theory under study is free.

agh	complete set of nontrivial representatives
$k+1$	$\overset{(k)}{\eta}{}^{*\alpha}$
$j=\overline{2,k}$	$\overset{(j-1)}{C'}{}^{*\mu_1\ldots\mu_{k-j+1}\|\alpha}$

Table 1: Structure of $\left(H_j^D(\delta|d)\right)_{j=\overline{2,k+1}}$ and $\left(H_j^{\mathrm{inv}D}(\delta|d)\right)_{j=\overline{2,k+1}}$.

3 Properties of the cohomology $H(s|d)$

We come back to the general setting of a normal gauge theory (see the beginning of section 2). The homological perturbation theory [12, 13] applied to the BRST cohomology $H(s)$ computed in any of the local form algebras Λ or Λ_x (without and respectively with an explicit dependence on x^μ) ensures that, irrespective of the structure of the decomposition of the BRST differential s along agh, $s = \delta + \gamma + \sum_{j\in\mathbb{N},j\geq 1} \overset{(j)}{s}$, only the Koszul–Tate differential and the exterior longitudinal derivative contribute to $H(s)$. More precisely, in this general framework are valid the isomorphisms mentioned in the former part of this paper (see the paragraph between formulas (35) and (36) from [1]). Similar arguments of homological perturbation theory lead to the next general results concerning the local BRST cohomology $H(s|d)$.

Theorem 9 *The local BRST cohomology computed in any of the algebras of local differential forms Λ or Λ_x is given by:*

$$H^l(s|d) \simeq H^l(\gamma|d, H_0(\delta)), \quad l \geq 0, \tag{76}$$

$$H^j(s|d) \simeq H_{-j}(\delta|d), \quad j < 0. \tag{77}$$

Just like in the first part of section 3 from [1], the values l and respectively j signify the degrees gh in relation with the local BRST cohomologies $H^l(s|d)$ and $H^j(s|d)$. On the contrary, l represents pgh with respect to the local cohomology of the exterior longitudinal derivative γ computed in $H_0(\delta)$, $H^l(\gamma|d, H_0(\delta))$, while $(-j)$ means the degree agh in connection with the local cohomology of the Koszul–Tate differential $H_{-j}(\delta|d)$. We recall that $H_0(\delta)$ means the entire cohomology algebra of δ since the Koszul–Tate differential is acyclic in strictly positive values of agh and γ may be in principle only a differential modulo d and not necessarily a true differential, as happens in the case of the $(k,1)$ theory. By means of the above theorem we remark that in gh $= l \geq 0$ the cohomological classes of $H^l(s|d)$ are completely antifield-independent, while in gh $= j < 0$ the classes of $H^j(s|d)$ are obtained by means of their minimum antighost components, $-j$, such that Theorem 1 guarantees that these components do not involve either the ghosts or their derivatives.

These general results get simplified in the context of the model describing a free, massless tensor field with the mixed symmetry $(k,1)$ due to the results analyzed in [1]. Indeed, on the one hand the decomposition $s = \delta + \gamma$ of the BRST differential in this case induces that the operator γ can be realized as a true differential, in which context the specific results contained in Table 1, formulas (67)–(68), Theorems 1 and 2, and Corollary 3 from [1] hold. On the other hand, the simple tensor gauge structure of this theory endows the differential δ with nice, lucrative properties and results, collected into Corollary 5, Theorem 7, Corollary 8, and Table 1. All these allow us to establish the next result, extremely useful at the effective computation of the general expression of the cocycles from the local BRST cohomology in maximum form degree.

Proposition 10 *From any cocycle of the local BRST cohomology corresponding to the free theory that describes a massless tensor field with the mixed symmetry* $(k,1)$ *in ghost number* $g \in \mathbb{Z}$ *and in maximum form degree,* $\overset{[D]}{a} \in H^{g,D}(s|d)$*, all components with* $\mathrm{agh} > k+1$ *can be eliminated by trivial redefinitions only. The terms from* $\overset{[D]}{a}$ *of maximum antighost number involve only nontrivial elements from the invariant characteristic cohomology.*

We stress that the term "trivial redefinition" is synonym with "transformation that does not change the equivalence classes from $H^{g,D}(s|d)$" and will be explained below. The proof of this proposition goes along the same line with the similar results from [7]–[11] and will not be given here. In the sequel we are mainly interested in the implications of this proposition. Let $\overset{[D]}{a} \in H^{g,D}(s|d)$ be a BRST cocycle of maximum form degree and with a fixed value g of the ghost number

$$s\overset{[D]}{a} + d\overset{[D-1]}{b} = 0, \tag{78}$$

$$\deg\left(\overset{[D]}{a}\right) = D, \quad \mathrm{gh}\left(\overset{[D]}{a}\right) = g \in \mathbb{Z}, \quad \deg\left(\overset{[D-1]}{b}\right) = D-1, \quad \mathrm{gh}\left(\overset{[D-1]}{b}\right) = g+1. \tag{79}$$

By trivial redefinition with respect to equation (78) we understand the simultaneous transformations

$$\overset{[D]}{a} \rightarrow \overset{[D]}{a'} = \overset{[D]}{a} + s\overset{[D]}{c} + d\overset{[D-1]}{e}, \qquad \overset{[D-1]}{b} \rightarrow \overset{[D-1]}{b'} = \overset{[D-1]}{b} + s\overset{[D-1]}{e} + d\overset{[D-2]}{f}, \tag{80}$$

$$\deg\left(\overset{[D]}{c}\right) = D, \quad \deg\left(\overset{[D-1]}{e}\right) = D-1, \quad \deg\left(\overset{[D-2]}{f}\right) = D-2, \tag{81}$$

$$\mathrm{gh}\left(\overset{[D]}{c}\right) = g-1, \quad \mathrm{gh}\left(\overset{[D-1]}{e}\right) = g, \quad \mathrm{gh}\left(\overset{[D-2]}{f}\right) = g+1, \tag{82}$$

such that

$$s\overset{[D]}{a'} + d\overset{[D-1]}{b'} \equiv s\overset{[D]}{a} + d\overset{[D-1]}{b} \tag{83}$$

via the nilpotency of the operators s and d ($s^2 = 0 = d^2$) and their anticommutation ($sd + ds = 0$) on the BRST algebra of local forms.

We decompose $\overset{[D]}{a}$ according to agh taking into account the fact that for any local form of simultaneously fixed degrees $\mathrm{gh} = g \in \mathbb{Z}$ and $\mathrm{agh} = j \geq 0$ the value of the pure ghost number is also fixed and, moreover, positive, $\mathrm{pgh} \equiv l = g + j \geq 0$, which further implies a restriction on the degree agh: $j \geq \max\{0, -g\}$. Similarly, related to $\overset{[D-1]}{b}$ there appears the restriction $j \geq \max\{0, -(g+1)\}$. Consequently, we infer the following decompositions at the level of $\overset{[D]}{a}$ and $\overset{[D-1]}{b}$

$$\overset{[D]}{a} = \sum_{j=\max\{0,-g\}}^{n} \overset{[D]}{a}_j, \quad \deg\left(\overset{[D]}{a}_j\right) = D, \quad \mathrm{agh}\left(\overset{[D]}{a}_j\right) = j, \quad \mathrm{gh}\left(\overset{[D]}{a}_j\right) = g, \tag{84}$$

$$\overset{[D-1]}{b} = \sum_{j=\max\{0,-(g+1)\}}^{n'} \overset{[D-1]}{b}_j, \ \deg\left(\overset{[D-1]}{b}_j\right) = D-1, \ \mathrm{agh}\left(\overset{[D-1]}{b}_j\right) = j, \ \mathrm{gh}\left(\overset{[D-1]}{b}_j\right) = g+1, \tag{85}$$

where n and n' are some finite natural numbers since we work in the space of local forms that are polynomials in antifields and their derivatives. The previous proposition states that we can eliminate all the components from (84) with $n > k+1$ and respectively from (85) with $n' > k+1$ meanwhile preserving the equivalence class from $H^{g,D}(s|d)$ and hence work with

$$\overset{[D]}{a} = \sum_{j=\max\{0,-g\}}^{k+1} \overset{[D]}{a}_j, \quad \overset{[D-1]}{b} = \sum_{j=\max\{0,-(g+1)\}}^{k+1} \overset{[D-1]}{b}_j. \tag{86}$$

In this way two distinct situations are met. If $g < -(k+1)$, then the range of the index j in the former relation from (86) is empty (the superior limit $(k+1)$ from the first expansion is strictly less than the inferior limit $\max\{0,-g\} = -g > k+1$) or, in other words, there are no nontrivial elements in $H^{g,D}(s|d)$. If $g \geq -(k+1)$, then formulas (86) apply.

Assume the second case and, moreover, that $g \geq 0$, which then further implies $\max\{0,-g\} = 0 = \max\{0,-(g+1)\}$, such that (86) becomes

$$\overset{[D]}{a} = \sum_{j=0}^{k+1} \overset{[D]}{a}_j, \quad \overset{[D-1]}{b} = \sum_{j=0}^{k+1} \overset{[D-1]}{b}_j. \tag{87}$$

By replacing the last two expansions in the cocycle condition (78) together with the decomposition $s = \delta + \gamma$ we generate the equivalent sequence of equations (ordered according to the decreasing values of agh)

$$\gamma \overset{[D]}{a}_{k+1} + d \overset{[D-1]}{b}_{k+1} = 0, \tag{88}$$

$$\delta \overset{[D]}{a}_{k+1} + \gamma \overset{[D]}{a}_k + d \overset{[D-1]}{b}_k = 0, \tag{89}$$

$$\vdots$$

$$\delta \overset{[D]}{a}_1 + \gamma \overset{[D]}{a}_0 + d \overset{[D-1]}{b}_0 = 0. \tag{90}$$

Corollary 3 from [1] applied to equation (88) allows us to transform $\overset{[D]}{a}_{k+1}$ such as to eliminate the component $\overset{[D-1]}{b}_{k+1}$ from (87) and replace this equation with $\gamma \overset{[D]}{a}_{k+1} = 0$ also without changing the class from $H^{g,D}(s|d)$. In this manner we reach the conclusion that in positive values of the ghost number the analysis of BRST cocycles with maximum form degree starts from assuming that $\overset{[D]}{a}$ and $\overset{[D-1]}{b}$ are expanded like

$$\overset{[D]}{a} = \sum_{j=0}^{k+1} \overset{[D]}{a}_j, \quad \overset{[D-1]}{b} = \sum_{j=0}^{k} \overset{[D-1]}{b}_j, \tag{91}$$

$$\deg\left(\overset{[D]}{a}_j\right) = D, \quad \mathrm{agh}\left(\overset{[D]}{a}_j\right) = j, \quad \mathrm{pgh}\left(\overset{[D]}{a}_j\right) = j + g, \tag{92}$$

$$\deg\left(\overset{[D-1]}{b}_j\right) = D - 1, \quad \mathrm{agh}\left(\overset{[D-1]}{b}_j\right) = j, \quad \mathrm{pgh}\left(\overset{[D-1]}{b}_j\right) = j + g + 1 \tag{93}$$

where their components are computed as solutions to the equations (equivalent to (78))

$$\gamma \overset{[D]}{a}_{k+1} = 0, \tag{94}$$

$$\delta \overset{[D]}{a}_{k+1} + \gamma \overset{[D]}{a}_k + d \overset{[D-1]}{b}_k = 0, \tag{95}$$

$$\vdots$$

$$\delta \overset{[D]}{a}_1 + \gamma \overset{[D]}{a}_0 + d \overset{[D-1]}{b}_0 = 0. \qquad (96)$$

(The properties of the local forms $\overset{[D]}{a}_j$ and $\overset{[D-1]}{b}_j$ related to agh and gh have been translated in terms of the degrees agh and pgh.) Equation (94) shows that the piece of maximum antighost number from the development of $\overset{[D]}{a}$ can be taken, without affecting the generality of the approach, as elements of the cohomology $H^{k+1+g,D}(\gamma)$ (with antighost number $(k+1)$), such that result (70) from [1] leads to its representation (up to γ-exact contributions) under the form

$$\overset{[D]}{a}_{k+1} = \sum_J \overset{[D]}{\alpha}_J\big([\Phi_A^*],[K]\big) e^J\Big(\overset{(1)}{\mathcal{F}}, \overset{(k)}{\eta}\Big), \qquad (97)$$

$$\deg(\overset{[D]}{\alpha}_J) = D, \quad \mathrm{agh}(\overset{[D]}{\alpha}_J) = k+1, \quad \mathrm{pgh}(e^J) = k+1+g. \qquad (98)$$

We recall that $\overset{[D]}{\alpha}_J$ are invariant polynomials, in this case of form degree D and of antighost number $(k+1)$ due to the first two properties from (92) in the particular case $j = k+1$, while e^J denote the elements of a basis in the space of polynomials in the tensor $\overset{(1)}{\mathcal{F}}_{\mu_1\ldots\mu_{k+1}}$ (see notation (49) from [1]) and also in the undifferentiated ghost $\overset{(k)}{\eta}$ of pure ghost number equal to $(k+1+g)$ (according to the third requirement from (92) with $j = k+1$). The final part of Proposition 10 states that the invariant polynomials present in (97) can be taken as (nontrivial) elements of the invariant characteristic cohomology,

$$\overset{[D]}{a}_{k+1} = \sum_J \overset{[D]}{\alpha}_J e^J\Big(\overset{(1)}{\mathcal{F}}, \overset{(k)}{\eta}\Big), \quad \overset{[D]}{\alpha}_J \in H_{k+1}^{\mathrm{inv}D}(\delta|d), \quad \mathrm{pgh}(e^J) = k+1+g. \qquad (99)$$

Finally, Table 1 at agh $= k+1$ provides the coefficients of the D-forms $\overset{[D]}{\alpha}_J$ as terms linear in the components of the undifferentiated antifield $\overset{(k)}{\eta}{}^*$.

4 Conclusions

The main conclusion of this two-part paper is that the systematic approach of the results exposed (here and in [1]) provides the recursive (according to the decreasing values of the antighost number), explicit computation of the BRST cocycles in maximum form degree. This technique in the particular case of ghost number 0 is required at the investigation of consistent interactions involving a massless tensor field with the mixed symmetry $(k,1)$ with the help of the deformation of the solution to the master equation [14]–[17]. Some examples in this sense can be found in [18, 19].

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