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Multiple Universes and Multiple Realities!?*

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Abstract

Adelic generalization of theoretical physics lead us to multiple wave functions of a quantum system, multiple realities and multiple universes, consequently. Our understanding of matter, especially its quantum aspects is still quite pure, but discretization appears as a quite favorable prediction of the theory, challenging our understanding of continuality and reality. It was shown that a general theory, based on number theory and nonarchimedean spaces, including multiple wave functions of the Universe, can be compatible with Cosmic Microwave Background observations. Recent investigations show that quantum tachyons could allow us to consider even more realistic inflationary models including quantum fluctuation. In this paper, we consider physical but also philosophical implications of this theoretical progress.

1 INTRODUCTION

Cosmology aims to explain the origin and evolution of the entire content of "our" Universe, its underlying physical processes, and thereby to obtain a deeper understanding of the laws of physics assumed to hold throughout the Universe. There is no other more promising field of science to help us in answering “Big Questions about the Universe“ Unfortunately, we have only one Universe to study, the one we live in, and we cannot experiment with it, just to observe. This puts serious limits on what we can learn about its origins. Moreover, our Universe might not be unique, but only one of many which have disappeared, still exist, or are yet to be created in a miraculous interplay of space-time, interactions, symmetries, numbers, etc. In this paper we are focused mainly on the very first moments in the evolution of the Universe, calculations and understanding of the wave function adjoint to the Universe as a quantum object, and on how to mathematically describe space-time around the shortest possible space-time distances - the Planck scale.

We start our paper with an overview of Classical Cosmology. Chapter 3 is devoted to a short description of the ways in which quantum physics can be involved in cosmological research, for decades reserved for a classical physics approach exclusively. Somehow "tangentially" to our main line of consideration, in chapter 4 we will advert to a period or, more generally, a concept of exponential expanding - inflation, triggered by a symmetry breaking. Despite some suspicions regarding inflation in the first few years after its

*Invited lecture at the International Conference “Big Questions about the Universe“, Cluj-Napoca, May 2009 appearance in the early 1980s, this concept has become a "paradigmatic" one in modern cosmology. It seems that inflation is an unavoidable scenario in contemporary cosmology, capable of solving or explaining some of the most bitter problems of the Classical Big Bang model(s). Chapters 5 and 6 are of crucial importance for an understanding of our approach to Cosmology, as well as to shad light on a new and promising test of relevance of the "p-adic", or more generally nonarchimedean, role in early inflation, for the first time linked to some concrete observational data. The origin of Inflation is probably one of the most intriguing questions in modern Cosmology. One of the possible candidates are quantum tachyons. Following some initial doubts regarding tachyonic inflation, because "real" (valued by means of real numbers) tachyons reach their stabile configuration too quickly to allow an explanation for the inflationary period, p-adic and, even more so, adelic valued tachyons still appear to be quite promising candidates for a realistic scenario.

This, of course, has opened many new questions. We will mention here just a few of them. For example: What is the physical meaning of infinitely many wave functions which appear in the p-adic approach; or, What is the real vacuum of a scalar field as a source of inflation; or, How many dimensions could "time" have, whatever "time" is really present, etc. Besides these questions we discuss some astonishing results, coming from a general adelic approach, which concern a very beauty of the discretization of space-time. We discuss connection of a multiple wave function of the Universe and a measurement of space-time distance and space-time structure, once we will be really able to perform such an experiment.

We end with a short conclusion and some short and quite subjective list of references for further reading. At least some of the questions touched upon in this article will be surely hot and open ones for a quite long time, and just a list of relevant references would require volumes and volumes of a journal.

2 CLASSICAL COSMOLOGY

The first theory of gravitation was put forward by Newton. In Newtonian mechanics the laws of physics were considered to operate in a flat Euclidean space, in which spatial distance could be measured on an infinite and immovable three-dimensional grid, and time was just a parameter marked out on a linear scale running from infinite past to infinite future. The foundations of modern cosmology were laid during the second and third decade of the 20th century: on the theoretical side by Einstein‘s Theory of General Relativity, which represented an indepth revision of current concepts; and on the observational side by Hubble‘s discovery of cosmic expansion, which ruled out a static Universe and set the primary requirement (expansion) on theory.

Realizing that the space we live in was not flat, except locally and approximately, Einstein proceeded to combine the principle of equivalence, gravitational interaction and accelerated system of reference with the requirement of general covariance of the fundamental physical laws [1], i.e. a form of the fundamental physical laws will be one and the same in all possible systems of reference. Einstein published his very first results in applying the General Theory of Relativity in Cosmology in 1917, but the only solution he found to the highly nonlinear differential equations was that of a static Universe. This was not entirely unsatisfactory though, because the then known Universe comprised only the stars in our Galaxy, which indeed was seen as static, and some nebulae of a controversial nature. Einstein firmly believed in a static Universe until he met Hubble in 1929 and was overwhelmed by the evidence for what was to be called Hubble‘s Law.

Immediately after general relativity became known, Willem de Sitter published (in 1917) another solution, for the case of empty space-time in an exponential state of expansion. Despite its “a priori“ nonphysical contents (empty space with a “phantom“ antigravitational force-energy, it happened that this model has persisted to be a very important and live one, after almost one century of its appearing. In 1922 Alexander Friedmann found a range of intermediate solutions to Einstein‘s equations which are the base of standard cosmology. Friedmann’s model(s) and the discovery of an expanding Universe was a natural introduction to Big Bang Theory proposed by Georges Lemaitre in 1927. The discovery of Cosmic Microwave Background (CMB) radiation by Penzias and Wilson in 1964 is acknowledged as the crucial confirmation of Big Bang Theory. Recent observations made by COBE and WMAP satellites observing this background radiation with enormous precision, have transformed cosmology from a highly speculative field into a predictive science. The newest surprise were Supernova Ia observations which show that the expansion of the Universe is accelerating [2], contrary to Friedmann cosmological models, with non-relativistic matter and radiation.

3 QUANTUM COSMOLOGY

The Universe as a whole seems classical and we can depict it from a classical perspective, using concepts and ideas typical of the macroscopic world, like space, time, geometry, gravitational forces, etc. But, we can get very interesting results if we start from the alternative description of cosmological evolution based on a quantum point of view, where the Universe can be represented as a wave propagating in abstract, multidimensional space, so-called superspace. The first models of quantum cosmology appear in the 1960s in the work of John Archibald Wheeler and Bryce DeWitt dealing with canonical quantum gravity [3]. Detailed models were formulated in the 1980‘s by Hartle, Hawking, Vilenkin, Linde, etc.

The first step in the formulation of Quantum Cosmology is the Hamiltonian formulation of General Relativity. Space time of General Relativity M is foliated in spatial hypersurfaces Σ for each value of time t, $M = \Sigma \times t$. For more details, see, for instance [4, 5, 6]. The three-dimensional Riemannian metric h_{ij} $(i, j = 1, 2, 3)$ of spatial hypersurfaces is taken as the new dynamic variable instead of the four-dimensional Lorentzian metric $g_{\mu\nu}$ $(\mu, \nu = 0, 1, 2, 3)$. The four dimensional metric $g_{\mu\nu}$ is defined by the equation

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{1}$$

The three-metric h_{ij}, appears explicitly in the metric form (the so-called Gaussian normal coordinates) as

$$ds^2 = -dt^2 + h_{ij} dx^i dx^j. \tag{2}$$

The equations of motion for h_{ij} and its conjugate momentum follow from Einstein’s equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu}, \tag{3}$$

for $g_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci tensor related to the Riemann curvature tensor, R is the curvature scalar, $T_{\mu\nu}$ is the energy-momentum tensor and k is a constant proportional to the (Newtonian) gravitational constant G and the speed of light c, i.e. $k = 8\pi G/c^4$. In addition to these dynamical variables, there are two parameters that describe how the different spatial hypersurfaces are glued together. These parameters are not dynamical and give rise to two constraints. The Hamiltonian constraint is equivalent to an identically zero Hamiltonian $H = 0$, i.e. the preservation of zero energy of the Universe.

If we want to apply this procedure to the Universe as a whole, the acceptable metrics of space-time are only those that fulfil the cosmological principle. The space of variables based on such metrics is called minisuperspace. The single degree of freedom in the h_{ij} becomes therefore the cosmological scale-factor a. In this case, the Hamiltonian reformulation of general relativity is much easier to perform than in the full theory.

In the Dirac quantization procedure, the configuration coordinate (scale factor a) and conjugate momentum became operators, $a \to a$ and $p \to -id/da$ ($\hbar = 1$). If one associates a wave function to the "baby" Universe, then the above-mentioned operators act on the wave function of the Universe $\Psi(a)$ and as a consequence of the Hamiltonian constraint, this leads to the equation

$$\hat{H}\Psi(a) = 0. \tag{4}$$

The evolution of a wave function, from one point to another, in minisuperspace represents the transition of the Universe from one geometrical "state" to another, and it is governed by this equation: Eq. (4), the so-called Wheeler-DeWitt equation. By developing Wheeler and DeWitt's idea of canonical quantum gravity, Misner initiated quantum cosmology by the application the Wheeler-DeWitt equation to a finite number of degrees of freedom [7].

As in non-relativistic quantum mechanics, canonical quantum gravity can be reformulated in terms of a path integral. This approach in quantum gravity is based on calculating transition probabilities between two spatial configurations h_{ij} $(i, j = 1, 2, 3)$, summing over all possible space-time geometries $g_{\mu\nu}$ $(\mu, \nu = 0, 1, 2, 3)$ and matter fields Φ (field histories) which interpolate between the initial and final intrinsic metrics h_{ij} and matter fields ϕ.

Historically speaking, Charles Misner introduced the "Feynman quantization of general relativity" in the form

$$\Psi \propto \int \exp\{(i/\hbar)(\text{Einstein-Hilbert action})\} d(\text{field histories}), \tag{5}$$

which connects the wave function of the Universe (and its evolution) with the Einstein-Hilbert action and the corresponding path integral.

As in the Wheeler-DeWitt approach, imposing initial conditions for the wave function is very important. There exit two proposals. In the Vilenkin proposal it is assumed that the initial condition is a vanishing spatial geometry, $h_{ij} = 0$ (or a point) and the final one is defined by some finite geometry h_{ij}. This state was called "nothing" by Vilenkin [8], but it would in fact correspond to the vacuum state of a theory more fundamental than general relativity. On the other hand, in the Hartle-Hawking proposal it is assumed that the initial condition is the set of all possible compact Euclidean geometries, such as, for example, a 4-sphere. Intuitively, this means connecting space-time with all geometries that close it smoothly below the Planck time, without tips or singularities at the beginning. To quote Stephen Hawking "The boundary condition of the universe is that it has no boundary" [9].

4 INFLATION

The standard Big Bang model describes an adiabatically expanding Universe, having "a birth" of space and time with nearly infinite temperature and density. But the Big Bang assumes very problematic initial conditions. There are also a number of problems caused by the expansion of space-time: the horizon problem related to its size at different epochs, the monopole problem associated with possible topological defects, and the flatness problem associated with its metric.

Physicists have developed a scenario, so-called cosmological inflation, to solve these problems [10]. Inflation is not a replacement for the Hot Big Bang theory, but rather an extra add-on idea which is supposed to apply during some very early stage of the Universe's expansion. Inflation is defined as a period in the evolution of the Universe during which the scale factor was accelerating

$$\ddot{a} > 0, a \propto e^t. \tag{6}$$

The first inflationary model was proposed by Alexei Starobinsky in 1979 and it was rather complicated. A much simpler model, with very clear physical motivation, was proposed by Alan Guth in 1981. His model, which is now called "old inflation", was based on the theory of supercooling during the cosmological phase transition. The end state of this model would be a highly chaotic universe, quite the opposite of what was intended. This model was therefore abandoned soon after it was suggested. The successor to old inflation was new inflation (Linde 1982, Albrecht and Steinhardt 1982) [10]. Thanks to Linde, we are now faced with a really challenging idea with a profound consequence for traditional philosophy-ontology and humankind‘s shared understanding of a unique Universe. Namely, our Universe, suggested Linde, is only a small causal part of a single bubble, and there are an infinity of bubbles. In fact, there is an infinity of universes like ours on a single bubble which is more like an expanding infinite sheet than a finite spherical surface.

Today, cosmological inflation has become an integral part of the standard model of the Universe. It provides important clues for structure formation in the Universe and is capable of removing the shortcomings of standard cosmology. Although inflation is usually described using canonical scalar fields, there also exist attempts to engage DBI-type (Dirac-Born-Infeld) scalar fields [11]. Looking for the "right" scenario of inflation, Gibbons emphasized the cosmological implication of tachyonic condensate rolling towards its ground state. Tachyonic matter, particles which travel faster than the speed of light or fast decaying unstable scalar fields, might provide an explanation for inflation and could contribute to a new form of dark matter (and energy)! We will return to this problem later on in this article in the context of p-adic inflation.

5 p-ADIC NUMBERS AND ADELES

Before saying anything about p-adic inflation, we will begin with a very brief survey of some basic properties of p-adic numbers and adeles. There are at least two reasons-motivations for the involvement of p-adic numbers (and adeles) in mathematical physics, more precisely in theoretical high energy physics (first of all in the string theory framework) and (quantum) cosmology. The first one, if we may be more pragmatical, is unavoidable and and appears to be rather general, appearing of divergencies in calculation of string amplitudes and their cross-sections the closer we are to the Planck scale. The second one has its origin in space-time structure and background mathematics. Let us list a few "facts": the field of rational numbers Q, which contains all observational and experimental numerical data, is a dense subfield not only in the field of real numbers R but also in the fields of p-adic numbers Q_p (where p is a prime number); general mathematical methods and fundamental physical laws should be invariant under an interchange of the number fields R and Q_p; there is a quantum gravity uncertainty in measuring distances around the Planck length which restricts the priority of Archimedean geometry based on real numbers and gives rise to the employment of non-Archimedean geometry related to p-adic numbers, etc.

As we have noted above, completion of the set of rational numbers Q with respect to the standard absolute value ($|\cdot|_\infty$) gives the fields of real numbers R, and an algebraic extension of R makes complex numbers C. According to the Ostrowski theorem [12] any non-trivial norm on the field of rational numbers Q is equivalent to the absolute value $|\cdot|_\infty$ or to the p-adic norm $|\cdot|_p$, where p is a prime number. The p-adic norm is a nonarchimedean (ultrametric) one and for a rational number, $0 \neq x \in Q$, $x = p^\nu \frac{m}{n}$, $0 \neq n, \nu, m \in Z$, has a value $|x|_p = p^{-\nu}$. Completion of Q with respect to the p-adic norm for a fixed p leads to the corresponding field of p-adic numbers Q_p. Completions of Q with respect to $|\cdot|_\infty$ and all $|\cdot|_p$ exhaust all possible completions of Q. p-Adic number $x \in Q_p$, in the canonical form, is an infinite expansion

$$x = p^\nu \sum_{i=0}^{\infty} x_i p^i, \qquad x_0 \neq 0, \quad 0 \leq x_i \leq p-1. \tag{7}$$

The norm of p-adic number x in (7) is $|x|_p = p^{-\nu}$ and satisfies not only the triangle inequality, but also the stronger one

$$|x+y|_p \leq \max(|x|_p, |y|_p). \tag{8}$$

Real and p-adic numbers are unified in the form of the adeles [13]. An adele is an infinite sequence

$$a = (a_\infty, a_2, ..., a_p, ...), \tag{9}$$

where $a_\infty \in Q_\infty$, and $a_p \in Q_p$, with restriction to $a_p \in Z_p$ ($Z_p = \{x \in Q_p : |x|_p \leq 1\}$) for all but a finite set S of primes p. If we introduce $\mathcal{A}(S) = Q_\infty \times \prod_{p \in S} Q_p \times \prod_{p \notin S} Z_p$ then the space of all adeles is $\mathcal{A} = \bigcup_S \mathcal{A}(S)$, which is a topological ring.

Even mathematical analysis over Q_p and $\mathcal{A}(S)$ (p-adic integration, Fourier transformation, p-adic series, etc.) is just a relatively small part of the whole of mathematics, or of general mathematical physics, and is still too comprehensive to be discussed here. We recommend reference [12] to a reader interested in this part of modern mathematical physics, for details and references therein.

6 p-ADIC INFLATION

Many string theorists and cosmologists have turned their attention to building and testing stringy models of inflation in recent years. The goals have been to find natural realizations of inflation within string theory, and novel features which would help to distinguish string-based models from their more conventional field theory counterparts. In most examples to date, string theory has been used to derive an effective 4D field theory operating at energies below the string scale and all inflationary predictions are made within the context of this low energy effective field theory. This is a perfectly valid approach to string cosmology, but at least a few problems still exist. For instance it is often very difficult to identify features of string theory inflation that cannot be reproduced in more conventional models. Thus, there is motivation to consider models in which inflation takes place at higher energy scales where stringy corrections to the low energy effective action play an important role. This is usually daunting since field theory description should be supplemented by an infinite number of higher dimensional operators at energies above the string scale, whose precise form is not known. Therefore, to study nonlocality (intimately connected with p-adic and nonarchimedean themes [14]) as ubiquitous in string field theory and to consider a broad class of nonlocal inflationary models is a quite interesting area of research.

As we have noted, Gibbons has emphasized the cosmological implication of tachyonic condensate rolling towards its ground state [11]. A recent paper on p-adic inflation gives rise to hopes that nonlocal inflation can succeed where real string theory fails [15]. p-Adic string theory, initiated by Volovich in his pioneering paper [16] and developed by Arefeva, Dragovich, Goshal, Frampton, Freund, Sen, Witten and many others, despite some open and serious problems, is an interesting and wide field of research.

Starting from the action of the p-adic string, with m_s the string mass scale and g_s the open string coupling constant,

$$S = \frac{m_s^4}{g_p^2} \int d^4x \left(-\frac{1}{2}\phi p^{-\frac{-\partial_t^2+\bigtriangledown^2}{2m_s^2}}\phi + \frac{1}{p+1}\phi^{p+1} \right), \quad \frac{1}{g_p^2} = \frac{1}{g_s^2}\frac{p^2}{p-1}, \tag{10}$$

for the open string tachyon scalar field $\phi(x)$, it has been shown that a p-adic tachyon drives a sufficiently long period of inflation while rolling away from the maximum of its potential. Even though this result is constrained by $p \gg 1$ and obtained by an approximation, it presents a good impetus for a consideration of p-adic inflation for different tachyonic potentials. In particular, it would be interesting to study p-adic inflation in the quantum regime and in the adelic framework to overcome the constraint $p \gg 1$ with an unclear physical meaning.

7 CLASSICAL AND QUANTUM TACHYONS

Tachyon fields represent a very interesting research topic in several branches of High Energy Physics. But our understanding of tachyons, in particular their quantum aspects, is quite pure. Here we would like to present or results related to the quantization of real and p-adic tachyons. A. Sen proposed a (DBI-type) field theory of tachyon matter a few years ago [17]. The action is given as:

$$S = -\int d^{D+1}x V(T)\sqrt{1+\eta^{ij}\partial_i T \partial_j T} \tag{11}$$

where $\eta_{00} = -1$ and $\eta_{\alpha\beta} = \delta_{\alpha\beta}$, $\alpha, \beta = 1, 2, 3, ..., D$, $T(x)$ is the scalar tachyon field and $V(T)$ is the tachyon potential which unusually appears in the action as a multiplicative factor and has (from string field theory arguments) exponential dependence with respect to the tachyon field $V(T) \sim e^{-\alpha T/2}$. It is very useful to understand and to investigate lower dimensional analogs of this tachyon field theory. The corresponding zero dimensional analog of a tachyon field can be obtained by the correspondence: $x^i \to t, T \to x, V(T) \to V(x)$. The action reads

$$S = -\int dt V(x)\sqrt{1-\dot{x}^2}. \tag{12}$$

In what follows, all variables and parameters can be treated as real or p-adic without a formal change in the obtained forms. It is not difficult to see that action (12), with some appropriate replacements leads to the equation of motion for a particle with mass m, under a constant external force, in the presence of quadratic damping:

$$m\ddot{y} + \beta\dot{y}^2 = mg. \tag{13}$$

This equation of motion can be obtained from two Lagrangians [18, 19, 20]:

$$L(y,\dot{y}) = \left(\frac{1}{2}m\dot{y}^2 + \frac{m^2 g}{2\beta}\right) e^{2\frac{\beta}{m}y}, \tag{14}$$

$$L(y,\dot{y}) = -e^{-\frac{\beta}{m}y}\sqrt{1 - \frac{\beta}{mg}\dot{y}^2}. \tag{15}$$

Despite the fact that different Lagrangians can give rise to nonequivalent quantization, we will choose the form (14) that can be handled easily. Using the first one is more convenient because of the presence of the square root in the second one. The general solution of the equation of motion is

$$y(t) = C_2 + \frac{m}{\beta}\ln[\cosh(\sqrt{\frac{g\beta}{m}}\,t + C_1)]. \tag{16}$$

For the initial and final conditions $y' = y(0)$ and $y'' = y(T)$, for the υ-adic classical action we obtain

$$\bar{S}_\upsilon(y'',T;y',0) = \frac{\sqrt{mg\beta}}{2\sinh(\sqrt{\frac{g\beta}{m}}T)}\left[(e^{\frac{2\beta}{m}y'} + e^{\frac{2\beta}{m}y''})\cosh(\sqrt{\frac{g\beta}{m}}T) - 2e^{\frac{\beta}{m}(y'+y'')}\right]. \tag{17}$$

In the p-adic case, we get a constraint which arises from the investigation of the domain of a convergence analytical function which appears during the derivation of the formulae (16).

By the transformation $X = \frac{m}{\beta}e^{\frac{\beta}{m}y}$, we can convert Lagrangian (14) in a more suitable, quadratic form

$$L(X,\dot{X}) = \frac{m\dot{X}^2}{2} + \frac{g\beta X^2}{2}. \tag{18}$$

For the conditions $X' = X(0)$, and $X'' = X(T)$, the action for the classical υ-adic solution $\bar{X}(t)$ is

$$\bar{S}_\upsilon(X'',T;X',0) = \frac{\sqrt{mg\beta}}{2\sinh(\sqrt{\frac{g\beta}{m}}T)}\left[(X'^2 + X''^2)\cosh(\sqrt{\frac{g\beta}{m}}T) - 2X'X''\right]. \tag{19}$$

Because the action (19) is quadratic one (with respect to the initial and final points), the corresponding kernel is [21]

$$\mathcal{K}_\upsilon(X'',T;X',0) = \lambda_\upsilon\left(\frac{1}{2h}\frac{\sqrt{g\beta m}}{\sinh(\sqrt{\frac{g\beta}{m}}T)}\right)\left|\frac{1}{h}\frac{\sqrt{g\beta m}}{\sinh(\sqrt{\frac{g\beta}{m}}T)}\right|_\upsilon^{1/2}\chi_\upsilon\left(-\frac{1}{h}\bar{S}_\upsilon\right), \tag{20}$$

where χ_υ is the υ-adic additive character. The resulting p-adic wave functions (for the ground state(s) and $p \neq 2$!) have the form

$$\Psi_p(X) = \Omega(|X|_p), \quad |T|_p \leq \left|\frac{m}{2h}\right|_p, \quad \left|\frac{g\beta m}{4h^2}\right|_p < 1 \tag{21}$$

$$\Psi_p(X) = \Omega(p^\nu |X|_p), \quad |T|_p \leq \left|\frac{m}{2h}\right|_p p^{-2\nu}, \quad \left|\frac{g\beta m}{4h^2}\right|_p \leq p^{3\nu} \tag{22}$$

$$\Psi_p(X) = \delta(p^\nu - |X|_p), \quad \left|\frac{T}{2}\right|_p \leq \left|\frac{m}{h}\right|_p p^{2\nu-2}, \quad \left|\frac{g\beta m}{h^2}\right|_p \leq p^{2-3\nu}. \tag{23}$$

The above conditions are in accordance with the conditions for the convergence of the p-adic analytical functions which appear in the solution of the equation of motion (16) and the classical action (17). We see there is a breadth of freedom in choosing the parameters of the model, such as mass of the tachyon field m, damping factor β, parameter g related to the "strength of the constant gravity", and cosmological constant Λ which appears in the de Sitter $(2+1)$ dimensional model. A relevant physical conclusion served from these relations still needs a more realistic model with tachyon matter and with a precise form of metrics. In any case, we see that there are many different wave functions describing the ground state. We can also conclude that because of quantum fluctuation a particle or "baby" universe can be described by many different functions and that there can exist many, (infinitely many, as infinitely many primes p exist, for instance) different quantum Universes in parallel.

8 CONCLUSION

In this paper, we have presented a brief overview of open problems in Cosmology and Partcile Physics, in addition to mentioning a few important results achieved in the course of the modern history of physics, in particular since 1915 and Einstein's General Relativity. We have also presented some applications of p-adic numbers and adeles in quantum cosmology. This provides plenty of new possibilities for the investigation of the structure of space-time at the Planck scale. All these models lead to the picture of space-time as a discrete one. Namely, for all the above models there exists the adelic wave function

$$\Psi(q^1, ..., q^n) = \prod_{\alpha=1}^{n} \Psi_\infty(q_\infty^\alpha) \prod_p \prod_{\alpha=1}^{n} \Omega(|q_p^\alpha|_p), \tag{24}$$

where $\Psi_\infty(q_\infty^\alpha)$ are the corresponding wave functions of the universe in standard cosmology. Adopting the usual probability interpretation of the wave function (24) in rational points of q^α, we have

$$\left|\Psi(q^1, ..., q^n)\right|_\infty^2 = \prod_{\alpha=1}^{n} |\Psi_\infty(q^\alpha)|_\infty^2 \prod_p \prod_{\alpha=1}^{n} \Omega(|q^\alpha|_p), \tag{25}$$

because $(\Omega(|q^\alpha|_p))^2 = \Omega(|q^\alpha|_p)$. As a consequence of Ω-function properties we have

$$\left|\Psi(q^1, \ldots, q^n)\right|_\infty^2 = \begin{cases} |\Psi_\infty(q^\alpha)|_\infty^2, & q^\alpha \in Z, \\ 0, & q^\alpha \in Q \backslash Z. \end{cases} \tag{26}$$

This result leads to the discretization of minisuperspace coordinates q^α, because probability is nonzero only in the integer points of q^α. Keeping in mind that Ω function is invariant with respect to the Fourier transform, this conclusion is also valid for momentum space. Note that this kind of discreteness depends on the adelic quantum state of the universe. When the system is in an excited state, the sharp discrete structure disappears and minisuperspace, as well as configuration space in quantum mechanics, demonstrate the usual properties of real space.

Recent results in nonlocal (p-adic) tachyon inflation [15], in which a p-adic tachyon drives a sufficiently long period of inflation while rolling away from the maximum of its potential, deserve much more attention. The classical p-adic models succeed with inflation where real string theory fails.

The obtained quantum propagator for the p-adic and adelic tachyons, as well as conditions for the existence of the vacuum state of p-adic and adelic tachyons, can be used as a basis for further investigation of (p-adic) quantum mechanical damped systems and corresponding wave functions of the universe in minisuperspace models based on tachyonic matter with different potentials. Further investigation should contribute to a better understanding of the quantum rolling tachyon scenario in a real and p-adic cases. And in this case we could continue, with much more confidence, in more challenging investigations related to questions such as: how and why did the universe begin, is time travel physically or logically possible, and many others.

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