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On the path integral quantization of the massive 4-forms

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Abstract

Massive 4-forms are analyzed from the point of view of the Hamiltonian quantization using the gauge-unfixing approach. This method finally output the manifestly Lorentz covariant path integral for 3- and 4-forms with Stückelberg coupling.

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1 Introduction

Antisymmetric tensor fields of various orders (p-form gauge fields) has been intensively studied as they play an important role in modern theories like string and superstring theory, supergravity and the gauge theory of gravity [1]–[5]. Particulary they are included within the supergravity multiplets of many supergravity theories [3]–[4]. Moreover, p-forms have a special place in the theory of p-branes [5], where $(p+1)$-forms couple naturally to p-branes. In fact, it is known that the configuration space for closed p-branes is nothing but the space of all closed p-manifolds embedded in space-time, in which background rank-$(p+1)$ antisymmetric tensor fields should be analyzed in connection with their geometric aspects.

The main aim of this paper is to quantize massive 4-forms using gauge-unfixing method [6]–[7]. This approach relies on separating the second-class constraints into two subsets, one of them being first-class and the other providing some canonical gauge conditions for the first-class subset. Starting from the canonical Hamiltonian of the original second-class system, one constructs a first-class Hamiltonian with respect to the first-class subset through an operator that projects any smooth function defined on the phase-space into a function that is in strong involution with the first-class subset. A systematic BRST treatment of the gauge-unfixed method has been realized in [8]–[9].

This paper is organized in four sections. In Section 2 we start from a bosonic second-class constrained system and briefly expose the above mentioned method of constructing first-class system equivalent with the original theory. In Section 3 we apply the gauge-unfixed method to massive 4-forms and meanwhile obtain the path integral corresponding to the first-class system associated with this model. After integrating out the auxiliary fields and performing some field redefinitions, we obtain nothing but the manifestly Lorentz covariant path integral corresponding to the Lagrangian formulation of the first-class system, which reduce to the Lagrangian path integral for Stückelberg-coupled 3- and 4-forms. Section 4 ends the paper with the main conclusions.

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2 Gauge unfixing (GU) method

The starting point is a bosonic dynamic system with the phase-space locally parameterized by n canonical pairs $z^a = (q^i,\ p_i)$, endowed with the canonical Hamiltonian H_c, and subject to the purely second-class constraints

$$\chi_{\alpha_0}(z^a) \approx 0, \qquad \alpha_0 = \overline{1, 2M_0}. \tag{1}$$

Assume that one can split the second-class constraint set (1) into two subsets

$$\chi_{\alpha_0}(z^a) \equiv \left(G_{\bar{\alpha}_0}(z^a), C^{\bar{\beta}_0}(z^a)\right) \approx 0, \qquad \bar{\alpha}_0, \bar{\beta}_0 = \overline{1, M_0}, \tag{2}$$

such that

$$\left[G_{\bar{\alpha}_0}, G_{\bar{\beta}_0}\right] = D^{\bar{\gamma}_0}_{\bar{\alpha}_0\bar{\beta}_0} G_{\bar{\gamma}_0}. \tag{3}$$

Relations (3) yield the subset

$$G_{\bar{\alpha}_0}(z^a) \approx 0 \tag{4}$$

to be first-class. The second-class behaviour of the overall constraint set ensures that

$$C^{\bar{\alpha}_0}(z^a) \approx 0 \tag{5}$$

may be regarded as some gauge-fixing conditions for first-class set (4).

We introduce an operator $\hat{X}$ [10]-[11] that associates with every smooth function F on the original phase-space an application $\hat{X}F$, which is in strong involution with the functions $G_{\bar{\alpha}_0}$,

$$\hat{X}F = F - C^{\bar{\alpha}_0}\left[G_{\bar{\alpha}_0}, F\right] + \frac{1}{2} C^{\bar{\alpha}_0} C^{\bar{\beta}_0}\left[G_{\bar{\alpha}_0}, \left[G_{\bar{\beta}_0}, F\right]\right] - \cdots, \tag{6}$$

$$\left[\hat{X}F, G_{\bar{\alpha}_0}\right] = 0. \tag{7}$$

With the help of this operator we construct a first-class (with respect to the constraints (4)) Hamiltonian $\hat{X}H_c$ starting from canonical Hamiltonian H_c.

If we denote by $\mathcal{S}_O$ and $\mathcal{S}_{GU}$ the original and respectively the gauge-unfixed system, then they are classically equivalent since they possess the same number of physical degrees of freedom

$$\mathcal{N}_O = \frac{1}{2}(2n - 2M_0) = \mathcal{N}_{GU} \tag{8}$$

and the corresponding algebras of classical observables are isomorphic

$$Phys(\mathcal{S}_O) = Phys(\mathcal{S}_{GU}). \tag{9}$$

Consequently, the two systems become also equivalent at the level of the path integral quantization, and we can to replace the Hamiltonian path integral of the original second-class theory

$$\begin{aligned} Z_O &= \int \mathcal{D}(z^a, \lambda^{\alpha_0}) \det\left(\left[G_{\bar{\alpha}_0}, C^{\bar{\beta}_0}\right]\right) \times \\ &\quad \exp\left[\mathrm{i}\int dt\left(\dot{q}^i p_i - H_c - \lambda^{\alpha_0}\chi_{\alpha_0}\right)\right], \end{aligned} \tag{10}$$

with the Hamiltonian path integral of the gauge-unfixed first-class system

$$\begin{aligned} Z_{GU} &= \int \mathcal{D}(z^a, \lambda^{\bar{\alpha}_0}) \left(\prod_{\bar{\alpha}_0} \delta(C^{\bar{\alpha}_0})\right)\left(\det\left(\left[G_{\bar{\alpha}_0}, C^{\bar{\beta}_0}\right]\right)\right) \times \\ &\quad \times \exp\left[\mathrm{i}\int dt\left(\dot{q}^i p_i - \hat{X}H_c - \lambda^{\bar{\alpha}_0} G_{\bar{\alpha}_0}\right)\right]. \end{aligned} \tag{11}$$

3 Massive 4-forms

We start from the Lagrangian action of massive 4-forms in $D \geq 5$ [12]–[13]

$$S_0^L\left[A_{\mu\nu\rho\lambda}\right] = \int d^D x \left(-\frac{1}{2\cdot 5!}F_{\mu\nu\rho\lambda\sigma}F^{\mu\nu\rho\lambda\sigma} - \frac{m^2}{2\cdot 4!}A_{\mu\nu\rho\lambda}A^{\mu\nu\rho\lambda}\right). \tag{12}$$

with m the mass of $A_{\mu\nu\rho\lambda}$ and $F_{\mu\nu\rho\lambda\sigma}$ the field strength of the 4-form, defined in the standard manner as $F_{\mu\nu\rho\lambda\sigma} = \partial_{[\mu}A_{\nu\rho\lambda\sigma]}$. Everywhere in this paper the notation $[\mu\nu\ldots\rho]$ signifies complete antisymmetry with respect to the (Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. We work with the Minkowski metric tensor of 'mostly minus' signature $\sigma_{\mu\nu} = \sigma^{\mu\nu} = \mathrm{diag}\,(+-\ldots-)$. In the sequel we denote by $\pi^{\mu\nu\rho\lambda}$ the canonical momenta respectively conjugated with $A_{\mu\nu\rho\lambda}$. The canonical analysis of this model [14]–[15] provides the constraints

$$\chi^{(1)}_{ijk} \equiv \pi_{0ijk} \approx 0, \tag{13}$$

$$\chi^{(2)ijk} \equiv 4\partial_l \pi^{lijk} - \frac{m^2}{3!}A^{0ijk} \approx 0, \tag{14}$$

and the canonical Hamiltonian

$$\begin{aligned} H_c(x^0) = & \int d^{D-1}x \left(-12\pi_{ijkl}\pi^{ijkl} + \frac{1}{2\cdot 5!}F_{ijkl}F^{ijkl}\right. \\ & \left. + \frac{m^2}{2\cdot 4!}A_{\mu\nu\rho\lambda}A^{\mu\nu\rho\lambda} - 4A_{0ijk}\partial_l\pi^{lijk}\right). \end{aligned} \tag{15}$$

The matrix of the Poisson brackets among the above constraints is expressed by

$$\left(\left[\chi_{\alpha_0}, \chi_{\beta_0}\right]\right) = \frac{m^2}{3!}\begin{pmatrix} \mathbf{0} & \frac{1}{3!}\delta^l_{[i}\delta^p_j\delta^q_{k]} \\ -\frac{1}{3!}\delta^i_{[l}\delta^j_p\delta^k_{q]} & \mathbf{0} \end{pmatrix} \tag{16}$$

and is easy to see that it is invertible. In consequence, the constraints (13) and (14) are second-class and irreducible [16].

According to the GU method we consider (14) as the first-class constraint set and the remaining constraints (13) as the corresponding canonical gauge conditions and redefine the first-class constraints as

$$G^{ijk} \equiv -\frac{1}{m^2}\left(4\partial_l\pi^{lijk} - \frac{m^2}{3!}A^{0ijk}\right) \approx 0. \tag{17}$$

The other choice, (13) as the first-class constraint set and the remaining constraints (14) as the corresponding canonical gauge conditions yields a path integral that cannot be written in a manifestly covariant form [17]–[18]. The first-class Hamiltonian with respect to (17) follows from relation (6) where $H_c(y^0)$ is displayed in (15)

$$\begin{aligned} &\hat{X}H_c(y^0) = H_c(y^0) - \int d^{D-1}y\chi^{(1)}_{ijk}(y)\left[G^{ijk}(y), H_c(y^0)\right] \\ &+\frac{1}{2}\int d^{D-1}yd^{D-1}z\chi^{(1)}_{ijk}(y)\,\chi^{(1)}_{lqp}\left(y^0,\mathbf{z}\right)\times \\ &\times\left[G^{ijk}(y)\left[G^{lqp}\left(y^0,\mathbf{z}\right), H_c\left(y^0\right)\right]\right] - \cdots \end{aligned}$$

$$= H_c(y^0) - \int d^{D-1}y \left[\frac{1}{4!}\partial_{[i}\pi_{jkl]0}A^{ijkl} - \frac{1}{m^2}\frac{1}{2\cdot 4!}\partial_{[i}\pi_{jkl]0}\partial^{[i}\pi^{jkl]0} \right]. \tag{18}$$

In order to obtain an manifestly Lorentz covariant path integral we pass to another first-class system equivalent with the original, second-class one at both classical and path integral levels. It is well known that any irreducible set of constraints can always be replaced by a reducible one by introducing constraints that are consequences of the ones already at hand [16]. In view of this, we supplement (17) with $G^{ij} \equiv -\frac{m^2}{3!}\partial_k G^{kij} \approx 0$, such that the new constraint set

$$G^{ijk} \equiv -\frac{1}{m^2}\left(4\partial_l \pi^{lijk} - \frac{m^2}{3!}A^{0ijk}\right) \approx 0, \tag{19}$$

$$G^{ij} \equiv -\frac{m^2}{3!}\partial_k A^{0kij} \approx 0, \tag{20}$$

remains first-class and, moreover, becomes off-shell third-order reducible, with first-order reducibility relations

$$Z^{lq}_{ijk}G^{ijk} + Z^{lq}_{ij}G^{ij} = 0, \qquad Z^{l}_{ij}G^{ij} = 0, \tag{21}$$

the second-order ones

$$Z^{p}_{lq}Z^{lq}_{ijk} = 0, \qquad Z^{p}_{lq}Z^{lq}_{ij} + Z^{p}_{l}Z^{l}_{ij} = 0, \qquad Z_l Z^{l}_{ij} = 0, \tag{22}$$

and respectively third-order reducibility relations

$$Z_p Z^{p}_{lq} = 0, \qquad Z_p Z^{p}_{l} + Z Z_l = 0. \tag{23}$$

The reducibility functions reads as

$$Z^{lq}_{ijk} = \frac{1}{3!}\delta^{l}_{[i}\delta^{q}_{j}\partial_{k]}, \qquad Z^{lq}_{ij} = \frac{1}{2m^2}\delta^{l}_{[i}\delta^{q}_{j]}, \qquad Z^{lq}_{ij} = \frac{1}{2}\delta^{l}_{[i}\partial_{j]}, \tag{24}$$

$$Z^{p}_{l} = -\frac{1}{m^2}\delta^{p}_{l}, \qquad Z_l = \partial_l, \qquad Z = \frac{1}{m^2}. \tag{25}$$

This procedure preserves the classical equivalence with the first-class theory from the GU method since the number of physical degrees of freedom or the algebra classical observables does not change, and keeps the first-class Hamiltonian, such that the evolution is not affected. The GU and third-order reducible first-class systems remain equivalent also at the level of the Hamiltonian path integral quantization. This further implies, given the established equivalence between the GU first-class system and the original second-class theory, that the third-order reducible first-class system is completely equivalent with the original second-class theory.

At this stage, it is useful to make the canonical transformation

$$A^{0ijk} \longrightarrow -\frac{1}{m^2}\Pi^{ijk}, \qquad \pi^{0ijk} \longrightarrow m^2 B^{ijk}. \tag{26}$$

The constraints (19) and (20) become

$$G^{ij} \equiv -\frac{1}{m^2}\left(4\partial_l \pi^{lijk} + \frac{1}{3!}\Pi^{ijk}\right) \approx 0, \tag{27}$$

$$G^{i} \equiv \frac{1}{3!}\partial_k \Pi^{kij} \approx 0, \tag{28}$$

and remain first-class and third-order reducible, while the first-class Hamiltonian (18) takes the form

$$\begin{aligned} H_{GU}(y^0) &= \int d^{D-1}y \left[-12\pi_{ijkl}\pi^{ijkl} + \frac{1}{2\cdot 5!}F_{ijklq}F^{ijklq} + \frac{m^2}{2\cdot 4!}A_{ijkl}A^{ijkl} \right. \\ &\quad + \frac{m^2}{4!}A_{ijkl}\partial^{[i}B^{jkl]} + \frac{m^2}{2\cdot 4!}\partial_{[i}B_{jkl]}\partial^{[i}B^{jkl]} \\ &\quad \left. - \frac{1}{m^2}\frac{1}{2\cdot 3!}\Pi_{ijk}\Pi^{ijk} + \frac{1}{m^2}\Pi_{ijk}\left(4\partial_l\pi^{lijk} + \frac{1}{3!}\Pi^{ijk}\right)\right]. \end{aligned} \tag{29}$$

Due to the equivalence between the third-order reducible first-class system and the original second-class theory, one can replace the Hamiltonian path integral of massive 4-forms with that associated with the reducible first-class system. The argument of the exponential from the Hamiltonian path integral of the reducible first-class system read as

$$\begin{aligned} S_{GU} &= \int d^D x \left[(\partial_0 A_{ijkl})\,\pi^{ijkl} + (\partial_0 B_{ijk})\,\Pi^{ijk} \right. \\ &\quad + 12\pi_{ijkl}\pi^{ijkl} - \frac{1}{2\cdot 5!}F_{ijklq}F^{ijklq} - \frac{m^2}{2\cdot 4!}A_{ijkl}A^{ijkl} \\ &\quad - \frac{m^2}{4!}A_{ijkl}\partial^{[i}B^{jkl]} - \frac{m^2}{2\cdot 4!}\partial_{[i}B_{jkl]}\partial^{[i}B^{jkl]} \\ &\quad + \frac{1}{m^2}\frac{1}{2\cdot 3!}\Pi_{ijk}\Pi^{ijk} - \frac{1}{m^2}\Pi_{ijk}\left(4\partial_l\pi^{lijk} + \frac{1}{3!}\Pi^{ijk}\right) \\ &\quad \left. + \frac{1}{m^2}\lambda_{ijk}\left(4\partial_l\pi^{lijk} + \frac{1}{3!}\Pi^{ijk}\right) - \frac{1}{3!}\lambda_{ij}\left(\partial_k\Pi^{kij}\right)\right]. \end{aligned} \tag{30}$$

If we perform the transformation

$$\Pi^{ijk} \longrightarrow \Pi^{ijk}, \qquad \lambda_{ijk} \longrightarrow \bar{\lambda}_{ijk} = \lambda_{ijk} - \Pi_{ijk} \tag{31}$$

in the path integral, the argument of the exponential becomes

$$\begin{aligned} S'_{GU} &= \int d^D x \left[(\partial_0 A_{ijkl})\,\pi^{ijkl} + (\partial_0 B_{ijk})\,\Pi^{ijk} \right. \\ &\quad + 12\pi_{ijkl}\pi^{ijkl} - \frac{1}{2\cdot 5!}F_{ijklq}F^{ijklq} - \frac{m^2}{2\cdot 4!}A_{ijkl}A^{ijkl} \\ &\quad - \frac{m^2}{4!}A_{ijkl}\partial^{[i}B^{jkl]} - \frac{m^2}{2\cdot 4!}\partial_{[i}B_{jkl]}\partial^{[i}B^{jkl]} + \frac{1}{m^2}\frac{1}{2\cdot 3!}\Pi_{ijk}\Pi^{ijk} \\ &\quad \left. + \frac{1}{m^2}\bar{\lambda}_{ijk}\left(4\partial_l\pi^{lijk} + \frac{1}{3!}\Pi^{ijk}\right) - \frac{1}{3!}\lambda_{ij}\left(\partial_k\Pi^{kij}\right)\right]. \end{aligned} \tag{32}$$

The reducible first-class system constructed in the above display the Hamiltonian path integral

$$Z_{GU} = \int \mathcal{D}\left(fields\right)\mu\left([A_{ijkl}],[B_{ijk}]\right)\exp\left(\mathrm{i}S'_{GU}\right), \tag{33}$$

where by '$fields$' we denoted the present fields, the associated momenta and the Lagrange multipliers, and by '$\mu\left([A_{ijkl}],[B_{ijk}]\right)$' the integration measure associated with the model subject to the reducible first-class constraints (19) and (20).

In order to infer from (33) a path integral that leads, after integrating out the auxiliary variables, a manifestly Lorentz covariant functional in its exponential, we enlarge the original phase-space with the Lagrange multipliers $\bar{\lambda}_{ijk}$ and λ_{ij} respectively associated with the first-class constraints (19) and (20) [16] and with their canonical momenta p^{ijk} and p^{ij}. In order to preserve the number of degrees of freedom we add the constraints

$$p^{ijk} \approx 0, \qquad p^{ij} \approx 0. \tag{34}$$

The argument of the exponential from the Hamiltonian path integral for the first-class theory with the phase-space locally parameterized by $\{A_{ijkl},\ B_{ijk},\ \bar{\lambda}_{ijk},\ \lambda_{ij},\ \pi^{ijkl},\ \Pi^{ijk},\ p^{ijk},\ p^{ij}\}$ and subject to the first-class constraints (27), (28), and (34) reads as

$$\begin{aligned} S''_{GU} = & \int d^D x \left[\left(\partial_0 A_{ijkl}\right) \pi^{ijkl} + \left(\partial_0 B_{ijk}\right) \Pi^{ijk} + 12 \pi_{ijkl} \pi^{ijkl} \right. \\ & - \frac{1}{2 \cdot 5!} F_{ijklq} F^{ijklq} - \frac{m^2}{2 \cdot 4!} A_{ijkl} A^{ijkl} \\ & - \frac{m^2}{4!} A_{ijkl} \partial^{[i} B^{jkl]} - \frac{m^2}{2 \cdot 4!} \partial_{[i} B_{jkl]} \partial^{[i} B^{jkl]} \\ & + \frac{1}{m^2} \frac{1}{2 \cdot 3!} \Pi_{ijk} \Pi^{ijk} + \frac{1}{m^2} \bar{\lambda}_{ijk} \left(4 \partial_l \pi^{lijk} + \frac{1}{3!} \Pi^{ijk} \right) \\ & \left. - \frac{1}{3!} \lambda_{ij} \left(\partial_k \Pi^{kij} \right) - \Lambda_{ijk} p^{ijk} - \Lambda_{ij} p^{ij} \right]. \end{aligned} \tag{35}$$

Performing in (35) the integration over $\{\pi^{ijkl},\ \Pi^{ijk},\ p^{ijk},\ p^{ij},\ \Lambda_{ijk},\ \Lambda_{ij}\}$ and making the notations

$$\frac{1}{m^2} \bar{\lambda}_{ijk} \equiv -\bar{A}_{ijk0}, \qquad \frac{1}{3 \cdot 3!} \lambda_{ij} \equiv -B_{ij0} \tag{36}$$

then (35) can be written as

$$\begin{aligned} S'''_{GU} = & \int d^D x \left[- \frac{1}{2 \cdot 5!} F_{ijklq} F^{ijklq} - \frac{1}{2 \cdot 4!} \left(\partial_0 A_{ijkl} + \partial_{[i} \bar{A}_{jkl]0} \right) \times \right. \\ & \times \left(\partial^0 A^{ijkl} + \partial^{[i} \bar{A}^{jkl]0} \right) - 3m^2 \left(\partial_0 B_{ijk} - \partial_{[i} B_{jk]0} \right) \left(\partial^0 B^{ijk} - \partial^{[i} B^{jk]0} \right) \\ & - \frac{m^2}{2 \cdot 3!} \bar{A}_{ijk0} \bar{A}^{ijk0} + m^2 \left(\partial_0 B_{ijk} - \partial_{[i} B_{jk]0} \right) \bar{A}^{ijk0} \\ & \left. - \frac{m^2}{2 \cdot 4!} A_{ijkl} A^{ijkl} - \frac{m^2}{4!} A_{ijkl} \partial^{[i} B^{jkl]} - \frac{m^2}{2 \cdot 4!} \partial_{[i} B_{jkl]} \partial^{[i} B^{jkl]} \right]. \end{aligned} \tag{37}$$

or, equivalently, as

$$\begin{aligned} S'''_{GU} = & \int d^D x \left[- \frac{1}{2 \cdot 5!} F_{ijklq} F^{ijklq} - \frac{1}{2 \cdot 4!} \bar{F}_{0ijkl} \bar{F}^{0ijkl} \right. \\ & - \frac{m^2}{2 \cdot 4!} F_{ijkl} F^{ijkl} - \frac{m^2}{2 \cdot 3!} F_{0ijk} F^{0ijk} - \frac{m^2}{2 \cdot 4!} A_{ijkl} A^{ijkl} \\ & \left. - \frac{m^2}{2 \cdot 3!} \bar{A}_{ijk0} \bar{A}^{ijk0} + \frac{m^2}{3!} F_{0ijk} \bar{A}^{0ijk} + \frac{m^2}{4!} A_{ijkl} F^{ijkl} \right]. \end{aligned} \tag{38}$$

where

$$\bar{F}_{0ijkl} = \partial_0 A_{ijkl} + \partial_{[i} \bar{A}_{jkl]0}, \tag{39}$$

$$F_{ijkl} = -\partial_{[i} B_{jkl]}, \qquad F_{0ijk} = -\frac{1}{3!} \left(\partial_0 B_{ijk} - \partial_{[i} B_{jk]0} \right). \tag{40}$$

The functional (38) associated with the reducible first-class system can be written in a manifestly Lorentz covariant form

$$S_{GU}^{\prime\prime\prime}\left[\bar{B}_{\mu\nu\rho},\bar{A}_{\mu\nu\rho\lambda}\right] = \int d^{D}x\left[-\frac{1}{2\cdot 5!}\bar{F}_{\mu\nu\rho\lambda\sigma}\bar{F}^{\mu\nu\rho\lambda\sigma} - \frac{1}{2\cdot 4!}\left(F_{\mu\nu\rho\lambda}-m\bar{A}_{\mu\nu\rho\lambda}\right)\left(F^{\mu\nu\rho\lambda}-m\bar{A}^{\mu\nu\rho\lambda}\right)\right], \tag{41}$$

with

$$\bar{A}_{\mu\nu\rho\lambda} \equiv \left(\bar{A}_{0ijk}, A_{ijkl}\right), \qquad \bar{F}_{\mu\nu\rho\lambda\sigma} = \partial_{[\mu}\bar{A}_{\nu\rho\lambda\sigma]}, \tag{42}$$

$$\bar{B}_{\mu\nu\rho} = \frac{1}{m}B_{\mu\nu\rho}, \qquad F_{\mu\nu\rho\lambda} = \partial_{[\mu}\bar{B}_{\nu\rho\lambda]}, \tag{43}$$

and describes precisely the (Lagrangian) Stückelberg [19] coupling between the 3-form $\bar{B}_{\mu\nu\rho}$ and 4-form $\bar{A}_{\mu\nu\rho\lambda}$.

4 Conclusion

In this paper we analyzed massive 4-form fields from the point of view of gauge-unfixing method. This approach (GU) relies on separating the (independent) second-class constraints into two subsets, of which one is first-class and the other a set of canonical gauge conditions. Starting from the original canonical Hamiltonian, we generated a first-class Hamiltonian with respect to the first-class constraint subset. Finally, we built the Hamiltonian path integral of the GU first-class system and then eliminated the auxiliary fields and performed some variable redefinitions such that the path integral finally takes a manifestly Lorentz covariant form. It is interesting to remark that this approaches require an appropriate extension of the phase-space in order to render a manifestly covariant path integral. The gauge-unfixed method allowed the identification of the Lagrangian path integral for Stückelberg-coupled 3- and 4-forms.

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