Physics AUC, vol. 19, 92-106 (2009)

No-go results for cross-couplings between a collection of tensors with the mixed symmetry $(3,1)$ and a Pauli–Fierz field

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Abstract

Under the hypotheses of analyticity, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the requirement that the interaction vertices contain at most two space-time derivatives of the fields, we investigate the consistent cross-couplings that can be added between a collection of massless tensor fields with the mixed symmetry $(3,1)$ and a Pauli–Fierz field. The computations are done with the help of the deformation theory based on a cohomological approach, in the context of the antifield-BRST formalism. Our final result is that no cross-couplings are possible.

PACS number: 11.10.Ef

During the last years tensor fields in exotic representations of the Lorentz group [1]–[7] have been extensively employed in many interesting problems, like the dual formulation of field theories of spin two or higher [8]–[14], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [15], or the derivation of some exotic gravitational interactions [16, 17]. An important matter related to mixed symmetry type tensor fields is the study of their consistent interactions, among themselves as well as with higher-spin gauge theories [18]–[26]. The most efficient approach to this problem is the cohomological one, based on the deformation of the solution to the master equation [27].

The purpose of this paper is to investigate the consistent cross-couplings between a collection of massless tensor gauge fields with the mixed symmetry $(3,1)$ and a Pauli–Fierz field. Our analysis relies on the deformation of the solution to the master equation by means of cohomological techniques with the help of the local BRST cohomology, whose component for a collection of $(3,1)$ fields has been considered in [28] and in the Pauli–Fierz sector has been investigated in [29]. Under the hypotheses of analyticity in the coupling constant, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the requirement that the maximum number of derivatives in each interaction vertex is equal to two, we find that no cross-couplings can be added to the original Lagrangian action.

The starting point is given by the Lagrangian action for a finite collection of free, massless tensor fields with the mixed symmetry $(3,1)$ and for a Pauli–Fierz field in $D \geq 5$

$$S_0\left[t^A_{\lambda\mu\nu|\alpha}, h_{\mu\nu}\right] = S_0\left[t^A_{\lambda\mu\nu|\alpha}\right] + S_0^{\mathrm{PF}}\left[h_{\mu\nu}\right], \tag{1}$$

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where

$$S_0^{\mathrm{t}}\left[t^A_{\lambda\mu\nu|\alpha}\right] = \int \left\{ \frac{1}{2}\left[\left(\partial^\rho t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial_\rho t^A_{\lambda\mu\nu|\alpha}\right) - \left(\partial_\alpha t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial^\beta t^A_{\lambda\mu\nu|\beta}\right)\right] \right.$$
$$- \frac{3}{2}\left[\left(\partial_\lambda t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial^\rho t^A_{\rho\mu\nu|\alpha}\right) + \left(\partial^\rho t_A^{\lambda\mu}\right)\left(\partial_\rho t^A_{\lambda\mu}\right)\right]$$
$$\left. +3\left[\left(\partial_\alpha t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial_\lambda t^A_{\mu\nu}\right) + \left(\partial_\rho t_A^{\rho\mu}\right)\left(\partial^\lambda t^A_{\lambda\mu}\right)\right] \right\} d^D x, \tag{2}$$

$$S_0^{\mathrm{PF}}\left[h_{\mu\nu}\right] = \int \left\{ -\frac{1}{2}\left[\left(\partial^\rho h^{\mu\nu}\right)\left(\partial_\rho h_{\mu\nu}\right) - \left(\partial^\rho h\right)\left(\partial_\rho h\right)\right] \right.$$
$$\left. \left(\partial_\rho h^{\rho\mu}\right)\left(\partial^\lambda h_{\lambda\mu}\right) - \left(\partial^\rho h\right)\left(\partial^\lambda h_{\lambda\rho}\right) \right\} d^D x. \tag{3}$$

Everywhere in this paper we employ the flat Minkowski metric of 'mostly plus' signature $\sigma^{\mu\nu} = \sigma_{\mu\nu} = (- + + + + \cdots)$. The uppercase indices A, B, etc. stand for the collection indices and are assumed to take discrete values 1, 2, ..., N. They are lowered with a symmetric, constant, and invertible matrix, of elements k_{AB}, and are raised with the help of the elements k^{AB} of its inverse. Each field $t^A_{\lambda\mu\nu|\alpha}$ is completely antisymmetric in its first three (Lorentz) indices and satisfies the identity $t^A_{[\lambda\mu\nu|\alpha]} \equiv 0$. The notation $t^A_{\lambda\mu}$ from (2) signifies the trace of $t^A_{\lambda\mu\nu|\alpha}$, defined by $t^A_{\lambda\mu} = \sigma^{\nu\alpha} t^A_{\lambda\mu\nu|\alpha}$. The trace components define an antisymmetric tensor, $t^A_{\lambda\mu} = -t^A_{\mu\lambda}$. The Pauli–Fierz field $h_{\mu\nu}$ is symmetric and h denotes its trace. A generating set of gauge transformations for action (1) can be chosen of the form

$$\delta_{\epsilon,\chi} t^A_{\lambda\mu\nu|\alpha} = 3\partial_\alpha \epsilon^A_{\lambda\mu\nu} + \partial_{[\lambda} \epsilon^A_{\mu\nu]\alpha} + \partial_{[\lambda} \chi^A_{\mu\nu]|\alpha}$$
$$= -3\partial_{[\lambda} \epsilon^A_{\mu\nu\alpha]} + 4\partial_{[\lambda} \epsilon^A_{\mu\nu]\alpha} + \partial_{[\lambda} \chi^A_{\mu\nu]|\alpha}, \tag{4}$$

$$\delta_\epsilon h_{\mu\nu} = \partial_{(\mu} \epsilon_{\nu)}, \tag{5}$$

where all the gauge parameters are arbitrary and bosonic, with $\epsilon^A_{\lambda\mu\nu}$ completely antisymmetric and $\chi^A_{\mu\nu|\alpha}$ with the mixed symmetry $(2,1)$. The generating set (4) and (5) is off-shell reducible of order two and the associated gauge algebra is Abelian. Consequently, the Cauchy order of this linear gauge theory is equal to four.

The most general quantities, invariant under the gauge transformations (4), are given by the components of the curvature tensors of mixed symmetry $(4,2)$ associated with each field from the collection

$$F_A^{\lambda\mu\nu\xi|\alpha\beta} = \partial^\alpha \partial^{[\lambda} t_A^{\mu\nu\xi]|\beta} - \partial^\beta \partial^{[\lambda} t_A^{\mu\nu\xi]|\alpha}, \tag{6}$$

of the linearized Riemann tensor

$$K^{\mu\nu|\alpha\beta} = -\frac{1}{2}\left(\partial^\mu \partial^\alpha h^{\nu\beta} - \partial^\nu \partial^\alpha h^{\mu\beta} - \partial^\mu \partial^\beta h^{\nu\alpha} + \partial^\nu \partial^\beta h^{\mu\alpha}\right), \tag{7}$$

together with their space-time derivatives.

The construction of the BRST symmetry for the free model under study debuts with the identification of the algebra on which the BRST differential s acts. The ghost spectrum comprises the fermionic ghosts $\left\{\eta^A_{\lambda\mu\nu}, \mathcal{G}^A_{\mu\nu|\alpha}, \eta_\mu\right\}$ respectively associated with the gauge parameters $\left\{\epsilon^A_{\lambda\mu\nu}, \chi^A_{\mu\nu|\alpha}, \epsilon_\mu\right\}$ from (4) and (5), the bosonic ghosts for ghosts $\left\{C^A_{\mu\nu}, G^A_{\nu\alpha}\right\}$

due to the first-order reducibility, and the fermionic ghosts for ghosts for ghosts C_{ν}^{A} corresponding to the maximum reducibility order (two). We ask that $\eta_{\lambda\mu\nu}^{A}$ and $C_{\mu\nu}^{A}$ are completely antisymmetric, $\mathcal{G}_{\mu\nu|\alpha}^{A}$ exhibit the mixed symmetry $(2,1)$, and $G_{\nu\alpha}^{A}$ are symmetric. The antifield spectrum comprises the antifields $\left\{t_{A}^{*\lambda\mu\nu|\alpha}, h^{*\mu\nu}\right\}$ associated with the original fields and those corresponding to the ghosts, $\left\{\eta_{A}^{*\lambda\mu\nu}, \mathcal{G}_{A}^{*\mu\nu|\alpha}, \eta^{*\mu}\right\}$, $\{C_{A}^{*\mu\nu}, G_{A}^{*\nu\alpha}\}$, and $C_{A}^{*\nu}$.

Since both the gauge generators and reducibility functions for this model are field-independent, it follows that the BRST differential s simply reduces to $s=\delta+\gamma$, where δ represents the Koszul–Tate differential and γ stands for the exterior longitudinal differential. According to the standard rules of the BRST method, the corresponding degrees of the generators from the BRST complex are valued like

$$\begin{gathered}
\mathrm{pgh}\left(t_{\lambda\mu\nu|\alpha}^{A}\right)=0=\mathrm{pgh}\left(h_{\mu\nu}\right), \\
\mathrm{pgh}\left(\eta_{\lambda\mu\nu}^{A}\right)=\mathrm{pgh}\left(\mathcal{G}_{\mu\nu|\alpha}^{A}\right)=\mathrm{pgh}\left(\eta_{\mu}\right)=1, \\
\mathrm{pgh}\left(C_{\mu\nu}^{A}\right)=\mathrm{pgh}\left(G_{\nu\alpha}^{A}\right)=2, \quad \mathrm{pgh}\left(C_{\nu}^{A}\right)=3, \\
\mathrm{pgh}\left(\text{anticampuri}\right)=0=\mathrm{agh}\left(\text{campuri/ghosturi}\right), \\
\mathrm{agh}\left(t_{A}^{*\lambda\mu\nu|\alpha}\right)=1=\mathrm{agh}\left(h^{*\mu\nu}\right), \\
\mathrm{agh}\left(\eta_{A}^{*\lambda\mu\nu}\right)=\mathrm{agh}\left(\mathcal{G}_{A}^{*\mu\nu|\alpha}\right)=\mathrm{agh}\left(\eta^{*\mu}\right)=2, \\
\mathrm{agh}\left(C_{A}^{*\mu\nu}\right)=\mathrm{agh}\left(G_{A}^{*\nu\alpha}\right)=3, \quad \mathrm{agh}\left(C_{A}^{*\nu}\right)=4.
\end{gathered}$$

The actions of δ and γ on the generators from the BRST complex are given by

$$\gamma t_{\lambda\mu\nu|\alpha}^{A}=-3\partial_{[\lambda}\eta_{\mu\nu\alpha]}^{A}+4\partial_{[\lambda}\eta_{\mu\nu]\alpha}^{A}+\partial_{[\lambda}\mathcal{G}_{\mu\nu]|\alpha}^{A}, \tag{8}$$

$$\gamma h_{\mu\nu}=\partial_{(\mu}\eta_{\nu)}, \quad \gamma\eta_{\lambda\mu\nu}^{A}=-\frac{1}{2}\partial_{[\lambda}C_{\mu\nu]}^{A}, \tag{9}$$

$$\gamma\mathcal{G}_{\mu\nu|\alpha}^{A}=2\partial_{[\mu}C_{\nu\alpha]}^{A}-3\partial_{[\mu}C_{\nu]\alpha}^{A}+\partial_{[\mu}G_{\nu]\alpha}^{A}, \quad \gamma\eta_{\mu}=0, \tag{10}$$

$$\gamma C_{\mu\nu}^{A}=\partial_{[\mu}C_{\nu]}^{A}, \quad \gamma G_{\nu\alpha}^{A}=-3\partial_{(\nu}C_{\alpha)}^{A}, \quad \gamma C_{\nu}^{A}=0, \tag{11}$$

$$\gamma t_{A}^{*\lambda\mu\nu|\alpha}=\gamma h^{*\mu\nu}=\gamma\eta_{A}^{*\lambda\mu\nu}=\gamma\mathcal{G}_{A}^{*\mu\nu|\alpha}=\gamma\eta^{*\mu}=0, \tag{12}$$

$$\gamma C_{A}^{*\mu\nu}=\gamma G_{A}^{*\nu\alpha}=\gamma C_{A}^{*\nu}=0, \tag{13}$$

$$\delta t_{\lambda\mu\nu|\alpha}^{A}=\delta h_{\mu\nu}=\delta\eta_{\lambda\mu\nu}^{A}=\delta\mathcal{G}_{\mu\nu|\alpha}^{A}=\delta\eta_{\mu}=0, \tag{14}$$

$$\delta C_{\mu\nu}^{A}=\delta G_{\nu\alpha}^{A}=\delta C_{\nu}^{A}=0, \tag{15}$$

$$\delta t_{A}^{*\lambda\mu\nu|\alpha}=T_{A}^{\lambda\mu\nu|\alpha}, \quad \delta h^{*\mu\nu}=2H^{\mu\nu}, \quad \delta\eta_{A}^{*\lambda\mu\nu}=-4\partial_{\alpha}t_{A}^{*\lambda\mu\nu|\alpha}, \tag{16}$$

$$\delta\mathcal{G}_{A}^{*\mu\nu|\alpha}=-\partial_{\lambda}\left(3t_{A}^{*\lambda\mu\nu|\alpha}-t_{A}^{*\mu\nu\alpha|\lambda}\right), \quad \delta\eta^{*\mu}=-2\partial_{\nu}h^{*\nu\mu}, \tag{17}$$

$$\delta C_{A}^{*\mu\nu}=3\partial_{\lambda}\left(\mathcal{G}_{A}^{*\mu\nu|\lambda}-\frac{1}{2}\eta_{A}^{*\lambda\mu\nu}\right), \quad \delta G_{A}^{*\nu\alpha}=\partial_{\mu}\mathcal{G}_{A}^{*\mu(\nu|\alpha)}, \tag{18}$$

$$\delta C_{A}^{*\nu}=6\partial_{\mu}\left(G_{A}^{*\mu\nu}-\frac{1}{3}C_{A}^{*\mu\nu}\right), \tag{19}$$

where $T_{A}^{\lambda\mu\nu|\alpha}=-\delta S_{0}^{\mathrm{t}}/\delta t_{\lambda\mu\nu|\alpha}^{A}$ and $H_{\mu\nu}=-(1/2)\delta S_{0}^{\mathrm{PF}}/\delta h^{\mu\nu}$ represent the components of the linearized Einstein tensor. We note that the action of the Koszul–Tate differential on the antifields with the antighost number equal to two and respectively three gains a simpler expression if we perform the changes of variables

$$\mathcal{G}'^{*\mu\nu|\alpha}_{A} = \mathcal{G}^{*\mu\nu|\alpha}_{A} + \frac{1}{4}\eta^{*\mu\nu\alpha}_{A}, \quad G'^{*\nu\alpha}_{A} = G^{*\nu\alpha}_{A} - \frac{1}{3}C^{*\nu\alpha}_{A}, \tag{20}$$

namely

$$\delta\mathcal{G}'^{*\mu\nu|\alpha}_{A} = -3\partial_{\lambda}t^{*\lambda\mu\nu|\alpha}_{A}, \quad \delta G'^{*\nu\alpha}_{A} = 2\partial_{\mu}\mathcal{G}'^{*\mu\nu|\alpha}_{A}, \quad \delta C^{*\nu}_{A} = 6\partial_{\mu}G'^{*\mu\nu}_{A}. \tag{21}$$

The same observation is valid with respect to γ if we make the changes

$$\mathcal{G}'^{A}_{\mu\nu|\alpha} = \mathcal{G}^{A}_{\mu\nu|\alpha} + 4\eta^{A}_{\mu\nu\alpha}, \quad G'^{A}_{\nu\alpha} = G^{A}_{\nu\alpha} - 3C^{A}_{\nu\alpha}, \tag{22}$$

in terms of which we can write

$$\gamma t^{A}_{\lambda\mu\nu|\alpha} = -\frac{1}{4}\partial_{[\lambda}\mathcal{G}'^{A}_{\mu\nu|\alpha]} + \partial_{[\lambda}\mathcal{G}'^{A}_{\mu\nu]|\alpha}, \gamma\mathcal{G}'^{A}_{\mu\nu|\alpha} = \partial_{[\mu}G'^{A}_{\nu]\alpha}, \gamma G'^{A}_{\nu\alpha} = -6\partial_{\nu}C^{A}_{\alpha}. \tag{23}$$

The transformed variables (20) and (22) form pairs that are conjugated in the antibracket.

The solution to the classical master equation for the free model under study reduces to the sum between the solutions in the two sectors

$$S = S^{\mathrm{t}} + S^{\mathrm{h}}, \tag{24}$$

where

$$\begin{aligned} S^{\mathrm{t}} = {} & S^{\mathrm{t}}_{0}\left[t^{A}_{\lambda\mu\nu|\alpha}\right] + \int \Big[t^{*\lambda\mu\nu|\alpha}_{A}\left(3\partial_{\alpha}\eta^{A}_{\lambda\mu\nu} + \partial_{[\lambda}\eta^{A}_{\mu\nu]\alpha} + \partial_{[\lambda}\mathcal{G}^{A}_{\mu\nu]|\alpha}\right) \\ & - \frac{1}{2}\eta^{*\lambda\mu\nu}_{A}\partial_{[\lambda}C^{A}_{\mu\nu]} + \mathcal{G}^{*\mu\nu|\alpha}_{A}\left(2\partial_{\alpha}C^{A}_{\mu\nu} - \partial_{[\mu}C^{A}_{\nu]\alpha} + \partial_{[\mu}G^{A}_{\nu]\alpha}\right) \\ & + C^{*\mu\nu}_{A}\partial_{[\mu}C^{A}_{\nu]} - 3G^{*\nu\alpha}_{A}\partial_{(\nu}C^{A}_{\alpha)}\Big]\, d^{D}x, \end{aligned} \tag{25}$$

$$S^{\mathrm{h}} = S^{\mathrm{PF}}_{0}\left[h_{\mu\nu}\right] + \int h^{*\mu\nu}\partial_{(\mu}\eta_{\nu)}d^{D}x. \tag{26}$$

The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution S to the master equation for the initial theory can be deformed into the solution $\bar{S}$ of the master equation for the interacting theory

$$S \longrightarrow \bar{S} = S + gS_1 + g^2S_2 + g^3S_3 + g^4S_4 + \cdots, \tag{27}$$

$$(S, S) = 0 \longrightarrow \left(\bar{S}, \bar{S}\right) = 0. \tag{28}$$

The projection of (28) for $\bar{S}$ on the various powers of the coupling constant induces the following tower of equations:

$$g^0 : (S, S) = 0, \tag{29}$$

$$g^1 : (S_1, S) = 0, \tag{30}$$

$$g^2 : (S_2, S) + \frac{1}{2}(S_1, S_1) = 0, \tag{31}$$

$$g^3 : (S_3, S) + (S_1, S_2) = 0, \tag{32}$$

$$\vdots$$

In the sequel we compute all consistent interactions that can be added to the free action (2) by solving the deformation equations (30)–(32), etc., by means of specific cohomological techniques, under the general hypotheses mentioned in the introductory paragraph.

In order to analyze equation (30) (that governs the first-order deformation) we make the notation $S_1 = \int a d^D x$ and write this equation in its local form and in dual notations, $sa = \partial_\mu m^\mu$. It is convenient to split the first-order deformation into

$$a = a^{\mathrm{h}} + a^{\mathrm{t}} + a^{\mathrm{int}}, \tag{33}$$

where a^{h} denotes the part responsible for the self-interactions of the Pauli–Fierz field, a^{t} is related to the deformations of the tensor fields $t^A_{\mu\nu|\alpha\beta}$, and a^{int} signifies the component that describes only the cross-interactions between $h_{\mu\nu}$ and $t^A_{\mu\nu|\alpha\beta}$. Then, a^{h} is completely known (for a detailed analysis, see for instance [29])

$$a^{\mathrm{h}} = a^{\mathrm{h}}_0 + a^{\mathrm{h}}_1 + a^{\mathrm{h}}_2, \tag{34}$$

where

$$a^{\mathrm{h}}_2 = \eta^{*\mu}\eta^\alpha \partial_\mu \eta_\alpha, \tag{35}$$

$$a^{\mathrm{h}}_1 = -h^{*\mu\nu}\eta^\alpha \left(\partial_\mu h_{\nu\alpha} + \partial_\nu h_{\mu\alpha} - \partial_\alpha h_{\mu\nu}\right), \tag{36}$$

and a^{h}_0 is the cubic vertex of the Einstein–Hilbert Lagrangian plus a cosmological term. The piece a^{t} has been computed in [28] and is given by

$$a^{\mathrm{t}} = 0, \tag{37}$$

In order to ensure the space-time locality of the deformations, from now on we work in the algebra of local differential forms with coefficients that are polynomial functions in the fields, ghosts, antifields, and their space-time derivatives (algebra of local forms), meaning that the non-integrated density of the first-order deformation, a, is a polynomial function in all these variables (algebra of local functions). Inserting (33) into the equation $sa = \partial_\mu m^\mu$ and using the fact that the first two components already obey the equations $sa^{\mathrm{h}} = \partial_\mu m^\mu_{\mathrm{h}}$ and $sa^{\mathrm{t}} = 0$, it follows that only a^{int} is unknown, being subject to the equation

$$sa^{\mathrm{int}} = \partial_\mu m^\mu_{\mathrm{int}}. \tag{38}$$

By taking into account the splitting $s = \delta + \gamma$ of the BRST differential, equation (38) becomes equivalent to a tower of local equations, corresponding to the different decreasing values of the antighost number

$$\gamma a^{\mathrm{int}}_I = 0, \quad I > 0 \tag{39}$$

$$\delta a^{\mathrm{int}}_I + \gamma a^{\mathrm{int}}_{I-1} = \partial_\mu \overset{(I-1)^\mu}{m}_{\mathrm{int}}, \tag{40}$$

$$\delta a^{\mathrm{int}}_k + \gamma a^{\mathrm{int}}_{k-1} = \partial_\mu \overset{(k-1)^\mu}{m}_{\mathrm{int}}, \quad I-1 \geq k \geq 1, \tag{41}$$

where $\left(\overset{(k)^\mu}{m}_{\mathrm{int}}\right)_{k=\overline{0,I}}$ are some local currents, with $\mathrm{agh}\left(\overset{(k)^\mu}{m}_{\mathrm{int}}\right) = k$. In conclusion, for $I > 0$ we have that $a^{\mathrm{int}}_I \in H^*(\gamma)$.

We have seen that the solution to equation (39) belongs to the cohomology of the exterior longitudinal differential computed in the algebra of local functions, such that we need to compute $H^*(\gamma)$ in order to construct the component of highest antighost number from the first-order deformation. We will see that we also need to compute the characteristic cohomology $H_I^D(\delta|d)$ (the local cohomology of the Koszul–Tate differential δ in antighost number I and in maximum form degree, computed in the algebra of local forms with the pure ghost number equal to zero).

Due to the fact that the exterior longitudinal differential γ splits as

$$\gamma = \gamma_{\mathrm{t}} + \gamma_{\mathrm{h}}, \tag{42}$$

where γ_{t} acts non-trivially only in the $(3,1)$ sector and γ_{h} does the same, but in the Pauli–Fierz sector, Künneth's Theorem for cohomologies ensure that

$$H^*(\gamma) = H^*(\gamma_{\mathrm{t}}) \otimes H^*(\gamma_{\mathrm{h}}). \tag{43}$$

Combining the results from [28] and [29] on $H^*(\gamma_{\mathrm{t}})$ and respectively on $H^*(\gamma_{\mathrm{h}})$, it follows that the general solution to (39) reads

$$a_I^{\mathrm{int}} = \alpha_I\left(\left[\pi^{*\Theta}\right], \left[F^A_{\lambda\mu\nu\xi|\alpha\beta}\right], \left[K_{\mu\nu|\alpha\beta}\right]\right) e^I\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, C^A_\nu, \mathcal{F}^A_{\lambda\mu\nu\alpha}\right), \tag{44}$$

where $\pi^{*\Theta}$ is a collective notation for all the antifields. The notation $f([q])$ means that f depends on q and its derivatives up to a finite order, while e^I denotes the elements of pure ghost number I (and antighost number zero) of a basis in the space of polynomials in η_μ, $\partial_{[\mu}\eta_{\nu]}$, $\mathcal{F}^A_{\lambda\mu\nu\alpha}$ and C^A_ν, which is finite dimensional since these variables anticommute. The objects α_I (obviously non-trivial in $H^0(\gamma)$) were taken to have a bounded number of derivatives, and therefore they are polynomials in the antifields $\pi^{*\Theta}$, in the curvature tensors $F^A_{\lambda\mu\nu\xi|\alpha\beta}$ and $K_{\mu\nu|\alpha\beta}$, as well as in their derivatives. They are nothing but the invariant polynomials of the theory (1) in form degree equal to zero.

Replacing solution (44) into equation (40) and taking into account definitions (8)–(19), we remark that a necessary (but not sufficient) condition for the existence of (non-trivial) solutions a_{I-1}^{int} is that the invariant polynomials α_I generate (non-trivial) objects from the characteristic cohomology $H_I^D(\delta|d)$ in antighost number $I>0$, maximum form degree, and pure ghost number equal to zero, $\alpha_I d^D x \in H_I^D(\delta|d)$. As the free model under study is a linear gauge theory of Cauchy order equal to four, the general results from [30] ensure that the entire characteristic cohomology is trivial in antighost numbers strictly greater than its Cauchy order

$$H_I^D(\delta|d) = 0, \quad I > 4. \tag{45}$$

Moreover, it is possible to show that the above result remains valid also in the algebra of invariant polynomials

$$H_I^{\mathrm{inv}D}(\delta|d) = 0, \quad I > 4, \tag{46}$$

where $H_I^{\mathrm{inv}D}(\delta|d)$ is known as the invariant characteristic cohomology. On account of the general results from [28] and [29] on the invariant characteristic cohomology, we are able to identify the non-trivial representatives of $\left(H_I^D(\delta|d)\right)_{I\geq 2}$, as well as of $\left(H_I^{\mathrm{inv}D}(\delta|d)\right)_{I\geq 2}$, under the form

$$\begin{array}{cc}
\text{agh} & H_I^D(\delta|d) \text{ and } H_I^{\mathrm{inv}D}(\delta|d) \\
I>4 & - \\
I=4 & f^A_\nu C^{*\nu}_A d^D x \\
I=3 & f^A_{\nu\alpha} G'^{*\nu\alpha}_A d^D x \\
I=2 & \left(f^A_{\mu\nu\alpha}\mathcal{G}'^{*\mu\nu|\alpha}_A + f_\mu \eta^{*\mu}\right) d^D x
\end{array}, \tag{47}$$

where all the coefficients denoted by f define some constant, non-derivative tensors. We remark that in $\left(H_I^D(\delta|d)\right)_{I\geq 2}$ and $\left(H_I^{\mathrm{inv}D}(\delta|d)\right)_{I\geq 2}$ there is no non-trivial element that effectively involves the curvatures $F^A_{\lambda\mu\nu\xi|\alpha\beta}$ or $K_{\mu\nu|\alpha\beta}$ and/or their derivatives, and the same stands for the quantities that are more than linear in the antifields and/or depend on their derivatives. In contrast to the groups $\left(H_I^D(\delta|d)\right)_{I\geq 2}$ and $\left(H_I^{\mathrm{inv}D}(\delta|d)\right)_{I\geq 2}$, which are finite-dimensional, the cohomology $H_1^D(\delta|d)$ at pure ghost number zero, that is related to global symmetries and ordinary conservation laws, is infinite-dimensional since the theory is free.

The previous results on $H_I^D(\delta|d)$ and $H_I^{\mathrm{inv}D}(\delta|d)$ are important because they control the obstructions to removing the antifields from the first-order deformation. Indeed, due to (46), it follows that we can successively eliminate all the pieces with $I > 4$ from the non-integrated density of the first-order deformation by adding only trivial terms, so we can take, without loss of non-trivial objects, the condition $I \leq 4$ in the first-order deformation. The last representative is of the form (44), where the invariant polynomials necessarily generate non-trivial objects from $H_I^{\mathrm{inv}D}(\delta|d)$ if $I = 2,3,4$ and respectively from $H_1^D(\delta|d)$ if $I = 1$.

For $I = 4$, the first-order deformation becomes

$$a^{\mathrm{int}} = a_0^{\mathrm{int}} + a_1^{\mathrm{int}} + a_2^{\mathrm{int}} + a_3^{\mathrm{int}} + a_4^{\mathrm{int}}, \tag{48}$$

with the γ-non-trivial part of a_4^{int} of the form (44) for $I = 4$ and the invariant polynomial $\alpha_4 d^D x$ a non-trivial object from $H_4^{\mathrm{inv}D}(\delta|d)$. We maintain the requirement on the maximum derivative order of a_0^{int} being equal to two and observe that, according to (47), α_4 can only be linear in the undifferentiated antifields $C_B^{*\nu}$. Consequently, we must select from the elements e^4 only those having at most one space-time derivative and at least one ghost field from the Pauli–Fierz sector

$$\begin{aligned}\text{eligible } e^4 : \bigl(&\eta_\mu\eta_\nu\eta_\rho\eta_\sigma, \eta_\mu\eta_\nu\eta_\rho\partial_{[\sigma}\eta_{\tau]},\\ &\eta_\mu\eta_\nu\eta_\rho\mathcal{F}^A_{\lambda'\mu'\nu'\rho'}, C^A_\mu\eta_\nu, C^A_\mu\partial_{[\nu}\eta_{\rho]}\bigr),\end{aligned} \tag{49}$$

which then yield

$$\begin{aligned}a_4^{\mathrm{int}} = C_\lambda^{*B}\Bigl[&\eta_\mu\eta_\nu\eta_\rho\left(f_{1B}^{\lambda\mu\nu\rho\sigma}\eta_\sigma + f_{2B}^{\lambda\mu\nu\rho\sigma\tau}\partial_{[\sigma}\eta_{\tau]} + f_{3BA}^{\lambda\mu\nu\rho\lambda'\mu'\nu'\rho'}\mathcal{F}^A_{\lambda'\mu'\nu'\rho'}\right)\\ &+C^A_\mu\left(f_{4BA}^{\lambda\mu\nu}\eta_\nu + f_{5BA}^{\lambda\mu\nu\rho}\partial_{[\nu}\eta_{\rho]}\right)\Bigr] + \gamma b_4.\end{aligned} \tag{50}$$

All the coefficients denoted by f must be constant (neither derivative nor depending on the space-time co-ordinates). Recalling that we work in $D \geq 5$ space-time dimensions, we find the following admitted representatives:

$$D = 5, \quad a_4^{(D=5)\mathrm{int}} = c_{1B}\varepsilon^{\lambda\mu\nu\rho\sigma}C_\lambda^{*B}\eta_\mu\eta_\nu\eta_\rho\eta_\sigma + \gamma b_4^{(D=5)}, \tag{51}$$

$$D = 6, \quad a_4^{(D=6)\mathrm{int}} = c_{2B}\varepsilon^{\lambda\mu\nu\rho\sigma\tau}C_\lambda^{*B}\eta_\mu\eta_\nu\eta_\rho\partial_{[\sigma}\eta_{\tau]} + \gamma b_4^{(D=6)}, \tag{52}$$

$$D = 8, \quad a_4^{(D=8)\mathrm{int}} = c_{3BA}\varepsilon^{\lambda\mu\nu\rho\lambda'\mu'\nu'\rho'}C_\lambda^{*B}\eta_\mu\eta_\nu\eta_\rho\mathcal{F}^A_{\lambda'\mu'\nu'\rho'} + \gamma b_4^{(D=8)}, \tag{53}$$

$$\begin{aligned}D \geq 5, \quad a_4^{(D)\mathrm{int}} = {}& C^{*B\lambda}\bigl(c'_{2B}\eta_\lambda\eta^\sigma\eta^\tau\partial_{[\sigma}\eta_{\tau]} + c'_{3BA}\eta^\mu\eta^\nu\eta^\rho\mathcal{F}^A_{\lambda\mu\nu\rho}\\ &+c'_{5BA}C^{A\mu}\partial_{[\lambda}\eta_{\mu]}\bigr) + \gamma b_4^{(D)}.\end{aligned} \tag{54}$$

Direct computation shows that the terms containing at least one undifferentiated Pauli–Fierz ghost cannot produce a consistent component of antighost number three in (48), irrespective of the γ-exact contribution from (51)–(54), and hence we must set

$$c_{1B} = c_{2B} = c_{3BA} = c'_{2B} = c'_{3BA} = 0, \tag{55}$$

which leaves us with a single candidate, namely,

$$a_4^{\text{int}} = c'_{5BA} C^{*B\lambda} C^{A\mu} \partial_{[\lambda} \eta_{\mu]} + \gamma b_4. \tag{56}$$

We will now show that (56) is not consistent at antighost number two, i.e., the equation

$$\delta a_4^{\text{int}} + \gamma a_3^{\text{int}} = \partial_\mu \overset{(3)}{m}^{\mu}, \tag{57}$$

possesses non-trivial solutions with respect to a_3^{int}, while the next equation

$$\delta a_3^{\text{int}} + \gamma a_2^{\text{int}} = \partial_\mu \overset{(2)}{m}^{\mu}, \tag{58}$$

exhibits no non-zero solutions for a_2^{int}. It is convenient (in order to simplify later developments) to fix the γ-exact term in (56) to the value

$$b_4 = c'_{5BA} C^{*B\lambda} C^{A\mu} h_{\lambda\mu}, \tag{59}$$

which further gives

$$a_4^{\text{int}} = 2c'_{5BA} C^{*B\lambda} C^{A\mu} \partial_\lambda \eta_\mu. \tag{60}$$

Inserting (60) in (57), straightforward calculations produce

$$a_3^{\text{int}} = 2c'_{5BA} G'^{*B\lambda\mu} \left[G'^{A}_{\lambda}{}^{\nu} \partial_\mu \eta_\nu + 3C^{A\nu} \left(\partial_{(\lambda} h_{\mu)\nu} - \partial_\nu h_{\lambda\mu} \right) \right], \tag{61}$$

and thus we then get

$$\begin{aligned} \delta a_3^{\text{int}} &= \partial_\mu \left[-4c'_{5BA} \mathcal{G}'^{*B\mu\nu|\lambda} \left(G'^{A}_{\nu}{}^{\rho} \partial_\lambda \eta_\rho + 3C^{A\rho} \left(\partial_{(\lambda} h_{\nu)\rho} - \partial_\rho h_{\lambda\nu} \right) \right) \right] \\ &\quad + \gamma \left[-2c'_{5BA} \mathcal{G}'^{*B\mu\nu|\lambda} \left(\mathcal{G}'^{A}_{\mu\nu|}{}^{\rho} \partial_\lambda \eta_\rho + G'^{A}_{\mu}{}^{\rho} \left(\partial_{(\lambda} h_{\nu)\rho} - \partial_\rho h_{\lambda\nu} \right) \right) \right] \\ &\quad - 12c'_{5BA} \mathcal{G}'^{*B\mu\nu|\alpha} K_{\mu\nu|\alpha\beta} C^{A\beta}, \end{aligned} \tag{62}$$

with $K_{\mu\nu|\alpha\beta}$ the linearized Riemann tensor (7) and the prime variables defined in (20) and (22). Comparing (58) with (62), we can state that (58) admits solutions for a_2^{int} if and only if

$$-12c'_{5BA} \mathcal{G}'^{*B\mu\nu|\alpha} K_{\mu\nu|\alpha\beta} C^{A\beta} = \gamma b_2 + \partial_\mu \overset{(2)}{m}^{\mu}, \tag{63}$$

for some b_2 and $\overset{(2)}{m}^{\mu}$ with the properties

$$\operatorname{agh}(b_2) = 2 = \operatorname{agh}\left(\overset{(2)}{m}^{\mu}\right), \quad \operatorname{pgh}(b_2) = 2, \quad \operatorname{pgh}\left(\overset{(2)}{m}^{\mu}\right) = 3. \tag{64}$$

The left-hand side of (63) is a non-trivial element from $H^3(\gamma)$ of antighost number two, with the accompanying invariant polynomial of the form

$$-12c'_{5BA} \mathcal{G}'^{*B\mu\nu|\alpha} K_{\mu\nu|\alpha\beta},$$

such that we must set $b_2 = 0$, while the same expression cannot be written like a divergence, so we also have that $\overset{(2)}{m}^{\mu} = 0$. The above observations lead to the conclusion that (63) holds if and only if

$$c'_{5BA} = 0, \tag{65}$$

which further implies, via (60),

$$a_4^{\text{int}} = 0. \tag{66}$$

Consequently, the first-order deformation that describes the cross-interactions between the Pauli–Fierz field and the mixed symmetry-type tensors $t^A_{\lambda\mu\nu|\alpha}$ can be taken to stop at an antighost number $I \leq 3$.

Assuming now that $I = 3$, we have that the first-order deformation reduces to

$$a^{\text{int}} = a_0^{\text{int}} + a_1^{\text{int}} + a_2^{\text{int}} + a_3^{\text{int}}, \tag{67}$$

where the last representative is of the form (44) for $I = 3$, with α_3 non-trivial in $H_3^{\text{inv}D}(\delta|d)$. We apply the result from (47), according to which α_3 is linear in the antifields $G'^{*B}_{\lambda\mu}$, such that the elements e^3 that can be used to construct an a_3^{int} with the desired properties (fulfilling the derivative order assumption and providing effective cross-interactions between the two types of tensor fields) are spanned by

$$\text{eligible } e^3 : \left(\eta_\nu\eta_\rho\eta_\sigma, \eta_\nu\eta_\rho\partial_{[\sigma}\eta_{\tau]}, \eta_\nu\eta_\rho\mathcal{F}^A_{\lambda'\mu'\nu'\rho'}\right), \tag{68}$$

so we obtain that

$$a_3^{\text{int}} = G'^{*B}_{\lambda\mu}\eta_\nu\eta_\rho\left(f_{1B}^{\lambda\mu\nu\rho\sigma}\eta_\sigma + f_{2B}^{\lambda\mu\nu\rho\sigma\tau}\partial_{[\sigma}\eta_{\tau]} + f_{3BA}^{\lambda\mu\nu\rho\lambda'\mu'\nu'\rho'}\mathcal{F}^A_{\lambda'\mu'\nu'\rho'}\right) + \gamma b_3. \tag{69}$$

All the coefficients denoted by f are restricted, as everywhere before, to be constant. By organizing the emerging acceptable (Lorentz-covariant and Poincaré-invariant) combinations according to the space-time dimension, we consequently arrive at:

$$D = 5, \quad a_3^{(D=5)\text{int}} = c_{1B}\varepsilon^{\lambda\mu\nu\rho\sigma}G'^{*B}_{\lambda\mu}\eta_\nu\eta_\rho\eta_\sigma + \gamma b_3^{(D=5)}, \tag{70}$$

$$D = 6, \quad a_3^{(D=6)\text{int}} = c_{2B}\varepsilon^{\lambda\mu\nu\rho\sigma\tau}G'^{*B}_{\lambda\mu}\eta_\nu\eta_\rho\partial_{[\sigma}\eta_{\tau]} + \gamma b_3^{(D=6)}, \tag{71}$$

$$D = 8, \quad a_3^{(D=8)\text{int}} = c_{3BA}\varepsilon^{\lambda\mu\nu\rho\lambda'\mu'\nu'\rho'}G'^{*B}_{\lambda\mu}\eta_\nu\eta_\rho\mathcal{F}^A_{\lambda'\mu'\nu'\rho'} + \gamma b_3^{(D=8)}, \tag{72}$$

$$\begin{aligned} D \geq 5, \quad a_3^{(D)\text{int}} = {} & G'^{*B\lambda\mu}\left(c'_{2B}\sigma_{\lambda\mu}\eta^\nu\eta^\rho\partial_{[\nu}\eta_{\rho]} + c''_{2B}\eta_\lambda\eta^\nu\partial_{[\mu}\eta_{\nu]}\right. \\ & \left. + c'_{3BA}\eta^\nu\eta^\rho\mathcal{F}^A_{\lambda\mu\nu\rho}\right) + \gamma b_3^{(D)}. \end{aligned} \tag{73}$$

Straightforward computation shows that (70)–(73) cannot be lifted to antighost number two, i.e., there are no solutions a_2^{int} to the equation $\delta a_3^{\text{int}} + \gamma a_2^{\text{int}} = \partial_\mu \overset{(2)}{m}{}^\mu$, irrespective of what γ-exact contributions we take in their right-hand sides, so we must set

$$c_{1B} = c_{2B} = c_{3BA} = c'_{2B} = c''_{2B} = c'_{3BA} = 0, \tag{74}$$

which leads to

$$a_3^{\text{int}} = 0, \tag{75}$$

and so the first-order deformation a^{int} cannot end at antighost number three either.

The next possible maximum value of the antighost number in a^{int} is $I = 2$, in which case a^{int} reads

$$a^{\text{int}} = a_0^{\text{int}} + a_1^{\text{int}} + a_2^{\text{int}}, \tag{76}$$

where a_2^{int} of the form (44) for $I = 2$ and α_2 is a non-trivial object from $H_2^{\text{inv}D}(\delta|d)$. According to (47) that $H_2^{\text{inv}D}(\delta|d)$ is spanned by the antifields $\mathcal{G}'^{*\mu\nu|\alpha}_A$ and $\eta^{*\mu}$. This is actually the first place where the Pauli–Fierz theory brings non-trivial contributions to the local cohomology of the Koszul–Tate differential. Taking into account the actions of δ on these antifields, we observe that e^2 cannot include more than one space-time derivative in order to ensure an a_0^{int} with at most two derivatives. Meanwhile, for a^{int} to describe cross-interactions, the terms proportional with $\mathcal{G}_B^{\prime*\mu\nu|\alpha}$ must involve at least one of the combinations η_μ or $\partial_{[\mu}\eta_{\nu]}$, while those linear in $\eta^{*\mu}$ are required to depend on $\mathcal{F}^A_{\lambda\mu\nu\alpha}$. The above considerations render the eligible e^2 like

$$\text{eligible } e^2 : \left(\eta_\nu\eta_\rho, \eta_\nu\partial_{[\rho}\eta_{\sigma]}, \eta_\nu\mathcal{F}^A_{\lambda'\mu'\nu'\rho'}\right), \tag{77}$$

which, appended to the general assumptions of Lorentz covariance and Poincaré invariance, then yield

$$\begin{aligned} a_2^{\text{int}} = {} & \mathcal{G}^{\prime*B}_{\lambda\mu|\alpha}\eta_\nu\left(f_{1B}^{\lambda\mu\alpha\nu\rho}\eta_\rho + f_{2B}^{\lambda\mu\alpha\nu\rho\sigma}\partial_{[\rho}\eta_{\sigma]} + f_{3BA}^{\lambda\mu\alpha\nu\lambda'\mu'\nu'\rho'}\mathcal{F}^A_{\lambda'\mu'\nu'\rho'}\right) \\ & + f_{4A}^{\lambda\mu\lambda'\mu'\nu'\rho'}\eta^*_\lambda\eta_\mu\mathcal{F}^A_{\lambda'\mu'\nu'\rho'} + \gamma b_2, \end{aligned} \tag{78}$$

where all the f's are constant. Structuring the independent possibilities like in the above, according to the space-time dimension, we get:

$$D = 5, \quad a_2^{(D=5)\text{int}} = c_{1B}\varepsilon^{\lambda\mu\alpha\nu\rho}\mathcal{G}^{\prime*B}_{\lambda\mu|\alpha}\eta_\nu\eta_\rho + \gamma b_2^{(D=5)}, \tag{79}$$

$$\begin{aligned} D = 6, \quad a_2^{(D=6)\text{int}} = {} & c_{2B}\varepsilon^{\lambda\mu\alpha\nu\rho\sigma}\mathcal{G}^{\prime*B}_{\lambda\mu|\alpha}\eta_\nu\partial_{[\rho}\eta_{\sigma]} \\ & + c_{4A}\varepsilon^{\lambda\mu\lambda'\mu'\nu'\rho'}\eta^*_\lambda\eta_\mu\mathcal{F}^A_{\lambda'\mu'\nu'\rho'} + \gamma b_2^{(D=6)}, \end{aligned} \tag{80}$$

$$D = 8, \quad a_2^{(D=8)\text{int}} = c_{3BA}\varepsilon^{\lambda\mu\alpha\nu\lambda'\mu'\nu'\rho'}\mathcal{G}^{\prime*B}_{\lambda\mu|\alpha}\eta_\nu\mathcal{F}^A_{\lambda'\mu'\nu'\rho'} + \gamma b_2^{(D=8)}, \tag{81}$$

$$\begin{aligned} D \geq 5, \quad a_2^{(D)\text{int}} = {} & \mathcal{G}^{\prime*B\lambda\mu|\alpha}\left(c'_{2B}\eta_\alpha\partial_{[\lambda}\eta_{\mu]} + c''_{2B}\sigma_{\rho\alpha}\eta^\rho\partial_{[\lambda}\eta_{\mu]}\right. \\ & \left.+ c'_{3BA}\eta^\nu\mathcal{F}^A_{\lambda\mu\alpha\nu}\right) + \gamma b_2^{(D)}. \end{aligned} \tag{82}$$

Due to the presence in all the previous representatives of $H^2(\gamma)$ of at least one undifferentiated Pauli–Fierz ghost, it results that they cannot be appropriately lifted to a_1^{int} as solution to the equation $\delta a_2^{\text{int}} + \gamma a_1^{\text{int}} = \partial_\mu \overset{(1)}{m}{}^\mu$, such that we are obliged to take

$$c_{1B} = c_{2B} = c_{4A} = c_{3BA} = c'_{2B} = c''_{2B} = c'_{3BA} = 0, \tag{83}$$

which further produces

$$a_2^{\text{int}} = 0, \tag{84}$$

so we conclude that the first-order deformation a^{int} can only stop at $I \leq 1$. Thus, there are no non-trivial cross-interactions between the tensor fields $t^A_{\lambda\mu\nu|\alpha}$ and the Pauli–Fierz field complying with all the above mentioned requirements that modify the original Abelian gauge algebra.

For $I = 1$, the first-order deformation a^{int} reduces to

$$a^{\text{int}} = a_0^{\text{int}} + a_1^{\text{int}}, \tag{85}$$

with a_1^{int} of the form(44) for $I = 1$ and α_1 necessarily an element from $H_1^D(\delta|d)$, such that

$$\alpha_1 = \alpha_1\left(\left[t_A^{*\lambda\mu\nu|\alpha}\right], \left[h^{*\mu\nu}\right], \left[F^A_{\lambda\mu\nu\xi|\alpha\beta}\right], \left[K_{\mu\nu|\alpha\beta}\right]\right), \quad \delta\alpha_1 = \partial_\mu t^\mu. \tag{86}$$

The elements of pure ghost number equal to one of a basis in the space of polynomials in η_μ, $\partial_{[\mu}\eta_{\nu]}$ and $\mathcal{F}^A_{\lambda\mu\nu\alpha}$ are spanned by

$$e^1 : \left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, \mathcal{F}^A_{\lambda\mu\nu\alpha}\right), \tag{87}$$

while the invariant polynomial $\alpha_1 d^D x \in H^D_1(\delta|d)$ is linear in the antifields $t^{*\lambda\mu\nu|\alpha}_B$, $h^{*\mu\nu}$ and in their derivatives up to some finite orders, as these are the only generators of antighost number equal to one from the BRST complex. The assumption on the maximum derivative order of a_0^{int} being equal to two restricts α_1 not to depend on either the curvatures or their derivatives. Along the same line, α_1 cannot contain more that the first-order derivatives of the antifields. Regarding the terms including the first-order derivatives of the antifields, we can always make (by an integration by parts in the corresponding functional) the derivative to act on e^1, and therefore they can be taken to be linear in the undifferentiated antifields. As the first-order derivatives of $\partial_{[\mu}\eta_{\nu]}$ and $\mathcal{F}^A_{\lambda\mu\nu\alpha}$ are γ-exact, the corresponding terms in a_1^{int} can be discarded since they are γ-trivial. The piece displaying the first-order derivatives of η_μ can always be made proportional with $\partial_{[\mu}\eta_{\nu]}$ by adding irrelevant, γ-exact objects, so it can be taken to depend only on the undifferentiated antifields. In consequence, the dependence of α_1 on the first-order derivatives of the antifields can be removed. Accordingly, we can write that

$$a_1 = \alpha_{1\text{lin}}\left(t^{*\lambda\mu\nu|\alpha}_A, h^{*\mu\nu}\right) e^1\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}, \mathcal{F}^A_{\lambda\mu\nu\alpha}\right) + \gamma b_1. \tag{88}$$

Selecting only the non-trivial terms that potentially lead to cross-couplings among the two types of tensor fields, we arrive at

$$a_1^{\text{int}} = t^{*A}_{\lambda\mu\nu|\alpha}\left(f^{\lambda\mu\nu\alpha\rho}_{1A}\eta_\rho + f^{\lambda\mu\nu\alpha\rho\sigma}_{2A}\partial_{[\rho}\eta_{\sigma]}\right) + f^{\lambda\mu\nu\rho\sigma\alpha}_{3A} h^*_{\lambda\mu}\mathcal{F}^A_{\nu\rho\sigma\alpha} + \gamma b_1, \tag{89}$$

where all the coefficients denoted by f are required to be constant. Taking into account the identity $t^{*A}_{[\lambda\mu\nu|\alpha]} \equiv 0$, it follows that any solution containing Levi–Civita symbols contracted on all the indices of this antifield vanish. Invoking in addition the symmetry of the Pauli–Fierz antifield, we remain with a single candidate in all $D \geq 5$ dimensions

$$a_1^{\text{int}} = c^A t^{*\lambda\mu}_A \partial_{[\lambda}\eta_{\mu]}. \tag{90}$$

For convenience, we took $b_1 = 0$ in (89). We observe that δa_1^{int} can be written under a divergence-like form

$$\delta a_1^{\text{int}} = \partial_\mu\left(2c^A\eta_\lambda T^{\lambda\mu}_A\right), \tag{91}$$

so the corresponding a_0^{int} can be taken equal to zero

$$a_0^{\text{int}} = 0, \tag{92}$$

so we obtain that the first-order deformation (85) reduces to its antighost number one component

$$a^{\text{int}} = a_1^{\text{int}} = c^A t^{*\lambda\mu}_A \partial_{[\lambda}\eta_{\mu]}. \tag{93}$$

This solution presents a strange behavior. It modifies the gauge transformations of the tensor fields $t^A_{\lambda\mu\nu|\alpha}$ at order one in the coupling constant by elements involving the Pauli–Fierz gauge parameter, but adds no coupling terms with the Pauli–Fierz field to the deformed Lagrangian action. Actually, solution (93) is purely trivial in $H^{0,D}(s|d)$

$$a^{\text{int}} = s\left(-\frac{2c^A}{3}\mathcal{G}'^{*\mu\nu|\alpha}_A\sigma_{\nu\alpha}\eta_\mu\right) + \partial_\mu\left(-2c^A t^{*\lambda\mu}_A\eta_\lambda\right), \tag{94}$$

such that it can be safely removed from the first-order deformation. In conclusion, the deformation procedure allows no non-trivial cross-couplings that change the original gauge transformations.

We are now left with one more case, where the first-order deformation $a^{\rm int}$ coincides with its antighost number zero component ($I=0$)

$$a^{\rm int} = a_0^{\rm int}\left(\left[t^A_{\lambda\mu\nu|\alpha}\right],\left[h_{\mu\nu}\right]\right), \tag{95}$$

with

$$\gamma a_0^{\rm int} = \partial_\mu \overset{(0)}{m}{}^\mu, \tag{96}$$

being understood that we discard the divergence-like solutions. There appear two different situations. The first one is associated with $\overset{(0)}{m}{}^\mu = 0$ in (96) and its solutions are constructed from the gauge-invariant quantities, which are the curvatures and their derivatives

$$\gamma a_0^{\prime\rm int} = 0 \Rightarrow a_0^{\prime\rm int} = a_0^{\prime\rm int}\left(\left[F^A_{\lambda\mu\nu\xi|\alpha\beta}\right],\left[K_{\mu\nu|\alpha\beta}\right]\right). \tag{97}$$

However, the cross-coupling component with the minimum number of derivatives from $a_0^{\prime\rm int}$ is of order four in the space-time derivatives, being proportional with $F^A_{\lambda\mu\nu\xi|\alpha\beta}K_{\mu'\nu'|\alpha'\beta'}$, and therefore the solutions (97) are not eligible, as they disagree with the assumption on the maximum derivative order of the interacting Lagrangian being equal to two.

The second situation corresponds to $\overset{(0)}{m}{}^\mu \neq 0$ in (96). Denoting the Euler–Lagrange derivatives of $a_0^{\rm int}$ by

$$B_A^{\lambda\mu\nu|\alpha} \equiv \frac{\delta a_0^{\rm int}}{\delta t^A_{\lambda\mu\nu|\alpha}}, \quad D^{\mu\nu} \equiv \frac{\delta a_0^{\rm int}}{\delta h_{\mu\nu}}, \tag{98}$$

equation (96) further implies the necessary conditions

$$\partial_\alpha B_A^{\lambda\mu\nu|\alpha} = 0, \quad \partial_\lambda B_A^{\lambda\mu\nu|\alpha} = 0, \quad \partial_\mu D^{\mu\nu} = 0. \tag{99}$$

The tensors $B_A^{\lambda\mu\nu|\alpha}$ have the same mixed symmetry like $t^A_{\lambda\mu\nu|\alpha}$ and $D^{\mu\nu}$ is symmetric. Moreover, $B_A^{\lambda\mu\nu|\alpha}$ and $D^{\mu\nu}$ must involve at least one Pauli–Fierz field, respectively, one tensor field $t^A_{\lambda\mu\nu|\alpha}$ in order to provide cross-couplings. The general solutions to equations (99) are of the type

$$\frac{\delta a_0^{\rm int}}{\delta t^A_{\lambda\mu\nu|\alpha}} \equiv B_A^{\lambda\mu\nu|\alpha} = \partial_\xi\partial_\beta\tilde{\Phi}_A^{\lambda\mu\nu\xi|\alpha\beta}, \quad \frac{\delta a_0^{\rm int}}{\delta h_{\mu\nu}} \equiv D^{\mu\nu} = \partial_\alpha\partial_\beta\bar{\Phi}^{\mu\alpha|\nu\beta}, \tag{100}$$

where $\tilde{\Phi}_A^{\lambda\mu\nu\xi|\alpha\beta}$ and $\bar{\Phi}^{\mu\alpha|\nu\beta}$ depend only on the undifferentiated fields $h_{\mu\nu}$ and $t^B_{\lambda\mu\nu|\alpha}$ (otherwise, the corresponding $a_0^{\rm int}$ would be more than second-order in the derivatives), with $\tilde{\Phi}_A^{\lambda\mu\nu\xi|\alpha\beta}$ having the mixed symmetry of the curvature tensors $F_A^{\lambda\mu\nu\xi|\alpha\beta}$ and $\bar{\Phi}^{\mu\alpha|\nu\beta}$ that of the linearized Riemann tensor. We introduce a derivation on the algebra of non-integrated densities depending on $t^A_{\lambda\mu\nu|\alpha}$, $h_{\mu\nu}$ and on their derivatives, that counts the powers of the fields and their derivatives

$$\bar{N} = \sum_{n\geq 0}\left[\left(\partial_{\mu_1\ldots\mu_n}t^A_{\lambda\mu\nu|\alpha}\right)\frac{\partial}{\partial\left(\partial_{\mu_1\ldots\mu_n}t^A_{\lambda\mu\nu|\alpha}\right)} + \left(\partial_{\mu_1\ldots\mu_n}h_{\mu\nu}\right)\frac{\partial}{\partial\left(\partial_{\mu_1\ldots\mu_n}h_{\mu\nu}\right)}\right], \tag{101}$$

and observe that the action of $\bar{N}$ on an arbitrary non-integrated density $\bar{u}$ is

$$N\bar{u} = t^A_{\lambda\mu\nu|\alpha}\frac{\delta\bar{u}}{\delta t^A_{\lambda\mu\nu|\alpha}} + h_{\mu\nu}\frac{\delta\bar{u}}{\delta h_{\mu\nu}} + \partial_\mu r^\mu, \tag{102}$$

where $\delta\bar{u}/\delta t^A_{\lambda\mu\nu|\alpha}$ and $\delta\bar{u}/\delta h_{\mu\nu}$ denote the variational derivatives of $\bar{u}$. In the case where $\bar{u}$ is an homogeneous polynomial of order $p > 0$ in the fields and their derivatives, we have that $\bar{N}\bar{u} = p\bar{u}$, and so

$$\bar{u} = \frac{1}{p}\left(t^A_{\lambda\mu\nu|\alpha}\frac{\delta\bar{u}}{\delta t^A_{\lambda\mu\nu|\alpha}} + h_{\mu\nu}\frac{\delta\bar{u}}{\delta h_{\mu\nu}}\right) + \partial_\mu\left(\frac{1}{p}r^\mu\right). \tag{103}$$

As a_0 can always be decomposed as a sum of homogeneous polynomials of various orders, it is enough to analyze the equation (96) for a fixed value of p. Putting $\bar{u} = a_0^{\rm int}$ in (103) and inserting (100) in the associated relation, we can write

$$a_0^{\rm int} = \frac{1}{p}\left(t^A_{\lambda\mu\nu|\alpha}\partial_\xi\partial_\beta\tilde{\Phi}_A^{\lambda\mu\nu\xi|\alpha\beta} + h_{\mu\nu}\partial_\alpha\partial_\beta\bar{\Phi}^{\mu\alpha|\nu\beta}\right) + \partial_\mu\bar{r}^\mu. \tag{104}$$

Integrating twice by parts in (104) and recalling the mixed symmetries of $\tilde{\Phi}_A^{\lambda\mu\nu\xi|\alpha\beta}$ and $\bar{\Phi}^{\mu\alpha|\nu\beta}$, we infer that

$$a_0^{\rm int} = k_1 F^A_{\lambda\mu\nu\xi|\alpha\beta}\tilde{\Phi}_A^{\lambda\mu\nu\xi|\alpha\beta} + k_2 K_{\mu\alpha|\nu\beta}\bar{\Phi}^{\mu\alpha|\nu\beta} + \partial_\mu\bar{l}^\mu, \tag{105}$$

with $k_1 = 1/8p$ and $k_2 = -1/2p$. By computing the action of γ on (105), we obtain that $\tilde{\Phi}^{\lambda\mu\nu\xi|\alpha\beta}$ and $\bar{\Phi}^{\mu\alpha|\nu\beta}$ are precisely of the type

$$\tilde{\Phi}_A^{\lambda\mu\nu\xi|\alpha\beta} = k'^B_A\Phi_B^{\lambda\mu\nu\xi|\alpha\beta}, \quad \bar{\Phi}^{\mu\alpha|\nu\beta} = k''\Phi^{\mu\alpha|\nu\beta}, \tag{106}$$

where k'^B_A and k'' are real constants, while $\Phi_B^{\lambda\mu\nu\xi|\alpha\beta}$ and $\Phi^{\mu\alpha|\nu\beta}$ are the tensors that define the free field equations. Accordingly, the admitted value of p is fixed to $p = 2$. Replacing (106) in (104) for $p = 2$, we finally arrive at

$$a_0^{\rm int}\left(\left[t^A_{\lambda\mu\nu|\alpha}\right], \left[h_{\mu\nu}\right]\right) = \frac{k'^B_A}{2}t^A_{\lambda\mu\nu|\alpha}T_B^{\lambda\mu\nu|\alpha} + \frac{k''}{2}h_{\mu\nu}H^{\mu\nu} + \partial_\mu\bar{r}^\mu. \tag{107}$$

The solution (107) is not eligible since it gives no cross-couplings between the two types of investigated tensor fields. Therefore, we can always discard it from the first-order deformation

$$a_0^{\rm int}\left(\left[t^A_{\lambda\mu\nu|\alpha}\right], \left[h_{\mu\nu}\right]\right) = 0. \tag{108}$$

In consequence, there are no cross-couplings invariant under the original gauge transformations (4) and (5) that can be added to the free action (1).

Putting together the results contained in this section, we can state that

$$S_1^{\rm int} = 0, \tag{109}$$

and so

$$S_1 = S_1^{\rm h}, \tag{110}$$

where $S_1^{\rm h}$ is the first-order deformation of the solution to the master equation for the Pauli–Fierz theory. The consistency of the deformed solution to the master equation at the second order in the coupling constant is governed by the equation (31), where

$\left(S_1^{\rm h}, S_1^{\rm int}\right) = 0 = \left(S_1^{\rm int}, S_1^{\rm int}\right)$, but $\left(S_1^{\rm h}, S_1^{\rm h}\right) \neq 0$, and thus we have that $S_2^{\rm int} = 0$, while $S_2^{\rm h}$ is highly non-trivial and is known to describe the quartic vertex of the Einstein–Hilbert action, as well as the second-order contributions to the gauge transformations and to the associated non-Abelian gauge algebra. The vanishing of $S_1^{\rm int}$ and $S_2^{\rm int}$ further leads, via the equations that stipulate the higher-order deformation equations, to the result that

$$S_k^{\rm int} = 0, \quad k \geq 1. \tag{111}$$

The main conclusion of this paper is that, under the general conditions of analyticity in the coupling constant, space-time locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the requirement that the interacting Lagrangian is at most second-order derivative, there are no consistent, non-trivial cross-couplings between the Pauli–Fierz field and a collection of massless tensor fields with the mixed symmetry $(3,1)$. The only pieces that can be added to action (1) are given by the self-interactions of the Pauli–Fierz field, which produce the Einstein–Hilbert action, invariant under diffeomorphisms.

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