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Power deposition profile in tokamak plasma with ICRH

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Abstract

Ion Cyclotron Resonance Heating (ICRH) is one of the usual schemes to auxiliary heat the plasma in tokamak fusion device. The profile of power absorption is of great importance both for the heating efficiency and the transport. This profile is actualy not a simple Gaussian, as is sometimes approximated, but a more complex function. An analytic expression is proposed here for a well description of the power deposition profile.

1 Introduction

Resonant interaction between ions and waves in the ion cyclotron range of frequencies leads both to heating the plasma and spatial transport of the resonating ions. The current and torque provided by ICRH depend on the concentrations of heated species and also on the positions of the resonance layers. Recently works analyses the resonant ion cyclotron interactions in tokamaks and influence of the radio frequency ponderomotive force on anomalous impurity transport in tokamaks, see for example [1], and [2]. In both issues the resonance layer position and the profile of power absorption plays an important role for stability and confinement properties of the plasma.

In the following we analyse the profile of power absorption by tritium in a deuterium-tritium plasma heated by ICRF, in particular the second harmonic resonance frequency for tritium, in a tokamak fusion device. In many works this profile is considered for simplicity as a Gaussian, but experiment measurements and numerical simulation reveal a more complex profile. In the section 2 we describe the geometry of the ICRH in a torus with circular cross-section of equilibrium magnetic surface. A realistic power deposition profile is described in section 3 and an analytic expression is proposed here for a well reproduction of the power deposition profile.

2 Ion cyclotron heating in tokamaks

In a homogeneous magnetic field the ion cyclotron heating by resonant absorption depends, through the Doppler frequency shift, only on the ion parallel velocity and $k_{||}$ and do not depend on the position (not localized). The collisional diffusion can alter the resonant condition: it can make an ion initially resonant to became not resonant and

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viceversa. By contrary, in inhomogenous configuration of the magnetic field the ion cyclotron resonant heating (ICRH) is also localized.

We assume here, for simplicity, an axisymmetric model of the equilibrium magnetic field for a large aspect ratio tokamak with circular concentric magnetic surfaces

$$\mathbf{B} = B_0 \frac{R_0}{R} \left(-\frac{r}{q_s R_0} \mathbf{e}_\theta + \mathbf{e}_\zeta \right) \tag{1}$$

where R_0 is the major radius, r the toroidal radial coordinate, $R = R_0 + r\cos\theta$ the cylindrical radial coordinate and $q_s(r) = |rB_\zeta / R_0 B_\theta|$ the safety factor (see for example *[3]*). The magnitude of the equilibrium magnetic field is then, see also [4]:

$$B(r) = \frac{\mathcal{B}_0(r)}{1+\eta\cos\theta} = \frac{B_0}{1+\eta\cos\theta}\sqrt{1+\eta^2 q^{-2}} \tag{2}$$

with $-\epsilon \le \eta \le \epsilon$, $\eta = r/R_0$, $\epsilon = a/R_0$ the inverse aspect ratio and $B_0 = B(r=0)$.

The cyclotron resonance condition is described using the δ-function

$$\delta\left(\omega - n\Omega_{ci} - k_{||}v_{||}\right)$$

For a given $k_{||}v_{||}$ the cyclotron resonance is localized on some surface $B(R) =$ constant, which here is a vertical cylindrical surface. In the case $k_{||}v_{||} = 0$ the distance r_0 from the center of toroidal cross section up to the resonant surface corresponds to

$$\omega = n\Omega_{ci,0} \equiv \omega_0$$

so

$$\omega_0 = n\frac{e_i}{m_i c}\frac{B_0}{1+r_0R_0^{-1}}\sqrt{1+r_0^2R_0^{-2}q^{-2}} \approx n\frac{e_i}{m_i c}\frac{B_0}{1+r_0R_0^{-1}} \tag{3}$$

where the approximation is valuable for a large aspect ratio tokamak.

When $k_{||}v_{||}$ has finite nonzero values the resonance condition leads to

$$\omega = n\Omega_{ci} + k_{||}v_{||}$$

or

$$\omega = n\frac{e_i}{m_i c}\frac{B_0}{1+rR_0^{-1}} + k_{||}v_{||} \tag{4}$$

where

$$r = r_0 + \triangle X$$

Hence the region of absorption is a vertical cylindrical layer of width $\triangle X$ determined by the Doppler shift of thermal particles. From (3) and (4) we evaluate $\triangle X$ as

$$\triangle X \approx R_0 \frac{k_{||}v_{||}}{n\Omega_{ci,0}} \tag{5}$$

Since resonant wave-ion particle interactions at the cyclotron frequency increase the average perpendicular energy of the resonant ions without appreciably affecting their parallel energy we can approximate $v_{||} \approx v_{thi}$, where v_{thi} is the ion thermal velocity, and obtain (see also [5])

$$\triangle X \simeq R_0 \frac{k_{||}v_{thi}}{n\Omega_{ci,0}} \tag{6}$$

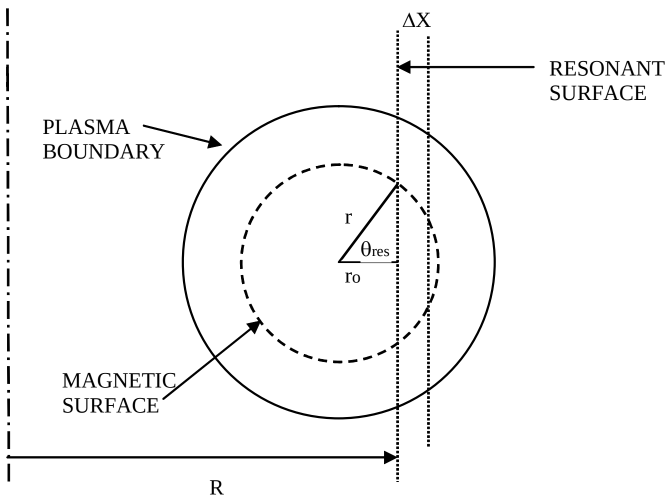

Fig.1. Geometry for ICRH in a torus with circular crossection.

If the ion velocity distribution is Maxwellian the intensity of absorption within this layer has a Gaussian profile. In this case the flux surface averaged absorbed rf power density, $\langle P\rangle$,

$$\langle P\rangle = P_0 \exp\left[-\frac{(r - a\cos\theta_{res})^2}{2(\triangle X)^2}\right]$$

where θ_{res} is the poloidal angle corresponding to the central vertical axes of the resonance layer and a the minor radius. We plot $\langle P\rangle / P_0$, see figure 2, in the case of ion cyclotron resonance waves with $\omega = 2\pi * 53$ MHz and $k_{||} = 27/R$ absorbed by Tritium in tokamak plasma with parameters $R_0 = 620$ cm, $a = 200$ cm, $\theta_{res} = 9\pi/19$.

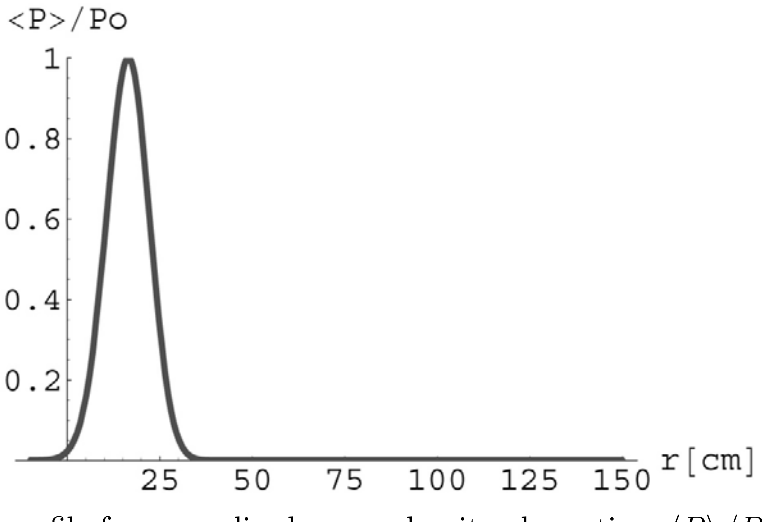

Fig.2. Gaussian profile for normalized power density absorption, $\langle P\rangle / P_0$, of ICRH with $\omega = 2\pi * 53MHz$ and $k_{||} = 27/R$ by Tritium in tokamak plasma with parameters $R_0 = 620cm$, $a = 200cm$, $\theta_{res} = 9\pi/19$.

3 Model for power density absorption profile

From the experiments evidences we conclude that the ion velocity distribution is not Maxwellian and the intensity of absorption has not a Gaussian profile. We give here, see Fig. 3, an example of the power density absorption profile given by a list of points obtained from numerical simulation - see [6].

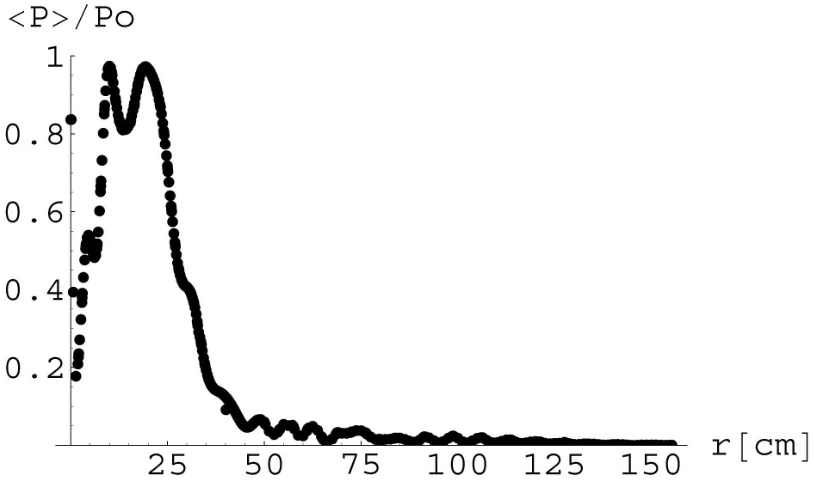

Fig 3. Profile for normalized power density absorbtion, $\langle P\rangle / P_0$, of ICRH with $\omega = 2\pi * 53MHz$ and $k_{||} = 27/R$ by Tritium in tokamak plasma obtained from numerical simulation - see [6]

The profile show in Fig. 3 can be well described through a superposition of 21 Gaussian functions:

$$\begin{aligned}(\langle P\rangle / P_0)_G = {} & 0.82 * \exp\left[-(r+0.18)^2\right] + 0.43 * \exp\left[-\frac{1}{5}(r-4.1)^2\right] \\ & + 0.59 * \exp\left[-\frac{1}{7.7}(r-9.6)^2\right] + 0.042 * \exp\left[-(r-13.3)^2\right] \\ & + 0.97 * \exp\left[-\frac{1}{10^2}(r-19.4)^2\right] + 0.16 * \exp\left[-\frac{(r-32)^2}{5.5}\right]\end{aligned} \tag{7}$$

$$+0.11 * \exp\left[-\frac{(r-40)^2}{30}\right] + 0.06 * \exp\left[-\frac{(r-49)^2}{7}\right] + 0.057 * \exp\left[-\frac{(r-56)^2}{10}\right]$$

$$+0.052 * \exp\left[-\frac{(r-63)^2}{7}\right] + 0.033 * \exp\left[-\frac{(r-70)^2}{6}\right] + 0.036 * \exp\left[-\frac{(r-74)^2}{8}\right]$$

$$+0.02 * \exp\left[-\frac{(r-80)^2}{4}\right] + 0.023 * \exp\left[-\frac{(r-85)^2}{4}\right] + 0.024 * \exp\left[-\frac{(r-90)^2}{3}\right]$$

$$+0.021 * \exp\left[-\frac{(r-95)^2}{3}\right] + 0.018 * \exp\left[-\frac{(r-100)^2}{5}\right] + 0.014 * \exp\left[-\frac{(r-105)^2}{5}\right]$$

$$+10^{-2} * \exp\left[-\frac{(r-110)^2}{5}\right] + 0.007 * \exp\left[-\frac{(r-115)^2}{5}\right] + 0.005 * \exp\left[-\frac{(r-120)^2}{5}\right]$$

In figure 4 we plot $(\langle P\rangle / P_0)_G$ given by (7) superposed on plot given in figure 3.

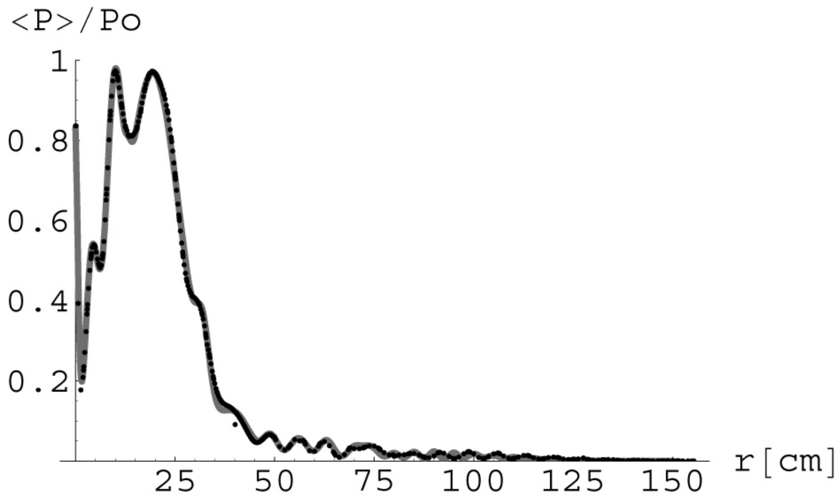

Fig. 4. Plot of $(\langle P\rangle / P_0)_G$ (continuos line) compared with $\langle P\rangle / P_0$ (points obtained by numerical simulations). A very good match result.

We propose here the following function to approximate the profile for $\langle P\rangle / P_0$:

$$\begin{aligned}(\langle P\rangle / P_0)_{app} &= 0.8 \exp\left[-\frac{(r-0.03R_0)^2}{5.6\,(\triangle X)}\right] + 0.21 \exp\left[-\frac{3\,(r-0.017R_0)^2}{(\triangle X)^2}\right] \\ &+ 0.3 \exp[-7r/a]\left(\sin\left[\exp\left[2.7/\left(1+r/a\right)^6\right]\right]\right)^2 \\ &+ 80\left(\frac{r}{a}\right)^2 \exp[-7r/a]\left[J_1\left(80r/a+3/2\right)\right]^2\end{aligned} \tag{8}$$

In Fig. 5 we plot both $(\langle P\rangle / P_0)_{app}$ given in eq.(8) and $\langle P\rangle / P_0$ given by points given in Fig.3. We remark a good agreement between the two but not an identity.

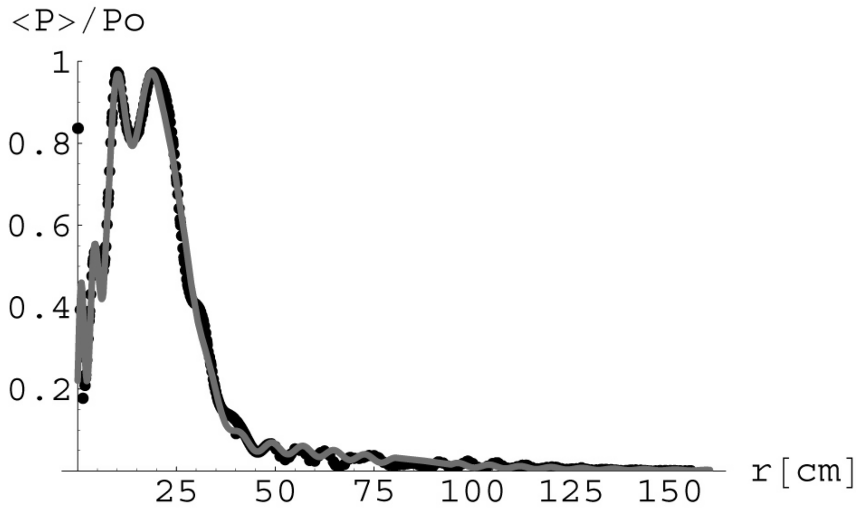

Fig. 5. Plot of $(\langle P\rangle / P_0)_{app}$ (continuos line) and $\langle P\rangle / P_0$ from numerical simulations (given in Fig.3)

By numerical integration on r we can verify that

$$\int_0^a \left(\langle P\rangle / P_0\right)_G dr \cong \int_0^a \left(\langle P\rangle / P_0\right)_{app} dr$$

which means the approximate same absorbed power given by the two equation (8) and (7).

4 Conclusion

The approximation given in (8) consist of superposition of two Gaussian with other two periodic functions. The expression (8) is more compact in rapport with (7) and, in consequence, easier to use in analytic analyses, both in kinetic and fluid transport theories. This model will be improved in next future by analyzing other profiles obtained by numerical methods or experimental measurements for other tokamak parameters.

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