Physics AUC, vol. 19, 1-21 (2009) PHYSICS AUC

Selfinteractions in collections of massless tensor fields with the mixed symmetry $(3,1)$ and $(2,2)$

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Abstract

Under the hypotheses of analyticity, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the requirement that the interaction vertices contain at most two spatiotemporal derivatives of the fields, we investigate the consistent selfinteractions that can be added to a collection of massless tensor fields with the mixed symmetry $(3,1)$ and respectively $(2,2)$. The computations are done with the help of the deformation theory based on a cohomological approach, in the context of the antifield-BRST formalism. Our result is that no selfinteractions that deform the original gauge transformations emerge. In the case of the collection of $(2,2)$ tensor fields it is possible to add a sum of cosmological terms to the free Lagrangian.

PACS number: 11.10.Ef

1 Introduction

Tensor fields in "exotic" representations of the Lorentz group, characterized by a mixed Young symmetry type [1, 2, 3, 4, 5, 6, 7], held the attention lately on some important issues, like the dual formulation of field theories of spin two or higher [8, 9, 10, 11, 12, 13, 14], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [15], a Lagrangian first-order approach [16, 17] to some classes of massless or partially massive mixed symmetry type tensor gauge fields, suggestively resembling to the tetrad formalism of General Relativity, or the derivation of some exotic gravitational interactions [18, 19]. An important matter related to mixed symmetry type tensor fields

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is the study of their consistent interactions, among themselves as well as with higher-spin gauge theories [20, 21, 22, 23, 24, 25, 26, 27, 28]. The most efficient approach to this problem is the cohomological one, based on the deformation of the solution to the master equation [29].

The purpose of this paper is to investigate the consistent selfinteractions in a collection of massless tensor gauge fields with the mixed symmetry of a two-column Young diagram of the type $(3,1)$, and respectively a collection of massless tensor gauge fields with the mixed symmetry $(2,2)$. It is worth mentioning the duality of a free massless tensor gauge field with the mixed symmetry $(3,1)$ to the Pauli–Fierz theory in $D=6$ dimensions and, in this respect, some developments concerning the dual formulations of linearized gravity from the perspective of M-theory [30, 31, 32]. Our analysis relies on the deformation of the solution to the master equation by means of cohomological techniques with the help of the local BRST cohomology, whose component in a single $(3,1)$ sector has been reported in detail in [33], while in a single $(2,2)$ sector has been considered in [34, 35]. Under the hypotheses of analyticity in the coupling constant, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field, we find that no selfinteractions that deform the original gauge transformations emerge. In the case of the collection of $(2,2)$ tensor fields it is possible to add a sum of cosmological terms to the free Lagrangian.

2 Brief review of the deformation procedure

There are three main types of consistent interactions that can be added to a given gauge theory: *(i)* the first type deforms only the Lagrangian action, but not its gauge transformations, *(ii)* the second kind modifies both the action and its transformations, but not the gauge algebra, and *(iii)* the third, and certainly most interesting category, changes everything, namely, the action, its gauge symmetries and the accompanying algebra.

The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution S to the master equation for the initial theory can be deformed into the solution $\bar{S}$ to the master equation for the interacting theory

$$S \longrightarrow \bar{S} = S + gS_1 + g^2 S_2 + g^3 S_3 + g^4 S_4 + \cdots , \tag{1}$$

$$(S,S)=0 \longrightarrow \left(\bar{S},\bar{S}\right)=0. \tag{2}$$

The projection of (2) for $\bar{S}$ on the various powers of the coupling constant induces the following tower of equations:

$$g^0 : (S,S)=0, \tag{3}$$

$$g^1 : (S_1,S)=0, \tag{4}$$

$$g^2 : (S_2,S)+\frac{1}{2}(S_1,S_1)=0, \tag{5}$$

$$g^3 : (S_3,S)+(S_1,S_2)=0, \tag{6}$$

$$g^4 : (S_4,S)+(S_1,S_3)+\frac{1}{2}(S_2,S_2)=0, \tag{7}$$

$$\vdots$$

The first equation is satisfied by hypothesis. The second one governs the first-order deformation of the solution to the master equation, S_1, and it expresses the fact that S_1 is a BRST co-cycle, $sS_1 = 0$, and hence it exists and is local. The remaining equations are responsible for the higher-order deformations of the solution to the master equation. No obstructions arise in finding solutions to them as long as no further restrictions, such as spatiotemporal locality, are imposed. Obviously, only non-trivial first-order deformations should be considered, since trivial ones ($S_1 = sB$) lead to trivial deformations of the initial theory, and can be eliminated by convenient redefinitions of the fields. Ignoring the trivial deformations, it follows that S_1 is a non-trivial BRST-observable, $S_1 \in H^0(s)$ (where $H^0(s)$ denotes the cohomology space of the BRST differential in ghost number zero). Once the deformation equations ((4)–(7), etc.) have been solved by means of specific cohomological techniques, from the consistent non-trivial deformed solution to the master equation one can extract all the information on the gauge structure of the resulting interacting theory.

3 Selfinteractions for a collection of massless tensor fields with the mixed symmetry $(3,1)$

3.1 Free model: Lagrangian formulation and BRST symmetry

The starting point is given by the Lagrangian action for a collection of free, massless tensor fields with the mixed symmetry $(3,1)$

$$\begin{aligned} S_0^{\mathrm{t}}\left[t_{\lambda\mu\nu|\alpha}^A\right] = & \int \left\{ \frac{1}{2}\left[\left(\partial^\rho t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial_\rho t_{\lambda\mu\nu|\alpha}^A\right) - \left(\partial_\alpha t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial^\beta t_{\lambda\mu\nu|\beta}^A\right)\right] \right. \\ & -\frac{3}{2}\left[\left(\partial_\lambda t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial^\rho t_{\rho\mu\nu|\alpha}^A\right) + \left(\partial^\rho t_A^{\lambda\mu}\right)\left(\partial_\rho t_{\lambda\mu}^A\right)\right] \\ & \left. +3\left[\left(\partial_\alpha t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial_\lambda t_{\mu\nu}^A\right) + \left(\partial_\rho t_A^{\rho\mu}\right)\left(\partial^\lambda t_{\lambda\mu}^A\right)\right]\right\} d^D x, \end{aligned} \tag{8}$$

in a Minkowski space-time of dimension $D \geq 5$. Everywhere in this paper we employ the flat Minkowski metric of 'mostly plus' signature $\sigma^{\mu\nu} = \sigma_{\mu\nu} = (- + + + + \cdots)$. The uppercase indices A, B, etc. stand for the collection indices and are assumed to take discrete values 1, 2, …, N. They are lowered with a symmetric, constant, and invertible matrix, of elements k_{AB}, and are raised with the help of the elements k^{AB} of its inverse. Each field $t_{\lambda\mu\nu|\alpha}^A$ is completely antisymmetric in its first three (Lorentz) indices and satisfies the identity $t_{[\lambda\mu\nu|\alpha]}^A \equiv 0$. Here and in the sequel the notation $[\lambda \cdots \alpha]$ signifies complete antisymmetry with respect to the (Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. The notation $t_{\lambda\mu}^A$ from (8) signifies the trace of $t_{\lambda\mu\nu|\alpha}^A$, defined by $t_{\lambda\mu}^A = \sigma^{\nu\alpha} t_{\lambda\mu\nu|\alpha}^A$. The trace components define an antisymmetric tensor, $t_{\lambda\mu}^A = -t_{\mu\lambda}^A$. A generating set of gauge transformations for action (8) can be chosen of the form

$$\begin{aligned} \delta_{\epsilon,\chi} t_{\lambda\mu\nu|\alpha}^A & = 3\partial_\alpha \epsilon_{\lambda\mu\nu}^A + \partial_{[\lambda}\epsilon_{\mu\nu]\alpha}^A + \partial_{[\lambda}\chi_{\mu\nu]|\alpha}^A \\ & = -3\partial_{[\lambda}\epsilon_{\mu\nu\alpha]}^A + 4\partial_{[\lambda}\epsilon_{\mu\nu]\alpha}^A + \partial_{[\lambda}\chi_{\mu\nu]|\alpha}^A, \end{aligned} \tag{9}$$

where the gauge parameters $\epsilon_{\lambda\mu\nu}^A$ are completely antisymmetric, and the gauge parameters $\chi_{\mu\nu|\alpha}^A$ (also bosonic) define a collection of tensor fields with the mixed symmetry $(2,1)$. It can be shown [33] that the generating set (9) is off-shell reducible of order two and the associated gauge algebra is Abelian. Consequently, the Cauchy order of this linear gauge theory is equal to four.

The most general quantities, invariant under the gauge transformations (9), are given by the components of the curvature tensors associated with each field from the collection

$$K_{A}^{\lambda\mu\nu\xi|\alpha\beta} = \partial^{\alpha}\partial^{[\lambda} t_{A}^{\mu\nu\xi]|\beta} - \partial^{\beta}\partial^{[\lambda} t_{A}^{\mu\nu\xi]|\alpha} \tag{10}$$

together with their space-time derivatives. It is easy to check that they display the mixed symmetry $(4,2)$.

The construction of the BRST symmetry for the free model under study debuts with the identification of the algebra on which the BRST differential s acts. The ghost spectrum comprises the fermionic ghosts $\left\{\eta_{\lambda\mu\nu}^{A}, \mathcal{G}_{\mu\nu|\alpha}^{A}\right\}$ respectively associated with the gauge parameters $\left\{\epsilon_{\lambda\mu\nu}^{A}, \chi_{\mu\nu|\alpha}^{A}\right\}$ from (9), the bosonic ghosts for ghosts $\left\{C_{\mu\nu}^{A}, G_{\nu\alpha}^{A}\right\}$ due to the first-order reducibility, and the fermionic ghosts for ghosts for ghosts C_{ν}^{A} corresponding to the maximum reducibility order (two). We ask that $\eta_{\lambda\mu\nu}^{A}$ and $C_{\mu\nu}^{A}$ are completely antisymmetric, $\mathcal{G}_{\mu\nu|\alpha}^{A}$ exhibit the mixed symmetry $(2,1)$, and $G_{\nu\alpha}^{A}$ are symmetric. The antifield spectrum comprises the antifields $t_{A}^{*\lambda\mu\nu|\alpha}$ associated with the original fields and those corresponding to the ghosts, $\left\{\eta_{A}^{*\lambda\mu\nu}, \mathcal{G}_{A}^{*\mu\nu|\alpha}\right\}$, $\left\{C_{A}^{*\mu\nu}, G_{A}^{*\nu\alpha}\right\}$, and $C_{A}^{*\nu}$.

Since both the gauge generators and reducibility functions for this model are field-independent, it follows that the BRST differential s simply reduces to $s = \delta + \gamma$, where δ represents the Koszul–Tate differential, graded by the antighost number agh ($\mathrm{agh}(\delta) = -1$), and γ stands for the exterior longitudinal differential, whose degree is named pure ghost number pgh ($\mathrm{pgh}(\gamma) = 1$). These two degrees do not interfere ($\mathrm{agh}(\gamma) = 0$, $\mathrm{pgh}(\delta) = 0$). The overall degree that grades the BRST complex is known as the ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(s) = \mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = 1$. According to the standard rules of the BRST method, the corresponding degrees of the generators from the BRST complex are valued like

$$\begin{gathered}
\mathrm{pgh}\left(t_{\lambda\mu\nu|\alpha}^{A}\right) = 0, \quad \mathrm{pgh}\left(\eta_{\lambda\mu\nu}^{A}\right) = \mathrm{pgh}\left(\mathcal{G}_{\mu\nu|\alpha}^{A}\right) = 1, \\
\mathrm{pgh}\left(C_{\mu\nu}^{A}\right) = \mathrm{pgh}\left(G_{\nu\alpha}^{A}\right) = 2, \\
\mathrm{pgh}\left(t_{A}^{*\lambda\mu\nu|\alpha}\right) = \mathrm{pgh}\left(\eta_{A}^{*\lambda\mu\nu}\right) = \mathrm{pgh}\left(\mathcal{G}_{A}^{*\mu\nu|\alpha}\right) = \mathrm{pgh}\left(C_{A}^{*\mu\nu}\right) = \mathrm{pgh}\left(G_{A}^{*\nu\alpha}\right) = 0, \\
\mathrm{agh}\left(t_{\lambda\mu\nu|\alpha}^{A}\right) = \mathrm{agh}\left(\eta_{\lambda\mu\nu}^{A}\right) = \mathrm{agh}\left(\mathcal{G}_{\mu\nu|\alpha}^{A}\right) = \mathrm{agh}\left(C_{\mu\nu}^{A}\right) = \mathrm{agh}\left(G_{\nu\alpha}^{A}\right) = 0, \\
\mathrm{agh}\left(t_{A}^{*\lambda\mu\nu|\alpha}\right) = 1, \quad \mathrm{agh}\left(\eta_{A}^{*\lambda\mu\nu}\right) = \mathrm{agh}\left(\mathcal{G}_{A}^{*\mu\nu|\alpha}\right) = 2, \\
\mathrm{agh}\left(C_{A}^{*\mu\nu}\right) = \mathrm{agh}\left(G_{A}^{*\nu\alpha}\right) = 3.
\end{gathered}$$

The Koszul–Tate differential is imposed to realize a homological resolution of the algebra of smooth functions defined on the stationary surface of field equations, while the exterior longitudinal differential is related to the gauge symmetries (see relations (9)) of action (8) through its cohomology at pure ghost number zero computed in the cohomology of δ, which is required to be the algebra of physical observables for the free model under consideration. The actions of δ and γ on the generators from the BRST complex, which enforce all the above mentioned properties, are given by

$$\gamma t_{\lambda\mu\nu|\alpha}^{A} = -3\partial_{[\lambda}\eta_{\mu\nu\alpha]}^{A} + 4\partial_{[\lambda}\eta_{\mu\nu]\alpha}^{A} + \partial_{[\lambda}\mathcal{G}_{\mu\nu]|\alpha}^{A}, \tag{11}$$

$$\gamma \eta^A_{\lambda\mu\nu} = -\frac{1}{2}\partial_{[\lambda} C^A_{\mu\nu]}, \tag{12}$$

$$\gamma \mathcal{G}^A_{\mu\nu|\alpha} = 2\partial_{[\mu} C^A_{\nu\alpha]} - 3\partial_{[\mu} C^A_{\nu]\alpha} + \partial_{[\mu} G^A_{\nu]\alpha}, \tag{13}$$

$$\gamma C^A_{\mu\nu} = \partial_{[\mu} C^A_{\nu]}, \qquad \gamma G^A_{\nu\alpha} = -3\partial_{(\nu} C^A_{\alpha)}, \qquad \gamma C^A_{\nu} = 0, \tag{14}$$

$$\gamma t^{*\lambda\mu\nu|\alpha}_A = \gamma \eta^{*\lambda\mu\nu}_A = \gamma \mathcal{G}^{*\mu\nu|\alpha}_A = \gamma C^{*\mu\nu}_A = \gamma G^{*\nu\alpha}_A = \gamma C^{*\nu}_A = 0, \tag{15}$$

$$\delta t^A_{\lambda\mu\nu|\alpha} = \delta \eta^A_{\lambda\mu\nu} = \delta \mathcal{G}^A_{\mu\nu|\alpha} = \delta C^A_{\mu\nu} = \delta G^A_{\nu\alpha} = \delta C^A_{\nu} = 0, \tag{16}$$

$$\delta t^{*\lambda\mu\nu|\alpha}_A = T^{\lambda\mu\nu|\alpha}_A, \qquad \delta \eta^{*\lambda\mu\nu}_A = -4\partial_\alpha t^{*\lambda\mu\nu|\alpha}_A, \tag{17}$$

$$\delta \mathcal{G}^{*\mu\nu|\alpha}_A = -\partial_\lambda \left(3t^{*\lambda\mu\nu|\alpha}_A - t^{*\mu\nu\alpha|\lambda}_A\right), \tag{18}$$

$$\delta C^{*\mu\nu}_A = 3\partial_\lambda \left(\mathcal{G}^{*\mu\nu|\lambda}_A - \frac{1}{2}\eta^{*\lambda\mu\nu}_A\right), \qquad \delta G^{*\nu\alpha}_A = \partial_\mu \mathcal{G}^{*\mu(\nu|\alpha)}_A, \tag{19}$$

$$\delta C^{*\nu}_A = 6\partial_\mu \left(G^{*\mu\nu}_A - \frac{1}{3}C^{*\mu\nu}_A\right), \tag{20}$$

where $T^{\lambda\mu\nu|\alpha}_A = -\delta S^{\mathrm{t}}_0/\delta t^A_{\lambda\mu\nu|\alpha}$ reads

$$\begin{aligned} T^{\lambda\mu\nu|\alpha}_A &= \Box t^{\lambda\mu\nu|\alpha}_A - \partial_\rho \left(\partial^{[\lambda} t^{\mu\nu]\rho|\alpha}_A + \partial^\alpha t^{\lambda\mu\nu|\rho}_A\right) + \partial^\alpha \partial^{[\lambda} t^{\mu\nu]}_A \\ &\quad + \sigma^{\alpha[\lambda} \left(\partial_\rho \left(\partial_\beta t^{\mu\nu]\rho|\beta}_A - \partial^\mu t^{\nu]\rho}_A\right) - \Box t^{\mu\nu]}_A\right). \end{aligned} \tag{21}$$

By convention, we take δ and γ to act like right derivations. We note that the action of the Koszul–Tate differential on the antifields with the antighost number equal to two and respectively three gains a simpler expression if we perform the changes of variables

$$\mathcal{G}'^{*\mu\nu|\alpha}_A = \mathcal{G}^{*\mu\nu|\alpha}_A + \frac{1}{4}\eta^{*\mu\nu\alpha}_A, \qquad G'^{*\nu\alpha}_A = G^{*\nu\alpha}_A - \frac{1}{3}C^{*\nu\alpha}_A. \tag{22}$$

The antifields $\mathcal{G}'^{*\mu\nu|\alpha}_A$ are still antisymmetric in their first two indices, but do not fulfill the identity $\mathcal{G}'^{*[\mu\nu|\alpha]}_A \equiv 0$, and $G'^{*\nu\alpha}_A$ have no definite symmetry or antisymmetry properties. With the help of relations (17)–(20), we find that δ acts on the transformed antifields through the relations

$$\delta \mathcal{G}'^{*\mu\nu|\alpha}_A = -3\partial_\lambda t^{*\lambda\mu\nu|\alpha}_A, \qquad \delta G'^{*\nu\alpha}_A = 2\partial_\mu \mathcal{G}'^{*\mu\nu|\alpha}_A, \qquad \delta C^{*\nu}_A = 6\partial_\mu G'^{*\mu\nu}_A. \tag{23}$$

The same observation is valid with respect to γ if we make the changes

$$\mathcal{G}'^A_{\mu\nu|\alpha} = \mathcal{G}^A_{\mu\nu|\alpha} + 4\eta^A_{\mu\nu\alpha}, \qquad G'^A_{\nu\alpha} = G^A_{\nu\alpha} - 3C^A_{\nu\alpha}, \tag{24}$$

in terms of which we can write

$$\gamma t^A_{\lambda\mu\nu|\alpha} = -\frac{1}{4}\partial_{[\lambda} \mathcal{G}'^A_{\mu\nu|\alpha]} + \partial_{[\lambda} \mathcal{G}'^A_{\mu\nu]|\alpha}, \ \gamma \mathcal{G}'^A_{\mu\nu|\alpha} = \partial_{[\mu} G'^A_{\nu]\alpha}, \ \gamma G'^A_{\nu\alpha} = -6\partial_\nu C^A_\alpha. \tag{25}$$

Again, $\mathcal{G}'^A_{\mu\nu|\alpha}$ are antisymmetric in their first two indices, but do not satisfy the identity $\mathcal{G}'^A_{[\mu\nu|\alpha]} \equiv 0$, while $G'^A_{\nu\alpha}$ have no definite symmetry or antisymmetry. We have deliberately chosen the same notations for the transformed variables (22) and (24) since they actually form pairs that are conjugated in the antibracket

$$\left(\mathcal{G}'^A_{\mu\nu|\alpha}, \mathcal{G}'^{*\mu_1\nu_1|\alpha_1}_B\right) = \frac{1}{2}\delta^A_B \delta^{[\mu_1}_\mu \delta^{\nu_1]}_\nu \delta^{\alpha_1}_\alpha,$$

$$\left(G'^{A}_{\nu\alpha}, G'^{*\nu_1\alpha_1}_{B}\right) = \delta^A_B\delta^{\nu_1}_{\nu}\delta^{\alpha_1}_{\alpha}.$$

The Lagrangian BRST differential admits a canonical action in a structure named antibracket and defined by decreeing the fields/ghosts conjugated with the corresponding antifields, $s\cdot = (\cdot, S)$, where $(,)$ signifies the antibracket and S denotes the canonical generator of the BRST symmetry. It is a bosonic functional of ghost number zero, involving both field/ghost and antifield spectra, that obeys the master equation $(S, S) = 0$. The master equation is equivalent with the second-order nilpotency of s, where its solution S encodes the entire gauge structure of the associated theory. Taking into account formulas (11)–(20) as well as the standard actions of δ and γ in canonical form, we find that the complete solution to the master equation for the free model under study is given by

$$\begin{aligned} S^{\mathrm{t}} = {} & S_0^{\mathrm{t}}\left[t^A_{\lambda\mu\nu|\alpha}\right] + \int \Big(t^{*\lambda\mu\nu|\alpha}_A \left(3\partial_\alpha \eta^A_{\lambda\mu\nu} + \partial_{[\lambda}\eta^A_{\mu\nu]\alpha} + \partial_{[\lambda}\mathcal{G}^A_{\mu\nu]|\alpha}\right) \\ & -\frac{1}{2}\eta^{*\lambda\mu\nu}_A \partial_{[\lambda}C^A_{\mu\nu]} + \mathcal{G}^{*\mu\nu|\alpha}_A \left(2\partial_\alpha C^A_{\mu\nu} - \partial_{[\mu}C^A_{\nu]\alpha} + \partial_{[\mu}G^A_{\nu]\alpha}\right) \\ & + C^{*\mu\nu}_A \partial_{[\mu}C^A_{\nu]} - 3G^{*\nu\alpha}_A \partial_{(\nu}C^A_{\alpha)}\Big)\, d^D x. \end{aligned} \tag{26}$$

3.2 Computation of basic cohomologies

In order to analyze equation (4) (that governs the first-order deformation) we make the notation $S_1 = \int a^{\mathrm{t}} d^D x$ and write this equation in its local form and in dual notations, $sa^{\mathrm{t}} = \partial_\mu m^\mu_{\mathrm{t}}$. Now, we approach the last equation in a standard manner, namely, we develop a^{t} according to the antighost number and assume that this expansion contains a finite number of terms, of maximum antighost number I. In order to ensure the space-time locality of the deformations, from now on we work in the algebra of local differential forms with coefficients that are polynomial functions in the fields, ghosts, antifields, and their space-time derivatives (algebra of local forms). This means that we assume the non-integrated density of the first-order deformation, a^{t}, to be a polynomial function in all these variables (algebra of local functions).

By taking into account the splitting $s = \delta + \gamma$ of the BRST differential, the equation $sa^{\mathrm{t}} = \partial_\mu m^\mu_{\mathrm{t}}$ becomes equivalent to a tower of local equations, corresponding to the different decreasing values of the antighost number

$$\gamma a^{\mathrm{t}}_I = \partial_\mu \overset{(I)}{m}{}^{\mu}_{\mathrm{t}}, \tag{27}$$

$$\delta a^{\mathrm{t}}_I + \gamma a^{\mathrm{t}}_{I-1} = \partial_\mu \overset{(I-1)}{m}{}^{\mu}_{\mathrm{t}}, \tag{28}$$

$$\delta a^{\mathrm{t}}_k + \gamma a^{\mathrm{t}}_{k-1} = \partial_\mu \overset{(k-1)}{m}{}^{\mu}_{\mathrm{t}}, \qquad I-1 \geq k \geq 1, \tag{29}$$

where $\left(\overset{(k)}{m}{}^{\mu}_{\mathrm{t}}\right)_{k=\overline{0,I}}$ are some local currents, with $\mathrm{agh}\left(\overset{(k)}{m}{}^{\mu}_{\mathrm{t}}\right) = k$. It can be proved that we can replace equation (27) at strictly positive antighost numbers with the homogeneous equation

$$\gamma a^{\mathrm{t}}_I = 0, \qquad I > 0. \tag{30}$$

The proof can be done like in the Appendix A, Corollary 1, from [33]. In conclusion, under the assumption that $I > 0$, the representative of highest antighost number from the non-integrated density of the first-order deformation can always be taken to be γ-closed, such that equation $sa^{\mathrm{t}} = \partial_\mu m^\mu_{\mathrm{t}}$, associated with the local form of the first-order deformation equation, is completely equivalent to the tower of equations given by (30) and (28)–(29).

Before proceeding to the analysis of the solutions to the first-order deformation equation, let us briefly comment on the uniqueness and triviality of such solutions. Due to the second-order nilpotency of γ ($\gamma^2 = 0$), the solution to the top equation, (30), is clearly unique up to γ-exact contributions, $a_I^{\mathrm{t}} \rightarrow a_I^{\mathrm{t}} + \gamma b_I$. Meanwhile, if a_I^{t} reduces to γ-exact terms only, $a_I^{\mathrm{t}} = \gamma b_I$, then it can be made to vanish, $a_I^{\mathrm{t}} = 0$. In other words, the non-triviality of the first-order deformation a^{t} is translated at its highest antighost number component into the requirement that $a_I^{\mathrm{t}} \in H^I(\gamma)$, where $H^I(\gamma)$ denotes the cohomology of the exterior longitudinal differential γ in pure ghost number equal to I computed in the algebra of local functions. At the same time, the general condition on the non-integrated density of the first-order deformation to generate an element $a^{\mathrm{t}} d^D x$ from a non-trivial cohomological class of $H^{0,D}(s|d)$ (the local cohomology of the BRST differential s — where d means the exterior space-time differential — in ghost number zero and in maximum form degree, computed in the algebra of local forms) shows on the one hand that the solution to equation $sa^{\mathrm{t}} = \partial_\mu m_{\mathrm{t}}^\mu$ is unique up to s-exact pieces plus total derivatives and, on the other hand, that if the general solution to this equation is completely trivial, $a^{\mathrm{t}} = sb + \partial_\mu n^\mu$, then it can be made to vanish, $a^{\mathrm{t}} = 0$.

We have seen that the solution to equation (30) belongs to the cohomology of the exterior longitudinal differential computed in the algebra of local functions, such that we need to compute $H^*(\gamma)$ in order to construct the component of highest antighost number from the first-order deformation. We will see that we also need to compute the characteristic cohomology $H_I^D(\delta|d)$ (the local cohomology of the Koszul–Tate differential δ in antighost number I and in maximum form degree, computed in the algebra of local forms with the pure ghost number equal to zero).

Acting like in [33], it is easy to see that $H^*(\gamma)$ is generated by the quantities

$$\begin{array}{ccc}
\mathrm{pgh} & \text{BRST generator} & \text{non}-\text{trivial objects from } H^*(\gamma) \\
0 & \left\{\begin{array}{c} \Pi^{*\Delta}, \partial\Pi^{*\Delta}, \ldots \\ t^A_{\lambda\mu\nu|\alpha}, \partial t^A_{\lambda\mu\nu|\alpha}, \ldots \end{array}\right. & \left\{\begin{array}{c} \Pi^{*\Delta}, \partial\Pi^{*\Delta}, \ldots \\ K^A_{\lambda\mu\nu\xi|\alpha\beta}, \partial K^A_{\lambda\mu\nu\xi|\alpha\beta}, \ldots \end{array}\right. \\
1 & \left\{\begin{array}{c} \eta^A_{\lambda\mu\nu}, \partial\eta^A_{\lambda\mu\nu}, \ldots \\ \mathcal{G}^A_{\mu\nu|\alpha}, \partial\mathcal{G}^A_{\mu\nu|\alpha}, \ldots \end{array}\right. & \mathcal{F}^A_{\lambda\mu\nu\alpha} = \partial_{[\lambda}\eta^A_{\mu\nu\alpha]}, \\
2 & \left\{\begin{array}{c} C^A_{\mu\nu}, \partial C^A_{\mu\nu}, \ldots \\ G^A_{\nu\alpha}, \partial G^A_{\nu\alpha}, \ldots \end{array}\right. & - \\
3 & C^A_\nu, \partial C^A_\nu, \ldots & C^A_\nu
\end{array}, \tag{31}$$

where $\Pi^{*\Delta}$ is a generic notation for all the antifields. So, the most general, non-trivial solution to the equation (30) (up to trivial, γ-exact contributions) reads

$$a_I^{\mathrm{t}} = \alpha_I\left(\left[\Pi^{*\Delta}\right], \left[K^A_{\lambda\mu\nu\xi|\alpha\beta}\right]\right)\omega^I\left(\mathcal{F}^A_{\lambda\mu\nu\alpha}, C^A_\nu\right). \tag{32}$$

The notation $f([q])$ means that f depends on q and its derivatives up to a finite order, while ω^I denotes the elements of pure ghost number I (and antighost number zero) of a basis in the space of polynomials in $\mathcal{F}^A_{\lambda\mu\nu\alpha}$ and C^A_ν, which is finite dimensional since these variables anticommute. The objects α_I (obviously non-trivial in $H^0(\gamma)$) were taken to have a bounded number of derivatives, and therefore they are polynomials in the antifields $\Theta^{*\Delta}$, in the curvature tensors $K^A_{\lambda\mu\nu\xi|\alpha\beta}$, as well as in their derivatives. They are nothing but the invariant polynomials of the theory described by formulas (8)–(9) in form degree equal to zero. At zero antighost number, the invariant polynomials are polynomials in the curvature tensors $K^A_{\lambda\mu\nu\xi|\alpha\beta}$ and in their derivatives.

Replacing solution (32) into equation (28) and taking into account definitions (16)–(20), we remark that a necessary (but not sufficient) condition for the existence of (non-trivial) solutions a^{t}_{I-1} is that the invariant polynomials α_I generate (non-trivial) objects from the characteristic cohomology $H^D_I(\delta|d)$ in antighost number $I>0$, maximum form degree, and pure ghost number equal to zero[1], $\alpha_I d^D x \in H^D_I(\delta|d)$. As the free model under study is a linear gauge theory of Cauchy order equal to four, the general results from [36] ensure that the entire characteristic cohomology is trivial in antighost numbers strictly greater than its Cauchy order

$$H^D_I(\delta|d)=0, \qquad I>4. \tag{33}$$

Moreover, it is possible to show that the above result remains valid also in the algebra of invariant polynomials

$$H^{\mathrm{inv}D}_I(\delta|d)=0, \qquad I>4, \tag{34}$$

where $H^{\mathrm{inv}D}_I(\delta|d)$ is known as the invariant characteristic cohomology. Looking at the definitions (23) involving the transformed antifields (22), we can organize the non-trivial, Poincaré-invariant representatives of $H^D_I(\delta|d)$ and $H^{\mathrm{inv}D}_I(\delta|d)$ (for $I\geq 2$) like:

$$\begin{array}{cc} \text{agh} & H^D_I(\delta|d) \text{ and } H^{\mathrm{inv}D}_I(\delta|d) \\ I>4 & - \\ I=4 & f^A_\nu C^{*\nu}_A d^D x \\ I=3 & f^A_{\nu\alpha} G'^{*\nu\alpha}_A d^D x \\ I=2 & f^A_{\mu\nu\alpha} \mathcal{G}'^{*\mu\nu|\alpha}_A d^D x \end{array}, \tag{35}$$

where all the coefficients denoted by f define some constant, non-derivative tensors. We remark that in $\left(H^D_I(\delta|d)\right)_{I\geq 2}$ and $\left(H^{\mathrm{inv}D}_I(\delta|d)\right)_{I\geq 2}$ there is no non-trivial element that effectively involves the curvatures $K^A_{\lambda\mu\nu\xi|\alpha\beta}$ and/or their derivatives, and the same stands for the quantities that are more than linear in the antifields and/or depend on their derivatives. In principle, one can construct from the above elements in (35) other non-trivial invariant polynomials from $H^D_I(\delta|d)$ or $H^{\mathrm{inv}D}_I(\delta|d)$, that depend on the space-time coordinates. For instance, it can be checked by direct computation that $\mathcal{G}'^{*\mu\nu|\alpha}_A f^A_{\mu\nu\alpha\rho} x^\rho d^D x$, with $f^A_{\mu\nu\alpha\rho}$ some completely antisymmetric and constant tensors, generate non-trivial representatives from both $H^D_2(\delta|d)$ and $H^{\mathrm{inv}D}_2(\delta|d)$. However, we will discard such candidates as they would break the Poincaré invariance of the deformations. In contrast to the groups $\left(H^D_I(\delta|d)\right)_{I\geq 2}$ and $\left(H^{\mathrm{inv}D}_I(\delta|d)\right)_{I\geq 2}$, which are finite-dimensional, the cohomology $H^D_1(\delta|d)$ at pure ghost number zero, that is related to global symmetries and ordinary conservation laws, is infinite-dimensional since the theory is free.

The previous results on $H^D_I(\delta|d)$ and $H^{\mathrm{inv}D}_I(\delta|d)$ are important because they control the obstructions to removing the antifields from the first-order deformation. Indeed, due to (34), it follows that we can successively eliminate all the pieces with $I>4$ from the non-integrated density of the first-order deformation by adding only trivial terms (the proof is similar to that from the Appendix C in [33]), so we can take, without loss of non-trivial objects, the condition $I\leq 4$ in the first-order deformation. The last representative is of the form (32), where the invariant polynomials necessarily generate non-trivial objects from $H^{\mathrm{inv}D}_I(\delta|d)$ if $I=2,3,4$ and respectively from $H^D_1(\delta|d)$ if $I=1$.

[1]We recall that the local cohomology $H^D_*(\delta|d)$ is completely trivial at both strictly positive antighost *and* pure ghost numbers (for instance, see [36], Theorem 5.4 and [37]).

3.3 Cohomological analysis of selfinteractions

Assuming $I = 4$, the non-integrated density of the first-order deformation becomes

$$a^{\mathrm{t}} = a_0^{\mathrm{t}} + a_1^{\mathrm{t}} + a_2^{\mathrm{t}} + a_3^{\mathrm{t}} + a_4^{\mathrm{t}}, \tag{36}$$

with a_4^{t} ($\gamma a_4^{\mathrm{t}} = 0$) a non-trivial element from $H^4(\gamma)$, and hence of the form (see (32))

$$a_4^{\mathrm{t}} = \alpha_4 \omega^4 \left(\mathcal{F}^A_{\lambda\mu\nu\alpha}, C^A_\nu \right), \tag{37}$$

and $\alpha_4 d^D x$ a non-trivial object from $H_4^{\mathrm{inv}D}(\delta|d)$. Since the elements of pure ghost number equal to four from the basis in the space of polynomials in $\mathcal{F}^A_{\lambda\mu\nu\alpha}$ and C^A_ν are spanned by the combinations

$$\omega^4 : \left(\mathcal{F}^B_{\lambda\mu\nu\alpha} C^C_\beta, \mathcal{F}^B_{\lambda\mu\nu\alpha} \mathcal{F}^C_{\lambda_1\mu_1\nu_1\alpha_1} \mathcal{F}^D_{\lambda_2\mu_2\nu_2\alpha_2} \mathcal{F}^E_{\lambda_3\mu_3\nu_3\alpha_3} \right), \tag{38}$$

with $\mathcal{F}^A_{\lambda\mu\nu\alpha}$ given in (31), and the non-trivial representatives of the space $H_4^{\mathrm{inv}D}(\delta|d)$ are generated by the antifields C^{*A}_ρ (see (35)), we obtain that the general form of the last term from the first-order deformation in the case $I = 4$ reads

$$\begin{aligned} a_4^{\mathrm{t}} \; &= \; C^{*A}_\rho \left(f_{1ABCDE}^{\rho\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1\lambda_2\mu_2\nu_2\alpha_2\lambda_3\mu_3\nu_3\alpha_3} \mathcal{F}^B_{\lambda\mu\nu\alpha} \mathcal{F}^C_{\lambda_1\mu_1\nu_1\alpha_1} \mathcal{F}^D_{\lambda_2\mu_2\nu_2\alpha_2} \mathcal{F}^E_{\lambda_3\mu_3\nu_3\alpha_3} \right. \\ &\quad \left. + f_{2ABC}^{\rho\lambda\mu\nu\alpha\beta} \mathcal{F}^B_{\lambda\mu\nu\alpha} C^C_\beta \right), \end{aligned} \tag{39}$$

where the coefficients denoted by f are some non-derivative constant tensors. The first term from the right-hand side of (39) (those containing homogeneous polynomials of degree four in the ghosts $\mathcal{F}^A_{\lambda\mu\nu\alpha}$), even if consistent, would lead to interaction vertices (in the corresponding a_0^{t}) of order five in the space-time derivatives of the fields, which disagrees with the hypothesis on the maximum derivative order of the interacting Lagrangian to be equal to two. For this reason, we eliminate this term from a_4^{t} by setting the associated coefficient to be equal to zero

$$f_{1ABCDE}^{\rho\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1\lambda_2\mu_2\nu_2\alpha_2\lambda_3\mu_3\nu_3\alpha_3} = 0, \tag{40}$$

such that

$$a_4^{\mathrm{t}} = f_{2ABC}^{\rho\lambda\mu\nu\alpha\beta} C^{*A}_\rho \mathcal{F}^B_{\lambda\mu\nu\alpha} C^C_\beta. \tag{41}$$

The requirements that the deformations are manifestly covariant and Poincaré invariant, the fact that we work in space-time dimensions $D \geq 5$, and the complete antisymmetry of $\mathcal{F}^B_{\lambda\mu\nu\alpha}$, provide a single non-trivial candidate, namely

$$D = 6, \qquad f_{2ABC}^{\rho\lambda\mu\nu\alpha\beta} = c_{ABC} \varepsilon^{\rho\lambda\mu\nu\alpha\beta}, \tag{42}$$

with c_{ABC} some real, arbitrary constants and $\varepsilon^{\rho\lambda\mu\nu\alpha\beta}$ the six-dimensional Levi–Civita symbol. As a consequence, we obtain

$$a_4^{\mathrm{t}} = c_{ABC} \varepsilon^{\rho\lambda\mu\nu\alpha\beta} C^{*A}_\rho \mathcal{F}^B_{\lambda\mu\nu\alpha} C^C_\beta. \tag{43}$$

If (43) is consistent, then it will produce a Lagrangian density at order one in the coupling constant, a_0^{t}, which breaks the PT invariance.

We will show that solution (43) is not consistent in antighost number two, meaning that it cannot provide a solution a_2^{t} to the equation (29) for $k = 3$. In view of this, we compute the remaining components from (36), which are subject to equations (28)–(29) for $I=4$

$$\delta a_4^{\mathrm{t}}+\gamma a_3^{\mathrm{t}} = \partial_\mu \overset{(3)^\mu}{m}_{\mathrm{t}}, \qquad \delta a_3^{\mathrm{t}}+\gamma a_2^{\mathrm{t}} = \partial_\mu \overset{(2)^\mu}{m}_{\mathrm{t}}, \tag{44}$$

$$\delta a_2^{\mathrm{t}}+\gamma a_1^{\mathrm{t}} = \partial_\mu \overset{(1)^\mu}{m}_{\mathrm{t}}, \qquad \delta a_1^{\mathrm{t}}+\gamma a_0^{\mathrm{t}} = \partial_\mu \overset{(0)^\mu}{m}_{\mathrm{t}}. \tag{45}$$

Replacing (43) into the former equation from (44) and using the first definition from (23), together with the results

$$\partial_\rho \mathcal{F}^A_{\lambda\mu\nu\alpha} = \gamma\left(\frac{1}{3}\partial_{[\lambda}t^A_{\mu\nu\alpha]|\rho}\right), \tag{46}$$

$$\partial_\mu C^A_\nu = \gamma\left(-\frac{1}{6}G'^A_{\mu\nu}\right), \tag{47}$$

we find that

$$a_3^{\mathrm{t}} = -c_{ABC}\varepsilon^{\rho\lambda\mu\nu\alpha\beta}G'^{*A\gamma}{}_\rho\left(8\left(\partial_\lambda t^B_{\mu\nu\alpha|\gamma}\right)C^C_\beta+\mathcal{F}^B_{\lambda\mu\nu\alpha}G'^C_{\gamma\beta}\right), \tag{48}$$

where $G'^C_{\gamma\beta}$ reads as in (24). In order to solve the latter equation from (44), we initially compute δa_3^{t} starting with (48) and using the second definition from (23), and then manipulate the resulting expression based on formulas (46), (47), and the second relation from (25), obtaining in the end

$$\begin{aligned}\delta a_3^{\mathrm{t}} = {} & \partial_\sigma\left(2c_{ABC}\varepsilon^{\rho\lambda\mu\nu\alpha\beta}\mathcal{G}'^{*A\sigma\gamma|}{}_\rho\left(8\left(\partial_\lambda t^B_{\mu\nu\alpha|\gamma}\right)C^C_\beta+\mathcal{F}^B_{\lambda\mu\nu\alpha}G'^C_{\gamma\beta}\right)\right)\\ & +\gamma\left(-c_{ABC}\varepsilon^{\rho\lambda\mu\nu\alpha\beta}\mathcal{G}'^{*A\sigma\gamma|}{}_\rho\left(\frac{4}{3}\left(\partial_\lambda t^B_{\mu\nu\alpha|[\sigma}\right)G'^C_{\gamma]\beta}+\mathcal{F}^B_{\lambda\mu\nu\alpha}\mathcal{G}'^C_{\sigma\gamma|\beta}\right)\right)\\ & -2c_{ABC}\varepsilon^{\rho\lambda\mu\nu\alpha\beta}\mathcal{G}'^{*A\sigma\gamma|}{}_\rho K^B_{\lambda\mu\nu\alpha|\sigma\gamma}C^C_\beta,\end{aligned} \tag{49}$$

where $K^B_{\lambda\mu\nu\alpha|\sigma\gamma}$ is precisely the curvature tensor (see (10)) and the transformed ghosts $\mathcal{G}'^C_{\sigma\gamma|\beta}$ are defined in (24). Comparing the latter equation from (44) with (49), we observe that a_3^{t} of the form (48) provides a consistent a_2^{t} if and only if

$$-2c_{ABC}\varepsilon^{\rho\lambda\mu\nu\alpha\beta}\mathcal{G}'^{*A\sigma\gamma|}{}_\rho K^B_{\lambda\mu\nu\alpha|\sigma\gamma}C^C_\beta = \gamma b_2+\partial_\sigma \overset{(2)^\sigma}{w}, \tag{50}$$

where b_2 and $\overset{(2)^\sigma}{w}$ must fulfill the properties

$$\mathrm{agh}\left(b_2\right)=2=\mathrm{agh}\left(\overset{(2)^\sigma}{w}\right), \qquad \mathrm{pgh}\left(b_2\right)=2, \qquad \mathrm{pgh}\left(\overset{(2)^\sigma}{w}\right)=3. \tag{51}$$

The above requirement takes place if and only if

$$c_{ABC}=0, \tag{52}$$

because the left-hand side of relation (50) contains only non-trivial elements of $H^3(\gamma)$ with the antighost number equal to two, where the role of invariant polynomials is played by

$$-2c_{ABC}\varepsilon^{\rho\lambda\mu\nu\alpha\beta}\mathcal{G}'^{*A\sigma\gamma|}{}_\rho K^B_{\lambda\mu\nu\alpha|\sigma\gamma},$$

which implies automatically $b_2=0$, and, on the other hand, this expression cannot be written in a divergence-like form, such that we must set $\overset{(2)^\sigma}{w}=0$. But $b_2=0$ and $\overset{(2)^\sigma}{w}=0$ simultaneously in (50) lead to (52), and in consequence to

$$a_4^{\mathrm{t}}=0. \tag{53}$$

In conclusion, under the hypothesis that the maximum derivative order of the interacting Lagrangian is equal to two, the first-order deformation can only stop at antighost numbers $I \leq 3$.

In the case $I = 3$ we have that

$$a^{\mathrm{t}} = a_0^{\mathrm{t}} + a_1^{\mathrm{t}} + a_2^{\mathrm{t}} + a_3^{\mathrm{t}}, \tag{54}$$

with $\gamma a_3^{\mathrm{t}} = 0$, such that we can write (see (32))

$$a_3^{\mathrm{t}} = \alpha_3 \omega^3 \left(\mathcal{F}^A_{\lambda\mu\nu\alpha}, C^A_\nu\right). \tag{55}$$

The consistency of a^{t} at antighost number two (the existence of a_2^{t} as solution to the equation $\delta a_3^{\mathrm{t}} + \gamma a_2^{\mathrm{t}} = \partial_\mu \overset{(2)}{m}{}^\mu_{\mathrm{t}}$) requires that $\alpha_3 d^D x$ is a non-trivial element from $H_3^{\mathrm{inv}D}(\delta|d)$. Because the elements with the pure ghost number equal to three of a basis in the space of polynomials in $\mathcal{F}^A_{\lambda\mu\nu\alpha}$ and C^A_ν are spanned by

$$\omega^3 : \left(C^B_\beta, \mathcal{F}^B_{\lambda\mu\nu\alpha}\mathcal{F}^C_{\lambda_1\mu_1\nu_1\alpha_1}\mathcal{F}^D_{\lambda_2\mu_2\nu_2\alpha_2}\right), \tag{56}$$

and the general, non-trivial representatives of $H_3^{\mathrm{inv}D}(\delta|d)$ are generated by the antifields $G'^{*A\nu\alpha}$ (see (35) for $I = 3$), we infer

$$a_3^{\mathrm{t}} = G'^{*A}_{\rho\sigma}\left(f_{1ABCD}^{\rho\sigma\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1\lambda_2\mu_2\nu_2\alpha_2}\mathcal{F}^B_{\lambda\mu\nu\alpha}\mathcal{F}^C_{\lambda_1\mu_1\nu_1\alpha_1}\mathcal{F}^D_{\lambda_2\mu_2\nu_2\alpha_2} + f_{2AB}^{\rho\sigma\beta}C^B_\beta\right), \tag{57}$$

where the coefficients denoted by f must be some non-derivative, constant tensors. The condition that the maximum derivative order of the interacting Lagrangian is equal to two imposes the restrictions

$$f_{1ABCD}^{\rho\sigma\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1\lambda_2\mu_2\nu_2\alpha_2} = 0, \tag{58}$$

since otherwise the corresponding interacting term from a_0^{t} would be of order four in the space-time derivatives of the fields, and hence we get

$$a_3^{\mathrm{t}} = f_{2AB}^{\rho\sigma\beta}G'^{*A}_{\rho\sigma}C^B_\beta. \tag{59}$$

Asking now that a_3^{t} is a Lorentz covariant and Poincaré invariant element defined on a space-time of dimension $D \geq 5$ leaves us with the trivial solution

$$f_{2AB}^{\rho\sigma\beta} = 0, \tag{60}$$

which further implies

$$a_3^{\mathrm{t}} = 0. \tag{61}$$

In conclusion, the first-order deformation cannot stop in a non-trivial manner also at the value $I = 3$ of the antighost number.

Next, we pass to the situation where the non-integrated density of the first-order deformation stops at antighost number two

$$a^{\mathrm{t}} = a_0^{\mathrm{t}} + a_1^{\mathrm{t}} + a_2^{\mathrm{t}}, \tag{62}$$

where $\gamma a_2^{\mathrm{t}} = 0$, and hence, in agreement with (32), we have that

$$a_2^{\mathrm{t}} = \alpha_2 \omega^2 \left(\mathcal{F}^A_{\lambda\mu\nu\alpha}\right). \tag{63}$$

(The ghosts C_{ν}^{A} no longer appear in ω^2 since their pure ghost number is equal to three, while $\mathrm{pgh}\,(\omega^2) = 2$). We recall that a necessary condition for the existence of (63) in antighost number one (the existence of a_1^{t} as solution to the equation $\delta a_2^{\mathrm{t}} + \gamma a_1^{\mathrm{t}} = \partial_\mu \overset{(1)}{m}{}_{\mathrm{t}}^{\mu}$) is that $\alpha_2 d^D x$ belongs to $H_2^{\mathrm{inv}D}\,(\delta|d)$. The elements of pure ghost number equal to two of a basis in the space of polynomials in $\mathcal{F}^{A}_{\lambda\mu\nu\alpha}$ are spanned by

$$\omega^2 : \left(\mathcal{F}^{A}_{\lambda\mu\nu\alpha}\mathcal{F}^{B}_{\lambda_1\mu_1\nu_1\alpha_1}\right), \tag{64}$$

and the general, non-trivial representatives of $H_2^{\mathrm{inv}D}\,(\delta|d)$ are built from the antifields $\mathcal{G}'^{*A\mu\nu|\alpha}$ (see (35) for $I = 2$), such that

$$a_2^{\mathrm{t}} = f_{1ABC}^{\rho\sigma\beta\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1}\mathcal{G}'^{*A}_{\rho\sigma|\beta}\mathcal{F}^{B}_{\lambda\mu\nu\alpha}\mathcal{F}^{C}_{\lambda_1\mu_1\nu_1\alpha_1}, \tag{65}$$

where the coefficients denoted by f must be some non-derivative, constant tensors. The derivative order hypothesis a_0^{t} requires

$$f_{1ABC}^{\rho\sigma\beta\lambda\mu\nu\alpha\lambda_1\mu_1\nu_1\alpha_1} = 0, \tag{66}$$

since otherwise, if consistent, component (65) would lead to an a_0^{t} with three space-time derivatives acting on the fields. Condition (66) further implies

$$a_2^{\mathrm{t}} = 0, \tag{67}$$

and hence we can take $I \leq 1$ in the first-order deformation. The result (67) emphasizes that the original, Abelian gauge algebra is rigid with respect to the deformation procedure (since the existence of non-trivial terms in a_2^{t} that are simultaneously linear in the antifields with the antighost number equal to two and quadratic in combinations of ghosts with the pure ghost number equal to one is not allowed in the first-order deformation), such that the resulting selfinteractions among the fields with the mixed symmetry $(3,1)$ might modify at most the original gauge transformations or the free Lagrangian.

For $I = 1$ the first-order deformation becomes

$$a^{\mathrm{t}} = a_0^{\mathrm{t}} + a_1^{\mathrm{t}}, \tag{68}$$

where the last component ($\gamma a_1^{\mathrm{t}} = 0$) takes the generic form (see (32))

$$a_1^{\mathrm{t}} = \alpha_1\left(\left[t_A^{*\lambda\mu\nu|\alpha}\right], \left[K^{A}_{\lambda\mu\nu\xi|\alpha\beta}\right]\right)\omega^1\left(\mathcal{F}^{A}_{\lambda\mu\nu\alpha}\right). \tag{69}$$

The invariant polynomial α_1 is linear in the antifields $t_A^{*\lambda\mu\nu|\alpha}$ and their derivatives (up to a finite order) since these are the only objects of antighost number equal to one from the BRST algebra, while

$$\omega^1 : \left(\mathcal{F}^{B}_{\lambda\mu\nu\alpha}\right). \tag{70}$$

We mentioned in the above (see the end of Section 3.2) that a necessary condition for the consistency of a^{t} is that $\alpha_1 d^D x$ is a non-trivial element of $H_1^D\,(\delta|d)$, which is infinite-dimensional. The impossible mission of computing $H_1^D\,(\delta|d)$ can be avoided if we demand from the start the hypothesis on a_0^{t} to be of maximum derivative order equal to two. This assumption is particularly useful at this stage since it forbids the invariant polynomial α_1 to depend on the curvature tensors $K^{A}_{\lambda\mu\nu\xi|\alpha\beta}$ or their space-time derivatives. Indeed, assuming that α_1 effectively depends on the curvature tensors, it follows that the component from (69) with the minimum number of derivatives will be linear in the undifferentiated antifields $t_A^{*\lambda\mu\nu|\alpha}$, in the undifferentiated curvature tensors, as well as in the elements (70), so it already contains three space-time derivatives. If consistent, it would produce an a_0^{t} of order four in the space-time derivatives of the fields. Therefore, we forbid the dependence on the curvature tensors and remain with

$$a_1^{\mathrm{t}} = \alpha_{1A}^{\mathrm{lin}}\left(\left[t_B^{*\lambda\mu\nu|\alpha}\right]\right)\mathcal{F}^A_{\lambda\mu\nu\alpha}. \tag{71}$$

Moreover, the invariant polynomial α_1^{lin} is further restricted not to depend on the derivatives of $t_B^{*\lambda\mu\nu|\alpha}$. This is because one can always move the derivatives (by making an integration by parts) such as to act on $\mathcal{F}^A_{\lambda\mu\nu\alpha}$, which provides purely trivial (γ-exact) contributions to a_1^{t} (see (46)), which can be eliminated from the first-order deformation.

The previous discussion allows us to state that the only eligible candidate to a_1^{t} is defined in $D=6$ and reads

$$a_1^{\mathrm{t}} \equiv a_1^{\mathrm{t}(D=6)} = c_{AB}\sigma^{\alpha\beta}\varepsilon^{\lambda\mu\nu\lambda'\mu'\nu'}t^{*A}_{\lambda\mu\nu|\alpha}\mathcal{F}^B_{\lambda'\mu'\nu'\beta}. \tag{72}$$

Let us investigate the solutions in antighost number zero

$$\delta a_1^{\mathrm{t}(D=6)} + \gamma a_0^{\mathrm{t}(D=6)} = \partial_\mu \overset{(0)}{m}{}^\mu_{\mathrm{t}}. \tag{73}$$

In order to evaluate $\delta a_1^{\mathrm{t}(D=6)}$, we use the identity

$$\varepsilon^{\lambda\mu\nu\lambda'\mu'\nu'}\sigma^{\alpha\beta}\overset{(1)}{T}{}^A_{\lambda\mu\nu|\alpha}\overset{(2)}{T}{}^B_{\lambda'\mu'\nu'\beta} = -\frac{3}{4}\varepsilon^{\lambda\mu\nu\lambda'\mu'\nu'}\sigma^{\alpha\beta}\overset{(1)}{T}{}^A_{\lambda\mu\beta|\alpha}\overset{(2)}{T}{}^B_{\nu\lambda'\mu'\nu'} \tag{74}$$

(that takes place for any tensor $\overset{(1)}{T}{}^A_{\lambda\mu\nu|\alpha}$ completely antisymmetric in its first three indices and for any completely antisymmetric tensor $\overset{(2)}{T}{}^B_{\lambda'\mu'\nu'\beta}$) together with the first definition from (17). After some computation, we obtain that

$$\begin{aligned}\delta a_1^{\mathrm{t}(D=6)} &= \gamma\left[\frac{c_{AB}}{2}(4-D)\,\varepsilon^{\lambda\mu\lambda'\mu'\nu'\rho'}t^A_{\lambda\mu(\rho|\alpha)}\partial_{\lambda'}\left(\sigma^{\alpha\rho}\partial^\beta t^B_{\mu'\nu'\rho'|\beta} - \partial^\alpha t^B_{\mu'\nu'\rho'|}{}^\rho\right)\right] \\ &+\partial_\rho j^\rho - \frac{c_{AB}}{2}(4-D)\,\varepsilon^{\lambda\mu\lambda'\mu'\nu'\rho'}\mathcal{T}^A_{\lambda\mu(\rho|\alpha)}\partial_{\lambda'}\left(\sigma^{\alpha\rho}\partial^\beta t^B_{\mu'\nu'\rho'|\beta} - \partial^\alpha t^B_{\mu'\nu'\rho'|}{}^\rho\right),\end{aligned} \tag{75}$$

where

$$\mathcal{T}^A_{\lambda\mu\nu|\alpha} \equiv 4\partial_{[\lambda}\eta^A_{\mu\nu]\alpha} + \partial_{[\lambda}\mathcal{G}^A_{\mu\nu]|\alpha}, \qquad \mathcal{T}^A_{\lambda\mu\nu|\alpha} = 3\mathcal{F}^A_{\lambda\mu\nu\alpha} + \gamma t^A_{\lambda\mu\nu|\alpha}. \tag{76}$$

Comparing (75) with (73), we observe that the existence of $a_0^{\mathrm{t}(D=6)}$ requires that the last terms from the right-hand side of (75) either vanish or reduce to a full divergence. It is clear from (76) that they cannot reduce to a divergence, and therefore must be set equal to zero, which further implies

$$c_{AB} = 0, \tag{77}$$

such that

$$a_1^{\mathrm{t}(D=6)} = 0. \tag{78}$$

Until now we showed that

$$a_1^{\mathrm{t}} = 0, \tag{79}$$

and hence the first-order deformation may contain at most terms of antighost number zero ($I=0$). The terms of antighost number one present in the solution to the master equation are known to control the gauge symmetries, such that (79) expresses the fact that there are no consistent selfinteractions in a collection of tensor fields $t^A_{\lambda\mu\nu|\alpha}$ that deform the original gauge transformations, given in (9).

In this manner, we are left with a sole possibility, namely that the first-order deformation reduces to the deformed Lagrangian at order one in the coupling constant

$$a^{\mathrm{t}} = a_0^{\mathrm{t}}\left(\left[t^A_{\lambda\mu\nu|\alpha}\right]\right), \tag{80}$$

and thus it is subject to the equation

$$\gamma a_0^{\mathrm{t}} = \partial_\mu \overset{(0)}{m}{}^{\mu}_{\mathrm{t}}. \tag{81}$$

Proceeding along a line similar to that employed in [33], it can be shown that the solution to (81) is purely trivial

$$a_0^{\mathrm{t}}\left(\left[t^A_{\lambda\mu\nu|\alpha}\right]\right) = 0. \tag{82}$$

Assembling the results expressed by (53), (61), (67), (79), and (82), we can state that

$$S_1 = 0, \tag{83}$$

such that we can also take

$$S_k = 0, \qquad k > 1. \tag{84}$$

Relations (83)–(84) emphasize the following main result of our paper: *under the hypotheses of analyticity of deformations in the coupling constant, space-time locality, Lorentz covariance, Poincaré invariance, and conservation of the number of derivatives on each field, there are no consistent selfinteractions in $D \geq 5$ for a collection of massless tensor fields with the mixed symmetry* $(3,1)$. In other words, the presence of the collection brings nothing new if compared to the case of a single tensor field $t_{\lambda\mu\nu|\alpha}$.

4 Selfinteractions for a collection of massless tensor fields with the mixed symmetry $(2,2)$

4.1 Free model: Lagrangian formulation and BRST symmetry

The starting point is given by the Lagrangian action for a finite collection of free, massless tensor fields with the mixed symmetry of the Riemann tensor in $D \geq 5$

$$\begin{aligned} S_0^{\mathrm{r}}\left[r^a_{\mu\nu|\alpha\beta}\right] = & \int \left(\frac{1}{8}\left(\partial^\lambda r_a^{\mu\nu|\alpha\beta}\right)\left(\partial_\lambda r^a_{\mu\nu|\alpha\beta}\right) - \frac{1}{2}\left(\partial_\mu r_a^{\mu\nu|\alpha\beta}\right)\left(\partial^\lambda r^a_{\lambda\nu|\alpha\beta}\right)\right. \\ & - \left(\partial_\mu r_a^{\mu\nu|\alpha\beta}\right)\left(\partial_\beta r^a_{\nu\alpha}\right) - \frac{1}{2}\left(\partial^\lambda r_a^{\nu\beta}\right)\left(\partial_\lambda r^a_{\nu\beta}\right) + \left(\partial_\nu r_a^{\nu\beta}\right)\left(\partial^\lambda r^a_{\lambda\beta}\right) \\ & \left. - \frac{1}{2}\left(\partial_\nu r_a^{\nu\beta}\right)\left(\partial_\beta r^a\right) + \frac{1}{8}\left(\partial^\lambda r_a\right)\left(\partial_\lambda r^a\right)\right) d^D x. \end{aligned} \tag{85}$$

Like in the previous section, we employ the flat Minkowski metric of 'mostly plus' signature $\sigma^{\mu\nu} = \sigma_{\mu\nu} = (-++++\cdots)$. The lowercase indices a, b, etc. stand for the collection indices and are assumed to take discrete values 1, 2, …, n. They are lowered with a symmetric, constant and invertible matrix, of elements k_{ab}, and are raised with the help of the elements k^{ab} of its inverse. Each tensor field $r^a_{\mu\nu|\alpha\beta}$ exhibits the mixed symmetry of the Riemann tensor, so it is separately antisymmetric in the pairs $\{\mu,\nu\}$ and $\{\alpha,\beta\}$, is symmetric under their permutation ($\{\mu,\nu\} \longleftrightarrow \{\alpha,\beta\}$), and satisfies the identity $r^a_{[\mu\nu|\alpha]\beta} \equiv 0$. The notations $r^a_{\nu\beta}$ signify the traces of $r^a_{\mu\nu|\alpha\beta}$, $r^a_{\nu\beta} = \sigma^{\mu\alpha} r^a_{\mu\nu|\alpha\beta}$, which are symmetric, $r^a_{\nu\beta} = r^a_{\beta\nu}$, while r^a represent their double traces , $r^a = \sigma^{\nu\beta} r^a_{\nu\beta}$, which are scalars. Action (85) admits a generating set of gauge transformations of the form

$$\delta_\xi r^a_{\mu\nu|\alpha\beta} = \partial_\mu \xi^a_{\alpha\beta|\nu} - \partial_\nu \xi^a_{\alpha\beta|\mu} + \partial_\alpha \xi^a_{\mu\nu|\beta} - \partial_\beta \xi^a_{\mu\nu|\alpha}, \tag{86}$$

where the gauge parameters $\xi^a_{\mu\nu|\alpha}$ are bosonic tensors, with the mixed symmetry $(2,1)$. Just like in the case of a single $(2,2)$ field [34], the gauge transformations from (86) are Abelian and off-shell, first-order reducible. Consequently, the Cauchy order of this linear gauge theory is equal to three.

Related to the generators of the BRST algebra, the ghost spectrum contains the fermionic ghosts $\mathcal{C}^a_{\alpha\beta|\mu}$ associated with the gauge parameters and the bosonic ghosts for ghosts $\mathcal{C}^a_{\mu\nu}$ corresponding to the first-order reducibility. Obviously, we will require that $\mathcal{C}^a_{\alpha\beta|\mu}$ preserve the mixed symmetry $(2,1)$ and the tensors $\mathcal{C}^a_{\mu\nu}$ remain antisymmetric. The antifield spectrum comprises the antifields $r_a^{*\mu\nu|\alpha\beta}$ associated with the original fields and those corresponding to the ghosts, $\mathcal{C}_a^{*\mu\nu|\alpha}$ and $\mathcal{C}_a^{*\mu\nu}$. The antifields $r_a^{*\mu\nu|\alpha\beta}$ still have the mixed symmetry $(2,2)$, $\mathcal{C}_a^{*\mu\nu|\alpha}$ the mixed symmetry $(2,1)$, and $\mathcal{C}_a^{*\mu\nu}$ are antisymmetric. Related to the traces of $r_a^{*\mu\nu|\alpha\beta}$, we will use the notations $r_a^{*\nu\beta} = \sigma_{\mu\alpha} r_a^{*\mu\nu|\alpha\beta}$ and $r_a^* = \sigma_{\nu\beta} r_a^{*\nu\beta}$.

The BRST differential decomposes like in the previous section, as $s = \delta + \gamma$, the corresponding degrees of the generators from the BRST complex being valued like

$$\begin{gathered}
\mathrm{pgh}\left(r^a_{\mu\nu|\alpha\beta}\right) = 0, \quad \mathrm{pgh}\left(\mathcal{C}^a_{\mu\nu|\alpha}\right) = 1, \quad \mathrm{pgh}\left(\mathcal{C}^a_{\mu\nu}\right) = 2, \\
\mathrm{pgh}\left(r_a^{*\mu\nu|\alpha\beta}\right) = \mathrm{pgh}\left(\mathcal{C}_a^{*\mu\nu|\alpha}\right) = \mathrm{pgh}\left(\mathcal{C}_a^{*\mu\nu}\right) = 0, \\
\mathrm{agh}\left(r^a_{\mu\nu|\alpha\beta}\right) = \mathrm{agh}\left(\mathcal{C}^a_{\mu\nu|\alpha}\right) = \mathrm{agh}\left(\mathcal{C}^a_{\mu\nu}\right) = 0, \\
\mathrm{agh}\left(r_a^{*\mu\nu|\alpha\beta}\right) = 1, \quad \mathrm{agh}\left(\mathcal{C}_a^{*\mu\nu|\alpha}\right) = 2, \quad \mathrm{agh}\left(\mathcal{C}_a^{*\mu\nu}\right) = 3.
\end{gathered}$$

The actions of δ and γ on the generators from the BRST complex, which enforce the standard BRST properties, are given by

$$\gamma r^a_{\mu\nu|\alpha\beta} = \partial_\mu \mathcal{C}^a_{\alpha\beta|\nu} - \partial_\nu \mathcal{C}^a_{\alpha\beta|\mu} + \partial_\alpha \mathcal{C}^a_{\mu\nu|\beta} - \partial_\beta \mathcal{C}^a_{\mu\nu|\alpha}, \tag{87}$$

$$\gamma \mathcal{C}^a_{\mu\nu|\alpha} = 2\partial_\alpha \mathcal{C}^a_{\mu\nu} - \partial_{[\mu} \mathcal{C}^a_{\nu]\alpha}, \qquad \gamma \mathcal{C}^a_{\mu\nu} = 0, \tag{88}$$

$$\gamma r_a^{*\mu\nu|\alpha\beta} = 0, \qquad \gamma \mathcal{C}_a^{*\mu\nu|\alpha} = 0, \qquad \gamma \mathcal{C}_a^{*\mu\nu} = 0, \tag{89}$$

$$\delta r^a_{\mu\nu|\alpha\beta} = 0, \qquad \delta \mathcal{C}^a_{\mu\nu|\alpha} = 0, \qquad \delta \mathcal{C}^a_{\mu\nu} = 0, \tag{90}$$

$$\delta r_a^{*\mu\nu|\alpha\beta} = \frac{1}{4} R_a^{\mu\nu|\alpha\beta}, \delta \mathcal{C}_a^{*\alpha\beta|\nu} = -4\partial_\mu r_a^{*\mu\nu|\alpha\beta}, \delta \mathcal{C}_a^{*\mu\nu} = 3\partial_\alpha \mathcal{C}_a^{*\mu\nu|\alpha}. \tag{91}$$

In the above $R_a^{\mu\nu|\alpha\beta}$ is defined by $\delta S_0^{\mathrm{r}} / \delta r_a^{\mu\nu|\alpha\beta} \equiv -(1/4)\, R^a_{\mu\nu|\alpha\beta}$ and reads as

$$\begin{aligned}
R^a_{\mu\nu|\alpha\beta} = \; & \Box r^a_{\mu\nu|\alpha\beta} + \partial^\rho \left(\partial_\mu r^a_{\alpha\beta|\nu\rho} - \partial_\nu r^a_{\alpha\beta|\mu\rho} + \partial_\alpha r^a_{\mu\nu|\beta\rho} - \partial_\beta r^a_{\mu\nu|\alpha\rho}\right) \\
& + \partial_\mu \partial_\alpha r^a_{\beta\nu} - \partial_\mu \partial_\beta r^a_{\alpha\nu} - \partial_\nu \partial_\alpha r^a_{\beta\mu} + \partial_\nu \partial_\beta r^a_{\alpha\mu} \\
& - \frac{1}{2} \partial^\lambda \partial^\rho \left(\sigma_{\mu\alpha}\left(r^a_{\lambda\beta|\nu\rho} + r^a_{\lambda\nu|\beta\rho}\right) - \sigma_{\mu\beta}\left(r^a_{\lambda\alpha|\nu\rho} + r^a_{\lambda\nu|\alpha\rho}\right)\right. \\
& \left. - \sigma_{\nu\alpha}\left(r^a_{\lambda\beta|\mu\rho} + r^a_{\lambda\mu|\beta\rho}\right) + \sigma_{\nu\beta}\left(r^a_{\lambda\alpha|\mu\rho} + r^a_{\lambda\mu|\alpha\rho}\right)\right)
\end{aligned}$$

$$
\begin{aligned}
&-\Box\left(\sigma_{\mu\alpha}r^a_{\beta\nu}-\sigma_{\mu\beta}r^a_{\alpha\nu}-\sigma_{\nu\alpha}r^a_{\beta\mu}+\sigma_{\nu\beta}r^a_{\alpha\mu}\right)\\
&+\partial^\rho\left(\sigma_{\mu\alpha}\left(\partial_\beta r^a_{\nu\rho}+\partial_\nu r^a_{\beta\rho}\right)-\sigma_{\mu\beta}\left(\partial_\alpha r^a_{\nu\rho}+\partial_\nu r^a_{\alpha\rho}\right)\right.\\
&\left.-\sigma_{\nu\alpha}\left(\partial_\beta r^a_{\mu\rho}+\partial_\mu r^a_{\beta\rho}\right)+\sigma_{\nu\beta}\left(\partial_\alpha r^a_{\mu\rho}+\partial_\mu r^a_{\alpha\rho}\right)\right)\\
&-\frac{1}{2}\left(\sigma_{\mu\alpha}\partial_\beta\partial_\nu-\sigma_{\mu\beta}\partial_\alpha\partial_\nu-\sigma_{\nu\alpha}\partial_\beta\partial_\mu+\sigma_{\nu\beta}\partial_\alpha\partial_\mu\right)r^a\\
&-\left(\sigma_{\mu\alpha}\sigma_{\nu\beta}-\sigma_{\mu\beta}\sigma_{\nu\alpha}\right)\left(\partial^\lambda\partial^\rho r^a_{\lambda\rho}-\frac{1}{2}\Box r^a\right). \qquad (92)
\end{aligned}
$$

The solution to the classical master equation for the free model under study is given by

$$
\begin{aligned}
S^{\mathrm{r}} &= S_0^{\mathrm{r}}\left[r^a_{\mu\nu|\alpha\beta}\right]+\int\left(r_a^{*\mu\nu|\alpha\beta}\left(\partial_\mu\mathcal{C}^a_{\alpha\beta|\nu}-\partial_\nu\mathcal{C}^a_{\alpha\beta|\mu}+\partial_\alpha\mathcal{C}^a_{\mu\nu|\beta}-\partial_\beta\mathcal{C}^a_{\mu\nu|\alpha}\right)\right.\\
&\quad\left.+\mathcal{C}_a^{*\mu\nu|\alpha}\left(2\partial_\alpha\mathcal{C}^a_{\mu\nu}-\partial_{[\mu}\mathcal{C}^a_{\nu]\alpha}\right)\right)d^Dx. \qquad (93)
\end{aligned}
$$

4.2 Computation of basic cohomologies

In order to analyze the local equation satisfied by the non-integrated density of the first-order deformation a^{r} ($S_1=\int a^{\mathrm{r}}d^Dx$), written in local form and dual language, $sa^{\mathrm{r}}=\partial_\mu m^\mu_{\mathrm{r}}$, we proceed like in the previous section. We ensure the space-time locality of the deformations by working in the algebra of local differential forms with coefficients that are polynomial functions in the fields, ghosts, antifields, and their space-time derivatives (algebra of local forms). This means that we assume the non-integrated density of the first-order deformation, a^{r}, to be a polynomial function in all these variables (algebra of local functions). Next, we develop a^{r} according to the antighost number and assume that this expansion contains a finite number of terms, with the maximum value of the antighost number equal to I. Due to the decomposition $s=\delta+\gamma$, this equation becomes equivalent to the chain

$$
\begin{aligned}
\gamma a^{\mathrm{r}}_I &= \partial_\mu \overset{(I)}{m}{}^\mu_{\mathrm{r}}, && (94)\\
\delta a^{\mathrm{r}}_I+\gamma a^{\mathrm{r}}_{I-1} &= \partial_\mu \overset{(I-1)}{m}{}^\mu_{\mathrm{r}}, && (95)\\
\delta a^{\mathrm{r}}_k+\gamma a^{\mathrm{r}}_{k-1} &= \partial_\mu \overset{(k-1)}{m}{}^\mu_{\mathrm{r}}, \qquad I-1\geq k\geq 1, && (96)
\end{aligned}
$$

where $\left(\overset{(k)}{m}{}^\mu_{\mathrm{r}}\right)_{k=\overline{0,I}}$ are some local currents, with $\mathrm{agh}\left(\overset{(k)}{m}{}^\mu_{\mathrm{r}}\right)=k$. Equation (94) can be replaced in strictly positive values of the antighost number (see [35], Corollary 3.1) with

$$
\gamma a^{\mathrm{r}}_I=0, \qquad I>0. \qquad (97)
$$

In conclusion, for $I>0$ we have that $a^{\mathrm{r}}_I\in H^I(\gamma)$. We maintain the considerations from the previous section on the uniqueness of a^{r}_I and a^{r}.

Thus, in order to solve equations (97) and (95)–(96), it is necessary to compute the cohomology $H^*(\gamma)$ in the algebra of local functions. Definitions (89) and (87) indicate that all the antifields

$$
\chi^{*\Delta}=\left(r_a^{*\mu\nu|\alpha\beta},\mathcal{C}_a^{*\mu\nu|\alpha},\mathcal{C}_a^{*\mu\nu}\right), \qquad (98)
$$

the curvature tensors

$$
F^a_{\mu\nu\lambda|\alpha\beta\gamma} = \partial_\lambda\partial_\gamma r^a_{\mu\nu|\alpha\beta}+\partial_\mu\partial_\gamma r^a_{\nu\lambda|\alpha\beta}+\partial_\nu\partial_\gamma r^a_{\lambda\mu|\alpha\beta}
$$

$$
\begin{aligned}
&+\partial_\lambda\partial_\alpha r^a_{\mu\nu|\beta\gamma}+\partial_\mu\partial_\alpha r^a_{\nu\lambda|\beta\gamma}+\partial_\nu\partial_\alpha r^a_{\lambda\mu|\beta\gamma}\\
&+\partial_\lambda\partial_\beta r^a_{\mu\nu|\gamma\alpha}+\partial_\mu\partial_\beta r^a_{\nu\lambda|\gamma\alpha}+\partial_\nu\partial_\beta r^a_{\lambda\mu|\gamma\alpha},
\end{aligned}
\tag{99}
$$

and all their space-time derivatives are non-trivial elements of $H^0(\gamma)$. The curvature tensors exhibit the mixed symmetry $(3,3)$. Simple computation shows that $H^1(\gamma)=0$ and, moreover,

$$
H^{2l+1}(\gamma)=0,\qquad l\geq 0. \tag{100}
$$

By means of the last definition from (88), we find that the ghosts for ghosts, $\mathcal{C}^a_{\mu\nu}$, are non-trivial objects from $H^*(\gamma)$. Consequently, their space-time derivatives are also γ-closed. From the first relation present in (88) it follows that

$$
\partial_{(\mu}\mathcal{C}^a_{\nu)\alpha}\equiv\gamma\left(-\frac{1}{3}\mathcal{C}^a_{\alpha(\mu|\nu)}\right). \tag{101}
$$

Formula (101) emphasizes that the quantities $\partial_{(\mu}\mathcal{C}^a_{\nu)\alpha}$ are trivial in $H^*(\gamma)$. Moreover, the objects $\partial_{[\mu}\mathcal{C}^a_{\nu\alpha]}$ are not γ-exact, and $\partial_{[\mu}\mathcal{C}^a_{\nu]\alpha}$ (for $\mu,\nu\neq\alpha$) belong to the same equivalence class from $H^*(\gamma)$ like $\partial_{[\mu}\mathcal{C}^a_{\nu\alpha]}$, such that they will also be non-trivial representatives of $H^*(\gamma)$. Meanwhile direct calculations produce the relations

$$
\partial_\alpha\partial_\beta\mathcal{C}^a_{\mu\nu}=\frac{1}{12}\gamma\left(3\left(\partial_\alpha\mathcal{C}^a_{\mu\nu|\beta}+\partial_\beta\mathcal{C}^a_{\mu\nu|\alpha}\right)+\partial_{[\mu}\mathcal{C}^a_{\nu]\,(\alpha|\beta)}\right), \tag{102}
$$

which show that all the space-time derivatives of the ghosts $\mathcal{C}^a_{\mu\nu}$ of order two or higher are trivial in $H^*(\gamma)$. In conclusion, the only non-trivial combinations from $H^*(\gamma)$ built from the ghosts for ghosts are polynomials in $\mathcal{C}^a_{\mu\nu}$ and $\partial_{[\mu}\mathcal{C}^a_{\nu\alpha]}$. Since $H^0(\gamma)$ is non-trivial, so far we proved that only the cohomological spaces $H^{2l}(\gamma)$, with $l\geq 0$, are non-trivial. Therefore, equation (97) possesses non-trivial solutions only for even values of I, $I=2J$, where the general form of a^{r}_{2J} is given by

$$
a^{\mathrm{r}}_I\equiv a^{\mathrm{r}}_{2J}=\alpha_{2J}\left(\left[\chi^{*\Delta}\right],\left[F^a_{\mu\nu\lambda|\alpha\beta\gamma}\right]\right)e^{2J}\left(\mathcal{C}^a_{\mu\nu},\partial_{[\mu}\mathcal{C}^a_{\nu\alpha]}\right),\qquad J>0. \tag{103}
$$

Notation $\chi^{*\Delta}$ follows from (98). The coefficients $\alpha_I([\chi^{*\Delta}],[F^a_{\mu\nu\lambda|\alpha\beta\gamma}])$ are nothing but the invariant polynomials (in form degree zero) of the theory (85)–(86).

Substituting solution (103) in equation (95) for $I=2J$ and taking into consideration definitions (87)–(88), we obtain that a necessary condition for the existence of non-trivial solutions a^{r}_{2J-1} is that the invariant polynomials α_{2J} present in (103) generate non-trivial elements from the characteristic cohomology in antighost number $2J>0$ computed in the algebra of local forms, $\alpha_{2J}d^Dx\in H^D_{2J}(\delta|d)$. As the free model under consideration is a linear gauge theory of Cauchy order equal to three, the general results from the literature [36] establish that

$$
H^D_k(\delta|d)=0,\qquad k>3. \tag{104}
$$

In addition, it can be shown that if the invariant polynomial α_k, with $\mathrm{agh}(\alpha_k)=k\geq 3$, defines a trivial element $\alpha_k d^Dx\in H^D_k(\delta|d)$, then this element can be taken to be trivial also in $H^{\mathrm{inv}D}_k(\delta|d)$. The above results ensure that

$$
H^{\mathrm{inv}D}_k(\delta|d)=0,\qquad k>3. \tag{105}
$$

Using definitions (91), we find that the non-trivial, Poincaré-invariant representatives of $\left(H^D_k(\delta|d)\right)_{k\geq 2}$ and $\left(H^{\mathrm{inv}D}_k(\delta|d)\right)_{k\geq 2}$ are linearly generated by the following invariant polynomials: for $k > 3$ — there are none; for $k = 3$ — $f^a_{\mu\nu}\mathcal{C}^{*\mu\nu}_a d^D x$; for $k = 2$ — $f^a_{\mu\nu\alpha}\mathcal{C}^{*\mu\nu|\alpha}_a d^D x$. In the above the coefficients denoted by f stand for the components of some constant, non-derivative tensors.

The previous results on $H^D_I(\delta|d)$ and $H^{\mathrm{inv}D}_I(\delta|d)$ allow us to eliminate successively all the terms of antighost number strictly greater than two from the non-integrated density of the first-order deformation. The last representative is of the form (103), where the invariant polynomials necessarily define non-trivial elements from $H^{\mathrm{inv}D}_I(\delta|d)$ if $I = 2$ or respectively from $H^D_1(\delta|d)$ if $I = 1$.

4.3 Cohomological analysis of selfinteractions

In order to develop the general method of construction of consistent selfinteractions that can be added to the free action (85), subject to the gauge symmetry (86), we initially solve equation (4), responsible for the first-order deformation, and then approach its consistency. We will work under the same hypotheses as before. The derivative order assumption restricts the interaction Lagrangian to contain only interaction vertices with maximum two space-time derivatives. Related to the non-integrated density of the first-order deformation, we have seen in the previous section that its component of highest antighost number, I, is constrained to satisfy the relation $I = 2J$ (see the result expressed by (100) on $H^*(\gamma)$). On the other hand, results (104) and (105) ensure that one can safely take $I \leq 2$.

In view of this, the first non-trivial situation is described by $I = 2J = 2 > 0$, in which case we can write

$$a^{\mathrm{r}} = a^{\mathrm{r}}_0 + a^{\mathrm{r}}_1 + a^{\mathrm{r}}_2, \tag{106}$$

where a^{r}_2 is the general, non-trivial solution to equation (97), and hence, in agreement with formula (103), has the expression

$$a^{\mathrm{r}}_2 = \alpha_2\left(\left[\chi^{*\Delta}\right], \left[F^a_{\mu\nu\lambda|\alpha\beta\gamma}\right]\right) e^2\left(\mathcal{C}^a_{\mu\nu}, \partial_{[\mu}\mathcal{C}^a_{\nu\alpha]}\right). \tag{107}$$

The elements e^2 are spanned by $\mathcal{C}^a_{\mu\nu}$ and $\partial_{[\mu}\mathcal{C}^a_{\nu\alpha]}$, and $\alpha_2 d^D x$ is a non-trivial element from $H^{\mathrm{inv}D}_2(\delta|d)$. Due to the fact that the general representative of $H^{\mathrm{inv}D}_2(\delta|d)$ is linear in the undifferentiated antifields $\mathcal{C}^{*a}_{\mu\nu|\alpha}$, we deduce that

$$a^{\mathrm{r}}_2 = \mathcal{C}^{*a}_{\mu\nu|\alpha}\left(f^{\mu\nu\alpha\beta\gamma}_{ab}\mathcal{C}^b_{\beta\gamma} + \bar{f}^{\mu\nu\alpha\beta\gamma\lambda}_{ab}\partial_{[\beta}\mathcal{C}^b_{\gamma\lambda]}\right), \tag{108}$$

where $f^{\mu\nu\alpha\beta\gamma}_{ab}$ and $\bar{f}^{\mu\nu\alpha\beta\gamma\lambda}_{ab}$ are some non-derivative, constant tensors. These constants cannot be simultaneously antisymmetric in the indices $\{\mu,\nu,\alpha\}$ since the identity $\mathcal{C}^{*[\mu\nu|\alpha]}_a \equiv 0$ would lead to $a^{\mathrm{r}}_2 = 0$. The last restriction (combined with the requirement $D \geq 5$) produces

$$f^{\mu\nu\alpha\beta\gamma}_{ab} = 0 = \bar{f}^{\mu\nu\alpha\beta\gamma\lambda}_{ab}, \tag{109}$$

and hence

$$a^{\mathrm{r}}_2 = 0, \tag{110}$$

so the first-order deformation cannot end non-trivially at antighost number two.

Due to the fact that the last representative a^{r}_I from the first-order deformation is subject to the condition $I = 2J$, we are left only with the case $I = 0$

$$a^{\mathrm{r}} = a^{\mathrm{r}}_0\left(\left[r^a_{\mu\nu|\alpha\beta}\right]\right), \tag{111}$$

where a_0^{r} satisfies equation (94) ($I = 0$, so equation (94) is no longer equivalent to (97))

$$\gamma a_0^{\mathrm{r}} = \partial_\mu \overset{(0)^\mu}{m}_{\mathrm{r}}. \tag{112}$$

Using a technique similar to that employed in [34], we find that the general solution to the last equation reduces to a linear combination of double traces of the undifferentiated tensor fields $r^a_{\mu\nu|\alpha\beta}$ (the analogue of the cosmological term for the Pauli–Fierz Lagrangian)

$$a_0^{\mathrm{r}} = \sum_{a=1}^{n} c_a r^a, \tag{113}$$

with c_a some real, arbitrary constants, such that

$$S_1 = \sum_{a=1}^{n} \int c_a r^a d^D x \tag{114}$$

represents the most general expression of the first-order deformation of the solution to the master equation for a collection of massless tensor fields with the mixed symmetry $(2,2)$. Moreover, this solution is already consistent to all orders in the coupling constant. Indeed, since $(S_1, S_1) = 0$, equation (5) is satisfied with the choice

$$S_2 = 0, \tag{115}$$

and similarly, all the higher-order equations are fulfilled for

$$S_3 = S_4 = \ldots = 0. \tag{116}$$

Relations (114)–(116) emphasize the following main result of our paper: *under the hypotheses of analyticity of deformations in the coupling constant, space-time locality, Lorentz covariance, Poincaré invariance, and conservation of the number of derivatives on each field, there are no consistent selfinteractions in $D \geq 5$ for a collection of massless tensor fields with the mixed symmetry of the Riemann tensor. The only terms that can be added to the free Lagrangian action are given by a sum of cosmological terms, whose existence does not modify the original gauge transformations.*

Acknowledgments

One of the authors (E.M.B.) acknowledges financial support from the contract 464/2009 in the framework of the programme IDEI of C.N.C.S.I.S. (Romanian National Council for Academic Scientific Research).

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