University of Craiova
Department of Applied Mathematics

NONLINEAR DYNAMICS

Workshop

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in the frame of the project FP7-PEOPLE-2012-IRSES-316338

"Dynamical systems and their applications"

and of the University of Craiova Grant 19C/27.01.2014

"Differential equations: qualitative study, efficient numerical processing and applications"

Abstracts

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Local Organizing Committee:

Associate Professor Dana Constantinescu
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Contents

Invited Lectures

Limit cycle bifurcations near a 2-polycycle or double 2-polycycle of planar systems

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Abstract. In this paper, we study the bifurcation problem of limit cycles for a near-Hamiltonian system near a 2-polycycle or double 2-polycycle. We obtain new results on the number of limit cycles by using the method of stability-changing of a homoclinic loop. This method also provides a new way to find alien limit cycles. Especially, we found some alien limit cycles of new type.

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On the study of two classes of singular nonlinear wave equations: dynamical system approach

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Abstract. Nonlinear wave phenomena are of great importance in the physical world and have been for a long time a challenging topic of research for both pure and applied mathematicians. There are numerous nonlinear evolution equations for which we need to analyze the properties of the solutions for time evolution of the system. As the first step, we should understand the dynamics of their travelling wave solutions.

There exists an enormous literature on the study of nonlinear wave equations, in which exact explicit solitary wave, kink wave, periodic wave solutions, bifurcations and dynamical stabilities of these waves are discussed. To find exact travelling wave solutions for a given nonlinear wave system, a lot of methods have been developed such as the inverse scattering method, Darboux transformation method, Hirota bilinear method, algebraic-geometric method, tanh method, etc. What is the dynamical behavior of these exact travelling wave solutions? How do the travelling wave solutions depend on the parameters of the system? What is the reason of the smoothness change of travelling wave solutions? How to understand the dynamics of the so-called compacton and peakon solutions? These are very interesting and important problems. In recent years, these topics have had significant advances and research is also very active.

The aim of this talk is to give a more systematic account for the bifurcation theory method of dynamical systems to find travelling wave solutions with an emphasis on singular waves and understand their dynamics for some classes of the well-posedness of nonlinear evolution equations.

Integrability and bifurcations in polynomial systems of ODEs

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Abstract. Consider polynomial vector fields in n-dimensional real space having a singularity at the origin with non-degenerate linear part. We discuss the problems of existence of analytic first integrals, center manifolds, periodic solutions and the problem of stability, and their interconnection. We also explicitly compute center manifolds and first integrals for several families of systems with quadratic and cubic higher order terms and describe computational methods for the study, in particular, an approach for solving systems of polynomials using modular calculations. Bifurcations of periodic solutions are discussed as well.

On solutions of Korteweg-de Vries equations of the higher order related to solutions of Painlevé equations

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Abstract. We consider hierarchies of P- type nonlinear ordinary differential equations (ODEs) which are generalizations of the Painlevé equations (P_1) - (P_6). They arise as symmetry reductions of PDEs hierarchies or as equations of isomonodromic deformation of linear systems. The Painlevé equations can be regarded as completely integrable equations, and they have hierarchies of algebraic solutions and one-parameter families of solutions expressed in terms of the classical special functions for special values of the parameters. Further the Painlevé equations admit symmetries under affine Weyl groups. In general, the Painlevé transcendent may be thought of nonlinear analogue of the classical special functions. We also consider hierarchies which arise as exact reduction of the higher analogue of the Korteveg de Vries equation with more general operator. These hierarchies contain the first and second Painlevé equations as special case. We show that equations of constructed hierarchies have the same properties as (P_1) and (P_2).

Applications of dynamical systems in neuronal models

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Abstract. Applications of dynamical system theory in Neuroscience are found especially in modelling various phenomena met in biological neurons. A well-known but crucial phenomenon for all living human beings (and animals) is the action potential, a transient phase of charging/ discharging neuron's membrane, when a signal is built and sent further within the central and peripheral nervous system. This phenomenon is quite well described by the ODE Huxley-Hodgkin (HH) neuronal model. In this talk, I will present several neuronal models used currently in this area of research. I will start with a brief description of the HH model and will continue with integrate-and-fire and other models.

Short Communications

Elliptic problems with variable exponents and no-flux boundary conditions

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Abstract. We are interested in the study of nonlinear elliptic problems involving generalized operators with variable exponents and no-flux boundary conditions. Thus, we introduce a general class of such problems and we search for weak solutions working in the framework of the anisotropic spaces with variable exponents. Using the critical point theory and relying on various variational means, we establish existence, uniqueness and multiplicity results.

The study of some dynamical processes from financial market

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Abstract. In this work we are studying some dynamical processes, that means the systems given by the difference equation $x_{n+1} = f(x_n)$, having the generators $(f_n)_{n\in\mathbb{N}}$.
The process

$$\begin{cases} x_{n+1} = (a_n \cdot y_n + b_n) \cdot x_n \cdot (1 - x_n) \\ y_{n+1} = (c_n \cdot x_n + d_n) \cdot y_n \cdot (1 - x_n) \end{cases} \tag{1}$$

describes the financial evolution of two interacting corporations (duopoly) in a market place with limited resources. Depending on the parameters a_n, b_n, c_n, d_n, associated with the mutual benefit or the mutual competitive interaction, the corporations are, at the time moment "n", in cooperation or in competition. We analyze the asymptotic behaviour of some processes becoming from (1).

Regular versus chaotic dynamics in some systems generated by area-preserving maps. Applications to the study of some transport phenomena equation

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Abstract. The study of dynamical systems generated by area-preserving maps is important from practical point of view because many area-preserving maps are Poincaré maps associated to 3/2 d.o.f. Hamiltonian

systems (with applications in astronomy, plasma physics, fluid dynamics, mechanics). From theoretical point of view the study is also interesting because the phase space of such systems is a complex mixture of invariant zones whose points have regular dynamics (the orbits are periodic or quasi-periodic), respectively chaotic dynamics (with strong mixing properties). The sudden separation of these zones is not yet fully understood.

In this work we propose general results concerning the existence and the localization of internal transport barriers (rotational invariant regular zones separating two chaotic zones) for a class of dynamical systems generated by area-preserving maps. We systematically study the influence of the parameters of the system on transport properties and we propose a method for building transport barriers in a prescribed zone of the phase space.

The results are applied for the study of the magnetic transport in tokamaks (toroidal devices used for obtaining energy through controlled thermonuclear fusion). In this case the formation of internal transport barriers is crucial for the (desired) plasma confinement because it prevents the radial transport of charged particles.

Bifurcation analysis in a 3D symmetric system of interest in plasma physics

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Abstract. The aim of this paper is the study of a 3D dynamical system which models some instabilities that occur in fusion plasma experiments in TOKAMAKs (toroidal devices for obtaining energy through controlled thermonuclear fusion). This model depends on three parameters. It was formulated for the description of a system with drive and relaxation processes which have different time scales. It has some properties analogue to Lorenz system, but it does not belong to the family of Lorenz-like systems. We systematically study the dynamics of the system for various values of the parameters and we provide analytical results concerning the bifurcations that occur in the parameters' space.

Opial-type inequalities in time scales with applications in dynamic equations

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Abstract. Opial's inequality and its generalizations play an important role in the study of differential equations. The real analysis based on the concept of time scale", which was introduced in 1988 by Stefan Hilger, has received a considerable attention in the last years. Among other things, the new theory unifies the study of differential and difference equations. Our goal is to review a number of remarkable extensions of Opial's inequality and to present their very recent versions on time scale. This kind of results finds important applications in the area of dynamic equations.

Such equations (defined on a given time scale $\mathbb{T}$) have the following general form

$$f\left(t, y\left(t\right), y^{\Delta}\left(t\right), ..., y^{\Delta^n}\left(t\right)\right) = 0,$$

where $y^{\Delta^k}\left(t\right)$ is the delta derivative of k-order at $t \in \mathbb{T}$. As in the classical case, these equations are treated under initial values and boundary conditions.

2000 Mathematics Classification: 34A40, 39A13, 26D15.

Key words and phrases: Time scales, Dynamic equations, Opial type inequalities.

Nonlinear dynamics of the mixing flow mathematical model. The parameter influence on the trajectory behavior

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Abstract. The mixing theory appears in an area with far from complete solving problems: the flow kinematics. Its methods and techniques developed the significant relation between turbulence and chaos. The turbulence is an important feature of dynamic systems with few freedom degrees, the so-called "far from equilibrium systems". These are widespread between the models of excitable media, and a recent challenge is to find a consistent and coherent theory to stand up that a mixing model in excitable media leads to a far from equilibrium model.

Studying a mixing for a flow implies the analysis of successive stretching and folding phenomena for its particles, the influence of parameters and initial conditions. In the previous works, the study of the 3D non-periodic models exhibited a quite complicated behavior. In agreement with experiments, they involved some significant events - the so-called "rare events". The variation of parameters had a great influence on the length and surface deformations.

In this paper there is continued the work started in recent years on turbulent mixing mathematical model. The comparisons of deformation (in length and surface) efficiency of 3D versus 2D case brought a very rich panel of random distributed events. An important partial conclusion was that the mixing, and especially the turbulent mixing, is introduced at irrational values of length and surface versors.

There is used a modern mathematical soft, MAPLE11, which has a lot of performing computational appliances. Crossing over from 2D to 3D case, it is easy to deduce the requirement of a special analysis of the influence of parameters on the behavior of this dynamical system. Since in the precedent work there were obtained numeric plots standing for the images of the deformation efficiency, in the present chapter the standpoint is a little changed. A logistic type term is tested for the 3D mixing mathematical model, and the trajectories behavior for this new model is analyzed.

Thus, the panel of random distributed events would be completed with new special events and new data would be collected for the qualitative analysis of this dynamical system.

2000 Mathematics Subject Classification: 76F25, 74A05, 70-08.

Key words and phrases: turbulent mixing, vortex flow, phase portrait, DEplot3d.

Using Matcont for bifurcations in neuronal models

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Abstract. Bifurcation analysis of a dynamical system based on differential equations is an important part to understanding the system's behavior. In general, analytical methods are difficult for describing bifurcations in complex systems. A good alternative is given by adequate software needed to obtain a bifurcation diagram,

based in general on principals of continuity. Such a software is Matcont. In this talk, I will present several bifurcations met in neuronal models using Matcont.

The lower bound of limit cycles appearing from the perturbation of a system with a multiple line of critical points

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Abstract. The authors of [1] studied a system with a multiple line of critical points and obtained an upper bound of the maximum number of limit cycles, which bifurcate from period annulus of the origin. Moreover, they proved the upper bound is reachable for suitable polynomials $P(x, y)$ and $Q(x, y)$. In this paper, we first give a counter example to show the upper bound is not obtained with the polynomials $P(x, y)$ and $Q(x, y)$, which were chosen in this paper. Then, we proved the upper bound is reachable for the cases $m = 1, 2, 3, 4$ under the quadric polynomial perturbations ($n = 2$).

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Computational analysis of conservation laws for mathematical models of the multi-species interactions

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Abstract. Our purpose is to study symmetries and conservation laws for the mathematical models of the multi-species interactions: prey-predator Lotka-Volterra equations and others systems. This work is a interplay between dynamical systems geometrical theory and computational calculus of dynamical systems, knowing that the theory provides a framework for interpreting numerical observations and foundations for efficient numerical algorithms. Also, we will use the Hamilton-Poisson realizations of 2D and 3D Volterra-Lotka systems for obtain new conservation laws.

Numerical simulations and some applications in the cortical bone behaviour and thermo ablation in living tissues

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Abstract. We intend to present in the current work a few numerical methods with applications in medicine. It concerns a mathematical model of human cortical bone based on the homogenization theory and the numerical simulation of thermoablation in a liver tumour. Both subjects are very important from a biomechanical point of view. The first one allows studying the mechanical behaviour of the heterogeneous and complex structure of the cortical bone, knowing the properties of its basics components (collagen, hydroxyapatite (Hap) and bony fluid) and its architectural configuration. The second application concerns the simulation in a 2D framework of the injection of hot vaporized water in a cancerous liver tissue.

For the first model, one uses the asymptotic homogenization method (AHM) [1] in a piezoelectric framework and we develop a finite element method for solving the local problems obtained by homogenization. The developed computational methods have been packed into a software (called SiNuPrOs) made on a Matlab platform; it allows a large number of predictive simulations corresponding to various different configurations [2]-[3]. In the long term this model could be helpful to understand the bone remodelling which obviously needs the knowledge of the mechanical information transfer.

The second modelization is based on a coupling of three equations: 1) a Darcy equation for the flow of the steam inside the tissue; 2) a transport equation for modelling the steam (and possibly of the therapeutic molecules); 3) a bioheat equation for the transfer of heat in the tissue. This method may be used for the validation of the TMT (Targeted Multi Therapy) technique [4] in a numerical way. The next step deals with the introduction of the therapeutic molecules instead of the water vapor and this is not difficult once one knows the properties of these molecules, but also integrating this modelization in the 3D framework with a virtual liver.

Key words and phrases: numerical simulations, cortical bone, homogenization, liver, thermoablation, tumour therapy.

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The controllability of a system modelling the motion of a swimmer moving in a viscous fluid

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Abstract. We study, from a control theoretic view point, a 1D model of fluid particle interaction. More precisely we consider a point mass moving in a pipe filled with a fluid. The fluid is modelled by the viscous

Burgers equation whereas the point mass obeys Newton's second law. The control variable is a force acting on the mass point. The main result of the paper asserts that for any initial data there exist a time T and a control such that, at the end of the control process, the particle reaches a point arbitrarily close to a given target, whereas the velocities of the fluid and of the point mass are driven exactly to zero. Therefore, within this simplified model, we can control simultaneously the fluid and the particle, by using inputs acting on the moving point only. Moreover, the main result holds without any smallness assumptions on the initial data.

Compatibility conditions for a potential on a dynamical system

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Abstract. In this paper we are investigating compatibility conditions between a dynamical system $\mathbb{D} = ((T, \alpha), S, \Phi)$ and a general potential $\gamma : T \times S \to C$. While we are focusing on continuous dynamical systems, there are some more general cases, where is no need for a differential structure on the space-time continuum.

Stability problems for a damped nonlinear oscillator

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Abstract. The stability of the null solution of nonlinear second order ODE

$$\ddot{x} + 2f(t)\dot{x} + \beta(t)x + g(t, x) = 0, \ t \geq 0,$$

is discussed. Under some unusual assumptions, we obtain new stability results for this classical oscillator equation. Our approach allows extensions to both the vector case and the case of the whole real line.

2000 Mathematics Subject Classification: 34D20.

Key words and phrases: oscillator, uniform stability, asymptotic stability.

Multiple positive periodic solutions for a delayed predator-prey system with Beddington-DeAngelis functional response and harvesting terms

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Abstract. In this paper, a delayed predator-prey system with Beddington-DeAngelis functional response and harvesting terms is studied. By using Mawhin's continuation theorem, the sufficient conditions are established for the existence of at least four positive periodic solutions. Finally, an example is presented to illustrate the effectiveness of the results.

Polynomial solutions of the polynomial-like iterative equation

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Abstract. In this report we discuss polynomial mappings which have iterative roots of polynomial form.

We apply the computer algebra system Singular to decompose algebraic varieties and finally find a condition under which polynomial functions have quadratic iterative roots of quadratic polynomial form. This condition is equivalent to but simpler than Schweizer and Sklar's and more convenient than Strycharz-Szembergand Szemberg's.

We further find all polynomial functions which have cubic iterative roots of quadratic polynomial form and compute all those iterative roots. Moreover, we give an algorithm for finding the simplest algebraic relations among coefficients of polynomials of degree m^n (the relations can guarantee these polynomials have the n-th iterative roots of polynomial form), and the explicit expressions of the iterative roots. And using the algorithm we obtain the quadratic iterative roots (polynomial form) of polynomials of degree 9 and 16 as examples. Finally, using the same ideal we find all 2-dimensional homogeneous polynomial mappings of degree 2 which have iterative roots of polynomial form and obtain expressions of some iterative roots.

Hopf bifurcation of a ratio-dependent predator-prey system with Holling type III functional response

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Abstract. In this paper a ratio-dependent predator-prey system with Holling type III functional response is discussed. We give parameter conditions for exact number of equilibria. The conditions for pitchfork bifurcation are given by the center manifold theory. Hopf bifurcations are discussed by computing Lyapunov coefficients. It is shown that at most one limit cycle occurs.