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Interactions in theories of spin 3/2 and spin 2 intermediated by topological BF models

–Summary of Ph.D. thesis–

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1 Main subjects approached in the thesis. Method employed. Working hypotheses

The main subjects approached in the thesis are the following: a) the derivation of the interactions between a BF model and a massless tensor field with mixed symmetry $(2,1)$; b) the construction of the couplings between a BF model and a massless Rarita-Schwinger field.

Our procedure is based on solving the equations that describe the deformation of the solution to the master equation [1] by means of specific cohomological techniques [2]–[4].

The interactions are obtained under the following hypotheses: locality, Lorentz covariance, Poincare invariance, analyticity of the deformations, and preservation of the number of derivatives on each field. The first three assumptions imply that the interacting theory is local in spacetime, Lorentz covariant and Poincare invariant. The analyticity of the deformations refers to the fact that the deformed solution to the master equation is analytical in the coupling constant and reduces to the original solution in the free limit. The conservation of the number of derivatives on each field with respect to the free theory means here that the following two requirements are simultaneously satisfied: (i) the derivative order of the equations of motion on each field is the same for the free and respectively for the interacting theory; (ii) the maximum number of derivatives in the interaction vertices is equal to two, i.e. the maximum number of derivatives from the free Lagrangian.

2 Results

In the sequel we will briefly present the main results of the thesis.

2.1 Interactions between a BF model and a massless tensor field with the mixed symmetry $(2,1)$

The starting point is a free theory in $D = 5$, whose Lagrangian action is written as the sum between the Lagrangian action of an Abelian BF model with a maximal field spectrum (a single scalar field φ, two types of one-forms H^{μ} and V_{μ}, two kinds of two-forms $B^{\mu\nu}$ and $\phi_{\mu\nu}$, and one three-form $K^{\mu\nu\rho}$) and the Lagrangian action of a free, massless tensor field with the mixed symmetry $(2,1)$ $t_{\mu\nu|\alpha}$ (meaning it is antisymmetric in its first two indices $t_{\mu\nu|\alpha}=-t_{\nu\mu|\alpha}$ and fulfills the identity $t_{[\mu\nu|\alpha]}\equiv 0$)

$$\begin{aligned} S_0\left[\Phi^{\alpha_0}\right] = \int d^5x\, \big(H^\mu\partial_\mu\varphi + \tfrac{1}{2}B^{\mu\nu}\partial_{[\mu}V_{\nu]} + \tfrac{1}{3}K^{\mu\nu\rho}\partial_{[\mu}\phi_{\nu\rho]} \\ -\tfrac{1}{12}\left(F_{\mu\nu\rho|\alpha}F^{\mu\nu\rho|\alpha} - 3F_{\mu\nu}F^{\mu\nu}\right)\big) \equiv \int d^5x\left(\mathcal{L}_0^{\mathrm{BF}} + \mathcal{L}_0^{\mathrm{t}}\right), \end{aligned} \tag{1}$$

where we used the notations

$$\Phi^{\alpha_0} = \left(\varphi, H^\mu, V_\mu, B^{\mu\nu}, \phi_{\mu\nu}, K^{\mu\nu\rho}, t_{\mu\nu|\alpha}\right), \tag{2}$$

$$F_{\mu\nu\rho|\alpha} = \partial_{[\mu}t_{\nu\rho]|\alpha}, \qquad F_{\mu\nu} = \sigma^{\rho\alpha}F_{\mu\nu\rho|\alpha}. \tag{3}$$

We work with a Minkowski-flat metric tensor of 'mostly minus' signature $\sigma^{\mu\nu}=\sigma_{\mu\nu}=(+----)$.

Action (1) is invariant under the gauge transformations

$$\delta_\Omega\varphi = 0,\ \delta_\Omega H^\mu = 2\partial_\nu\epsilon^{\mu\nu},\ \delta_\Omega V_\mu = \partial_\mu\epsilon,\ \delta_\Omega B^{\mu\nu} = -3\partial_\rho\epsilon^{\mu\nu\rho}, \tag{4}$$

$$\delta_\Omega\phi_{\mu\nu} = \partial_{[\mu}\xi_{\nu]},\ \delta_\Omega K^{\mu\nu\rho} = 4\partial_\lambda\xi^{\mu\nu\rho\lambda},\ \delta_\Omega t_{\mu\nu|\alpha} = \partial_{[\mu}\theta_{\nu]\alpha} + \partial_{[\mu}\chi_{\nu]\alpha} - 2\partial_\alpha\chi_{\mu\nu}, \tag{5}$$

where all the gauge parameters are bosonic, with $\epsilon^{\mu\nu}$, $\epsilon^{\mu\nu\rho}$, $\xi^{\mu\nu\rho\lambda}$, and $\chi_{\mu\nu}$ completely antisymmetric and $\theta_{\mu\nu}$ symmetric. By Ω we denoted collectively all the gauge parameters as

$$\Omega^{\alpha_1} \equiv \left(\epsilon^{\mu\nu}, \epsilon, \epsilon^{\mu\nu\rho}, \xi_\mu, \xi^{\mu\nu\rho\lambda}, \theta_{\mu\nu}, \chi_{\mu\nu}\right). \tag{6}$$

The gauge transformations given by (4)–(5) are off-shell reducible of order three (the reducibility relations hold everywhere in the space of field histories).

The first main result is given by the following two theorems.

Theorem 1
A) Under the above mentioned hypotheses, the most general form of the action that describes the interacting theory reads as

$$\begin{aligned}\bar{S}_0\left[\Phi^{\alpha_0}\right] = & \int d^5x \left\{ H_\mu \partial^\mu \varphi + \tfrac{1}{2} B^{\mu\nu} \partial_{[\mu} V_{\nu]} + \tfrac{1}{3} K^{\mu\nu\rho} \partial_{[\mu} \phi_{\nu\rho]} \right. \\ & -\tfrac{1}{12}\left(F_{\mu\nu\rho|\alpha}F^{\mu\nu\rho|\alpha} - 3F_{\mu\nu}F^{\mu\nu}\right) \\ & +g\left[W_1 V_\mu H^\mu + W_2 B_{\mu\nu}\phi^{\mu\nu} - W_3 \phi_{[\mu\nu} V_{\rho]} K^{\mu\nu\rho} + \bar{M}(\varphi)\right. \\ & \left. +\varepsilon^{\alpha\beta\gamma\delta\varepsilon}\left(9W_4 V_\alpha \tilde{K}_{\beta\gamma}\tilde{K}_{\delta\varepsilon} + \tfrac{1}{4}W_5 V_\alpha \phi_{\beta\gamma}\phi_{\delta\varepsilon} + W_6 B_{\alpha\beta} K_{\gamma\delta\varepsilon}\right)\right] \\ & \left. +g\left(k_1\phi^{\mu\nu} - \tfrac{k_2}{20}\tilde{K}^{\mu\nu}\right)\left[F_{\mu\nu} + \tfrac{3g}{2}\left(k_1\phi_{\mu\nu} - \tfrac{k_2}{20}\tilde{K}_{\mu\nu}\right)\right]\right\}, \end{aligned} \quad (7)$$

where k_1 and k_2 are constants, while $\bar{M}$, W_1, $\ldots$, W_5 and W_6 are functions depending only on the undifferentiated scalar field. The notation g signifies the coupling constant (deformation parameter).
B) The constants k_1 and k_2 and the functions $\bar{M}$, W_1, $\ldots$, W_5 and W_6 satisfy the equations

$$\frac{d\bar{M}(\varphi)}{d\varphi}W_1(\varphi) = 0, \qquad W_1(\varphi)W_2(\varphi) = 0, \quad (8)$$

$$W_1(\varphi)\frac{dW_2(\varphi)}{d\varphi} - 3W_2(\varphi)W_3(\varphi) + 6W_5(\varphi)W_6(\varphi) = 0, \quad (9)$$

$$W_2(\varphi)W_3(\varphi) + W_5(\varphi)W_6(\varphi) = 0, \quad (10)$$

$$W_1(\varphi)\frac{dW_6(\varphi)}{d\varphi} + 3W_3(\varphi)W_6(\varphi) - 6W_2(\varphi)W_4(\varphi) = 0, \quad (11)$$

$$W_1(\varphi)W_6(\varphi) = 0, \qquad W_2(\varphi)W_4(\varphi) + W_3(\varphi)W_6(\varphi) = 0, \quad (12)$$

$$W_2(\varphi)W_5(\varphi) = 0, \qquad W_4(\varphi)W_6(\varphi) = 0, \quad (13)$$

$$k_1W_3 + \tfrac{k_2}{60}W_5 = 0, \qquad k_1W_4 + \tfrac{k_2}{2\cdot 5!}W_3 = 0, \qquad k_1W_6 + \tfrac{k_2}{5!}W_2 = 0. \quad (14)$$

C) Equations (8)–(14) possess solutions.

We observe that the cross-interaction terms,

$$g\left(k_1\phi^{\mu\nu} - \tfrac{k_2}{20}\tilde{K}^{\mu\nu}\right)F_{\mu\nu}, \quad (15)$$

are only of order one in the deformation parameter and couple the tensor field $t_{\lambda\mu|\alpha}$ to the two-form $\phi_{\mu\nu}$ and to the three-form $K^{\mu\nu\rho}$ from the BF sector.

Also, it is interesting to see that the interaction components

$$\tfrac{3g^2}{2}\left(k_1\phi^{\mu\nu}-\tfrac{k_2}{20}\tilde{K}^{\mu\nu}\right)\left(k_1\phi_{\mu\nu}-\tfrac{k_2}{20}\tilde{K}_{\mu\nu}\right) \tag{16}$$

which describe self-interactions in the BF sector, are strictly due to the presence of the tensor $t_{\lambda\mu|\alpha}$ (in its absence $k_1=k_2=0$, so they would vanish).

Theorem 2
A) Action (7) is invariant under the deformed gauge transformations

$$\bar{\delta}_\Omega\varphi=-gW_1\epsilon, \tag{17}$$

$$\begin{aligned}\bar{\delta}_\Omega H^\mu &= 2D_\nu\epsilon^{\mu\nu}+g\left(\frac{dW_1}{d\varphi}H^\mu-3\frac{dW_3}{d\varphi}K^{\mu\nu\rho}\phi_{\nu\rho}\right)\epsilon-3g\frac{dW_2}{d\varphi}\phi_{\nu\rho}\epsilon^{\mu\nu\rho}\\ &+2g\left(\frac{dW_2}{d\varphi}B^{\mu\nu}-3\frac{dW_3}{d\varphi}K^{\mu\nu\rho}V_\rho\right)\xi_\nu+12g\frac{dW_3}{d\varphi}V_\nu\phi_{\rho\lambda}\xi^{\mu\nu\rho\lambda}\\ &+2g\frac{dW_6}{d\varphi}B^{\mu\nu}\varepsilon_{\nu\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta}+3gK^{\mu\nu\rho}\left(4\frac{dW_4}{d\varphi}V_\nu\varepsilon_{\rho\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta}-\frac{dW_6}{d\varphi}\varepsilon_{\nu\rho\alpha\beta\gamma}\epsilon^{\alpha\beta\gamma}\right)\\ &+g\varepsilon^{\mu\nu\rho\lambda\sigma}\left[\tfrac{1}{4}\frac{dW_4}{d\varphi}\varepsilon_{\nu\rho\alpha\beta\gamma}K^{\alpha\beta\gamma}\varepsilon_{\lambda\sigma\alpha'\beta'\gamma'}K^{\alpha'\beta'\gamma'}\epsilon-\frac{dW_5}{d\varphi}\phi_{\nu\rho}\left(V_\lambda\xi_\sigma-\tfrac{1}{4}\phi_{\lambda\sigma}\epsilon\right)\right],\end{aligned} \tag{18}$$

$$\bar{\delta}_\Omega V_\mu=\partial_\mu\epsilon-2gW_2\xi_\mu-2g\varepsilon_{\mu\nu\rho\lambda\sigma}W_6\xi^{\nu\rho\lambda\sigma}, \tag{19}$$

$$\begin{aligned}\bar{\delta}_\Omega B^{\mu\nu} &= -3\partial_\rho\epsilon^{\mu\nu\rho}-2gW_1\epsilon^{\mu\nu}+6gW_3\left(2\phi_{\rho\lambda}\xi^{\mu\nu\rho\lambda}+K^{\mu\nu\rho}\xi_\rho\right)\\ &\quad+g\left(12W_4K^{\mu\nu\rho}\varepsilon_{\rho\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta}-W_5\varepsilon^{\mu\nu\rho\lambda\sigma}\phi_{\rho\lambda}\xi_\sigma\right),\end{aligned} \tag{20}$$

$$\begin{aligned}\bar{\delta}_\Omega\phi_{\mu\nu} &= D^{(-)}_{[\mu}\xi_{\nu]}+3g\left(W_3\phi_{\mu\nu}\epsilon-2W_4V_{[\mu}\varepsilon_{\nu]\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta}\right)\\ &\quad+3g\varepsilon_{\mu\nu\rho\lambda\sigma}\left(2W_4K^{\rho\lambda\sigma}\epsilon+W_6\epsilon^{\rho\lambda\sigma}-\tfrac{k_2}{180}\partial^{[\rho}\chi^{\lambda\sigma]}\right),\end{aligned} \tag{21}$$

$$\begin{aligned}\bar{\delta}_\Omega K^{\mu\nu\rho} &= 4D^{(+)}_\lambda\xi^{\mu\nu\rho\lambda}-3g\left(W_2\epsilon^{\mu\nu\rho}+W_3K^{\mu\nu\rho}\epsilon\right)-2gk_1\partial^{[\mu}\chi^{\nu\rho]}\\ &\quad-g\varepsilon^{\mu\nu\rho\lambda\sigma}W_5\left(V_\lambda\xi_\sigma-\tfrac{1}{2}\phi_{\lambda\sigma}\epsilon\right),\end{aligned} \tag{22}$$

$$\bar{\delta}_\Omega t_{\mu\nu|\alpha}=\partial_{[\mu}\theta_{\nu]\alpha}+\partial_{[\mu}\chi_{\nu]\alpha}-2\partial_\alpha\chi_{\mu\nu}+gk_1\sigma_{\alpha[\mu}\xi_{\nu]}-\tfrac{gk_2}{5!}\sigma_{\alpha[\mu}\varepsilon_{\nu]\beta\gamma\delta\varepsilon}\xi^{\beta\gamma\delta\varepsilon}, \tag{23}$$

where

$$D_\nu = \partial_\nu - g\frac{dW_1}{d\varphi}V_\nu, \qquad D_\nu^{(\pm)} = \partial_\nu \pm 3gW_3V_\nu. \tag{24}$$

B) The algebra of the gauge transformations (17)–(23) is non-Abelian and closes on-shell, where on-shell means on the stationary surface of field equations for action (7).
C) The reducibility relations corresponding to the gauge transformations (17)–(23) hold on-shell.

2.2 Couplings between a BF model and a massless Rarita-Schwinger field

We start from a free four-dimensional theory whose Lagrangian action is written as the sum between the action for a massless Rarita-Schwinger field and the action for a topological BF theory involving one scalar field, two one-forms and one two-form

$$\begin{aligned} S_0[\psi_\mu, A^\mu, H^\mu, \varphi, B^{\mu\nu}] &= \int d^4x \left(-\frac{\mathrm{i}}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho + H_\mu\partial^\mu\varphi + \frac{1}{2}B^{\mu\nu}\partial_{[\mu}A_{\nu]}\right) \\ &\equiv \int d^4x \left(\mathcal{L}_0^{\mathrm{RS}} + \mathcal{L}_0^{\mathrm{BF}}\right). \end{aligned} \tag{25}$$

We work with a Minkowski-flat metric tensor of 'mostly minus' signature $\sigma^{\mu\nu} = \sigma_{\mu\nu} = (+---)$ and employ the Majorana representation of the Clifford algebra

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\sigma_{\mu\nu}\mathbf{1}. \tag{26}$$

This means that all the γ-matrices are purely imaginary, with γ_0 Hermitian and γ_i anti-Hermitian

Action (25) is found invariant under the gauge transformations

$$\delta_\epsilon A^\mu = \partial^\mu\epsilon, \qquad \delta_\epsilon H^\mu = 2\partial_\nu\epsilon^{\mu\nu}, \qquad \delta_\epsilon B^{\mu\nu} = -3\partial_\rho\epsilon^{\mu\nu\rho}, \tag{27}$$

$$\delta_\epsilon\varphi = 0, \qquad \delta_\chi\psi_\mu = \partial_\mu\chi, \tag{28}$$

where the gauge parameters ϵ, $\epsilon^{\mu\nu}$, and $\epsilon^{\mu\nu\rho}$ are bosonic, with $\epsilon^{\mu\nu}$ and $\epsilon^{\mu\nu\rho}$ completely antisymmetric. In addition, the gauge parameter χ is a Majorana spinor. The gauge transformations (27)–(28) are Abelian and off-shell, second-order reducible.

The second main result is synthesized by the following theorems.

Theorem 3
Under the above mentioned hypotheses, the most general form of the action that describes the interacting theory reads as

$$\tilde{S}_0[A^\mu, H^\mu, \varphi, B^{\mu\nu}, \psi_\mu] = \int d^4x \, [H_\mu \left(\partial^\mu \varphi - gW(\varphi) A^\mu\right) \\ + \frac{1}{2} B^{\mu\nu} \partial_{[\mu} A_{\nu]} - \frac{\mathrm{i}}{2} \left(\bar{\psi}_\mu \gamma^{\mu\nu\rho} \left(\partial_\nu + g\mathrm{i}\gamma_5 A_\nu U_1\right) \psi_\rho\right) \Big], \tag{29}$$

where U_1 is an arbitrary function depending only on the undifferentiated scalar field.

We remark that the interaction vertices from (29), namely,

$$\mathrm{i}g\bar{\psi}_\mu \gamma^{\mu\nu\rho} \gamma_5 A_\nu U_1 \psi_\rho, \tag{30}$$

are of order one in the deformation parameter g.

Theorem 4
A) Action (29) is invariant under the deformed gauge transformations

$$\bar{\delta}_\epsilon \varphi = gW(\varphi) \epsilon, \tag{31}$$

$$\bar{\delta}_{\epsilon,\chi} H^\mu = 2 \left(\partial_\nu + g\frac{dW}{d\varphi} A_\nu\right) \epsilon^{\mu\nu} + g \left[\left(\frac{1}{2} \frac{dU_1}{d\varphi} \bar{\psi}_\nu \gamma^{\mu\nu\rho} \gamma_5 \psi_\rho \right.\right. \\ \left.\left. - \frac{dW}{d\varphi} H^\mu\right) \epsilon + \frac{dU_1}{d\varphi} A_\rho \bar{\psi}_\nu \gamma^{\mu\nu\rho} \gamma_5 \chi\right], \tag{32}$$

$$\bar{\delta}_\epsilon A^\mu = \partial^\mu \epsilon, \tag{33}$$

$$\bar{\delta}_{\epsilon,\chi} B^{\mu\nu} = -3\partial_\rho \epsilon^{\mu\nu\rho} + 2gW(\varphi) \epsilon^{\mu\nu} - gU_1(\varphi) \bar{\psi}_\rho \gamma^{\mu\nu\rho} \gamma_5 \chi, \tag{34}$$

$$\bar{\delta}_{\epsilon,\chi} \psi_\mu = \partial_\mu \chi - \mathrm{i}gU_1(\varphi) \left(\gamma_5 \psi_\mu \epsilon - \gamma_5 \chi A_\mu\right). \tag{35}$$

B) The algebra of the gauge transformations (31)–(35) is non-Abelian and closes on-shell, where on-shell means on the stationary surface of field equations for action (29).
C) The reducibility relations corresponding to the gauge transformations (31)–(35) hold on-shell.

2.3 Published papers

The main results of the thesis are published in the papers [5]–[10].

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