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Summary of Ph.D. THESIS

COHOMOLOGICAL APPROACHES TO EINSTEIN-HILBERT GRAVITY

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SUMMARY

Theories involving one or several spin-two fields have raised a constant interest over the last thirty years, especially at the level of direct or intermediated graviton interactions. In this context more results on the impossibility of consistent cross-couplings among different gravitons have been obtained, either without other fields or in the presence of a scalar field, or respectively of a Dirac spinor. All these no-go results have been deduced under some specific hypotheses, always including the preservation of the derivative order of each field equation with respect to its free limit (derivative order assumption). Through their implications, these findings support the common belief that the only consistent interactions in graviton theories require a single spin-two field and are subject to the standard prescriptions of General Relativity (meaning diffeomorphisms for the gauge transformations of the graviton and diffeomorphism algebra for the gauge algebra of the interacting theory). This idea is also strengthened by the confirmation of the uniqueness of Einstein-Hilbert action having the Pauli-Fierz model as its free limit or the uniqueness of $N = 1$, $D = 4$ SUGRA action allowing for a Pauli-Fierz field and a massless Rarita-Schwinger spinor as the corresponding uncoupled limit. Our work submits to the same topic, of constructing spin-two field(s) couplings. We employ a systematic approach to the construction of interactions in gauge theories, based on the cohomological reformulation of Lagrangian BRST symmetry. In this approach interactions result from the analysis of consistent deformations of the generator of the Lagrangian BRST symmetry (known as the solution of the master equation) by means of specific cohomological techniques, relying on local BRST cohomology. The emerging deformations, and hence also the interactions, are constructed under the general hypotheses of locality, smoothness in the coupling constant, Poincaré invariance, Lorentz covariance, and derivative order assumption.

The thesis is structured in 6 chapters.

The first chapter is an introductive one and presents briefly the importance of the subject under study and gives a general overview of the original results derived in the next chapters.

The second chapter presents the esence of the mathematical method used, the basic working equations and their interpretation. Here we aproached briefly the problem of constructing consistent interactions between gauge theories by deforming the solution to the master equation. The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution to the master equation for the initial theory can be deformed into the solution of the master equation for the interacting theory such that the master equation remains valid. This chapter serves as a reference for the next three chapters.

Our third chapter submits to the topic of constructing spin-two field(s) couplings, initially in the presence of a massless vector field and then of a p-form, with $p > 1$. We employ a systematic approach to the construction of interactions in gauge theories, based on the cohomological reformulation of Lagrangian BRST symmetry. In this approach interactions result from the analysis of consistent deformations of the generator of the Lagrangian BRST symmetry (known as the solution of the master equation) by means of specific cohomological techniques, relying on local BRST cohomology. The emerging deformations, and hence also the interactions, are constructed under the general hypotheses of locality, smoothness in the coupling constant, Poincaré invariance, Lorentz covariance, and derivative order assumption.

In this specific situation the derivative order assumption requires that the interaction vertices contain at most two spacetime derivatives of the fields, but does not restrict the polynomial order in the undifferentiated fields either in the Lagrangian or in the gauge symmetries. Our analysis envisages three steps, which introduce gradually the situations under investigation, according to the complexity of their cohomological content.

Initially, we consider the case of couplings between a single Pauli-Fierz field and a massless vector field. In this setting we compute the coupling terms to order two in the coupling constant k and find two distinct solutions. The first solution leads to the full cross-coupling Lagrangian in all $D > 2$

$$\begin{aligned} \mathcal{L}_{\mathrm{I}}^{(\mathrm{int})} &= -\frac{1}{4}\sqrt{-g}g^{\mu\nu}g^{\rho\lambda}\bar{F}_{\mu\rho}\bar{F}_{\nu\lambda} + k\left(q_1\delta_3^D\varepsilon^{\mu_1\mu_2\mu_3}\bar{V}_{\mu_1}\bar{F}_{\mu_2\mu_3}\right. \\ &\left. +q_2\delta_5^D\varepsilon^{\mu_1\mu_2\mu_3\mu_4\mu_5}\bar{V}_{\mu_1}\bar{F}_{\mu_2\mu_3}\bar{F}_{\mu_4\mu_5}\right), \end{aligned}$$

which respects the standard rules of General Relativity. The second solution is more unusual: it 'lives' only in $D = 3$, produces polynomials of order two in the coupling constant (and not series, like in the first case), and the the couplings are mixing-component terms that can be written in terms of a deformed field strength (of the massless vector-field) as

$$\mathcal{L}_{\mathrm{II}}^{(\mathrm{int})} = -\frac{1}{4}F'^{\mu\nu}F'_{\mu\nu}, \qquad F'_{\mu\nu} = F_{\mu\nu} + 2k\varepsilon_{\mu\nu\rho}\partial^{[\theta}h^{\rho]}{}_{\theta}.$$

By contrast to General Relativity, where all the gauge symmetries are deformed, here only those of the vector field are modified by terms of order one in the coupling constant that involve the Pauli-Fierz gauge parameters, while the spin-two field keeps its original gauge symmetries, namely the linearized version of diffeomorphisms. To our knowledge, *this is the first situation where the linearized version of the spin-two field allows for non-trivial couplings, other than those subject to General Relativity, which fulfill all the working hypotheses, including that on the derivative order.*

Next, we focus on the investigation of cross-interactions among different gravitons intermediated by a massless vector field. In view of this, we start from a finite sum of Pauli-Fierz actions with a positively defined metric in internal space and a massless vector field. The cohomological analysis reveals again two cases. The former is related to the standard graviton-vector field interactions from General Relativity and exhibits no consistent cross-interactions among different gravitons (with a positively defined metric in internal space) in the presence of a massless vector field. At most one graviton can be coupled to the vector field via a Lagrangian similar to $\mathcal{L}_{\mathrm{I}}^{(\mathrm{int})}$, while each of the other spin-two fields may interact only with itself through an Einstein-Hilbert action with a cosmological term. The latter case seems to describe some new type of couplings in $D = 3$, which appear to allow for cross-couplings among different gravitons. The coupled Lagrangian is, like in the case of a single graviton, a polynomial of order two in the coupling constant, obtained by deforming the vector field strength

$$\hat{\mathcal{L}}_{\mathrm{II}}^{(\mathrm{int})} = -\frac{1}{4}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu}, \qquad \hat{F}^{\mu\nu} = F^{\mu\nu} + 2k\varepsilon^{\mu\nu\rho}\sum_{A=1}^{n}\left(y_3^A\partial_{[\theta}h_{\rho]}^{A\ \theta}\right),$$

where y_3^A are some arbitrary, nonvanishing real constants. Nevertheless, these cross-couplings can be decoupled through an orthogonal, linear transformation of the spin-two fields, in terms of which $\hat{\mathcal{L}}_{\rm II}^{(\rm int)}$ becomes nothing but $\mathcal{L}_{\rm II}^{(\rm int)}$, with $h_{\mu\nu}$ replaced for instance by the first transformed spin-two field from the collection. In consequence, these case also leads to no indirect cross-couplings between different gravitons.

Then, we show that all the new results obtained in the case a massless vector field can be generalized to an arbitrary p-form. More precisely, if one starts from a free action describing an Abelian p-form and a single Pauli-Fierz field, then *it is possible to construct some new deformations in* $D = p + 2$ *that are consistent to all orders in the coupling constant and are not subject to the rules of General Relativity.* It is important to remark that all the working hypotheses, including the derivative order assumption, are fulfilled. There are several physical consequences of these couplings, such as the appearance of a constant linearized scalar curvature if one allows for a cosmological term or the modification of the initial $(p+1)$-order conservation law for the p-form by terms containing the spin-two field. Regarding a collection of spin-two fields, we find that the deformed Lagrangian does not allow for cross-couplings between different gravitons intermediated by a p-form, either in the setting of General Relativity or in the special, $(p+2)$-dimensional situation. The original results are contained in papers [2] and [6].

In the fourth chapter we focused on the investigation of the consistent interactions between a collection of massless tensor gauge fields, each with the mixed symmetry of a two-column Young diagram of the type $(3,1)$, and one vector field, respectively one p-form gauge field. It is worth mentioning the duality of a free massless tensor gauge field with the mixed symmetry $(3,1)$ to the Pauli-Fierz theory in $D = 6$ dimensions. This paper generalizes our results regarding the cross-interactions between a single massless $(3,1)$ field and a vector field. We find a deformation of the solution to the master equation that provides nontrivial cross-couplings. This case corresponds to a $p+4$-dimensional spacetime and is described by a deformed solution that stops at order two in the coupling constant. The interacting Lagrangian action contains only mixing-component terms of order one and two in the coupling constant, *but only one mixed symmetry tensor field from the collection gets coupled to the p-form, while the others remain free.* At the level of the gauge transformations, only those of the p-form are modified at order one in the coupling constant with a term linear in the antisymmetrized first-order derivatives of a single gauge parameter from the $(3,1)$ sector such that the gauge algebra and the reducibility structure of the coupled model are not modified during the deformation procedure, being the same like in the case of the starting free action. Our result is interesting since it *exhibits strong similarities to the Einstein gravitons from General Relativity, in the sense that no nontrivial cross-couplings between different fields with the mixed symmetry* $(3,1)$ *are allowed, neither direct nor intermediated by a p-form.* The original results can be found in papers [3] and [5].

The main aim of the fifth chapter is to investigate the cross-couplings among several massless spin-two fields (described in the free limit by a sum of Pauli-Fierz actions) in the presence of a massive Rarita-Schwinger field. More precisely, under the hypotheses of locality, smoothness of the interactions in the coupling constant, Poincaré invariance, (background) Lorentz invariance, and the preservation of the number of derivatives on each field, we prove that there are no consistent cross-interactions among different gravitons with a positively defined metric in internal space in the presence of a massive Rarita-Schwinger field. This result is obtained by using the deformation technique combined with the local BRST cohomology. It is well-known the fact that the spin-two field in metric formulation

(Einstein-Hilbert theory) cannot be coupled to a spin-3/2 field. However, as it will be shown below, if we decompose the metric like $g_{\mu\nu} = \sigma_{\mu\nu} + \lambda h_{\mu\nu}$, where $\sigma_{\mu\nu}$ is the flat metric and λ is the coupling constant, then we can indeed couple the massive spin-3/2 field to $h_{\mu\nu}$ in the space of formal series with the maximum derivative order equal to one in $h_{\mu\nu}$. Thus, our approach envisages two different aspects. One is related to the couplings between the spin-two fields and one massive Rarita-Schwinger field, while the other focuses on proving the impossibility of cross-interactions among different gravitons via a single massive Rarita-Schwinger field. In order to make the analysis as clear as possible, we initially consider the case of the couplings between a single Pauli-Fierz field and a massive Rarita-Schwinger field. In this setting we compute the interaction terms to order two in the coupling constant. Next, we prove the isomorphism between the local BRST cohomologies corresponding to the Pauli-Fierz theory and respectively to the linearized version of the vierbein formulation of the spin-two field. Since the deformation procedure is controlled by the local BRST cohomology of the free theory (in ghost number zero and one), the previous isomorphism allows us to translate the results emerging from the Pauli-Fierz formulation into the vierbein version and conversely. In this manner we obtain that the first two orders of the interacting Lagrangian resulting from our setting originate in the development of the full interacting Lagrangian

$$
\begin{aligned}
\mathcal{L}^{(\mathrm{int})} &= \frac{e}{2}\left(-\mathrm{i}\bar{\psi}_{\mu} e_{a}{}^{\mu} e_{b}{}^{\nu} e_{c}{}^{\rho} \gamma^{abc} D_{\nu}\psi_{\rho} + m\bar{\psi}_{\mu} e_{a}{}^{\mu}\gamma^{ab} e_{b}{}^{\nu}\psi_{\nu}\right) \\
&+\lambda\left[eV\left(X,Y,Z\right) + d_{1}\left(X,Y,Z\right) e_{a}{}^{\nu}\bar{\psi}_{\nu}\gamma^{a} D_{\mu}\left(e\psi^{\mu}\right)\right. \\
&\left.+ed_{2}\left(X,Y,Z\right)\left(\bar{\psi}^{\mu}\gamma^{b} + e_{a}{}^{\mu} e^{b}{}_{\rho}\bar{\psi}^{\rho}\gamma^{a}\right) D_{\mu}\left(e_{b}{}^{\nu}\psi_{\nu}\right)\right].
\end{aligned}
$$

Here, $e_{a}{}^{\mu}$ represent the vierbein fields, e is the inverse of their determinant, $e = \left(\det\left(e_{a}{}^{\mu}\right)\right)^{-1}$, D_{μ} signifies the full covariant derivative, and γ^{a} stand for the flat Dirac matrices. The fields ψ_{ν} denote the (curved) Rarita-Schwinger spinors ($\psi_{\nu} = e^{a}{}_{\nu}\psi_{a}$). The quantities denoted by V, d_1, and d_2 are arbitrary polynomials of $X \equiv \bar{\psi}_{a}\psi^{a}$, $Y \equiv \bar{\psi}_{a}\gamma^{ab}\psi_{b}$, and $Z = \mathrm{i}\bar{\psi}_{a}\gamma_{5}\psi^{a}$. Here and in the sequel λ is the coupling constant (deformation parameter). We observe that the first two terms in $\mathcal{L}^{(\mathrm{int})}$ describe the standard minimal couplings between the spin-two and massive Rarita-Schwinger fields. The last terms from $\mathcal{L}^{(\mathrm{int})}$, namely those proportional with V, d_1, or d_2, produce non-minimal couplings. To our knowledge, these non-minimal interaction terms are not discussed in the literature. However, they are consistent with the gauge symmetries of the Lagrangian $\mathcal{L}_2 + \mathcal{L}^{(\mathrm{int})}$, where $\mathcal{L}_2$ is the full spin-two Lagrangian in the vierbein formulation. With this result at hand, we start from a finite sum of Pauli-Fierz actions with a positively defined metric in internal space and a massive Rarita-Schwinger field, and prove that there are no consistent cross-interactions between different gravitons in the presence of such a fermionic matter field. The original results briefly described above are treated in papers [1] and [4].

The last chapter draws the conclusions of the thesis.

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