Multisoliton solutions for the KMN Equation

Rodica Cimpoiasu, Aurelia Florian
Department of Physics, University of Craiova
13 A.I.Cuza, 200585 Craiova, Romania

27 August 2020

Abstract

In this paper we are using two important methods that could offer information on the dynamics of systems described through nonlinear differential equations. One of the method is the Lie symmetry method and the other one is an expansion method in terms of the well-known solutions for some auxiliary equations. Both methods will be illustrated on the Kundu-Mukherjee-Naskar (KMN) model, a nonlinear partial differential equation which appears in many fields, as optics, hydrodynamics or superfluid films. New solutions of soliton type for the model could be generated using the two methods.

1 Introduction

The nonlinear phenomena are most frequent in nature as the linear behavior. They are usualy described by nonlinear differential equations, so that solving nonlinear partial differential equations (NPDEs) is a quite important issue. There are not clear prescriptions on how to solve such equations, and, moreover, it is not clear if a considered equation can be integrated or not. There are many approaches related to the integrability: the inverse scattering transformation [**?**], the Hirota bilinear method [**?**], and, not at the end, the symmetry analysis [**?**, **?**, **?**]. The NPDs could accept many types of solutions, depending on the initial conditions or on the values of the parameters appearing in the equation. Ones of these solutions are known as "traveling wave solutions" and there are many methods that can be used to get them. All these methods suppose three main steps: (i) transformation of the NPDE into an nonlinear ordinary differential equation (NODE) by passing to the wave variable; (ii) choice of an auxiliary equation, that is an ODE with well-known solutions; (iii) looking to a solution of the NODE in terms of the solutions of the auxiliary equation. It is important to underline that the step (ii) could generate a dependence of the traveling wave solutions on the choice of the auxiliary equation, while step (iii) could give different solutions depending on the solving method, that is on the mathematical form we chose for the NODE solutions.

Speaking about auxiliary equations, the most frequent choice is the Riccati equation. Other options are represented by various other ODEs, linear or nonlinear, of first or of higher order of differentiability. Here we will consider three such equations: two first order equations and one of the second order. Their mathematical form and their solutions are presented in the forthcoming section of the paper.

Related to the solving methods, there are many direct methods allowing to get traveling wave solutions, as for example: *the new extended direct algebraic method* [6, **?**], *the novel* (G'/G)*-expansion method* [7, **?**], *generalized* (G'/G)*-expansion method* [8], *the Jacobi elliptic functions* [10], *the extended trial equation method* [11, **?**], *the modified Khater method* [**?**], *the generalized Kudryashov method* [**?**, **?**], *the exp-function method* [9], or *the functional expansion* introduced in [**?**]. This paper will deal with the last two methods, both of them supposing to find solutions of a given NODE as expansion of the known solutions of an auxiliary equation. The main features of the two methods will be underlined in the third section of the paper.

The aim of this paper consists in discussing how solutions could depend on the two important factors we mentioned: the choices of the auxiliary equation and the form of the expansion considered for expressing the solution of the investigated NPDE. We will analyze the most appropiate way that should be used to solve a given NPDE, and we will compare the solutions obtained by using the two methods and the three auxiliary equations. The model to be effectively investigated will be the Kundu-Mukherjee-Naskar (KMN) model. The KMN model appears in many fields and describes many types of phenomena, from optics to the flows in superfluid films [**?**]. It is given by the following equation:

$$iq_t + \alpha q_{xy} + i\beta q(qq_x^* - q^* q_x) = 0, \tag{1}$$

where $q(x, y, t)$ represents an complex variable defined in a $(2D+1)$ space-time. The equation was investigated through many of the before mentioned methods, as for example: through the extended trial function method [**?**], the modified simple equation approach [**?**], the new extended algebraic method [**?**], the multiplier and sine-Gordon expansion methods [**?**], and through the novel extended rational sinh-Gordon equation expansion technique [**?**]. Similar approaches can be mentioned for other equations, with important application in physics Similar approaches can be mentioned for other equations, with important application in physicsQTOunrecognized[**?**, **?**,]

T. B. Benjamin, J. L Bona, J. J. .Mahony, "Model equations for long waves in nonlinear dispersive systems", Society Philosophical Transactions Mathematical Physical & Engineering Sciences, (1972) Vol. 272.

References

[1] A.M. Wazwaz, "New travelling wave solutions of different physical structures to generalized BBM equation", Physics Letters A: General, Atomic and Solid State Physics, (2006) Vol. 355, 358-362.

[2] W. Hereman, "Shallow water waves and solitary waves," Mathematics of Complexity and Dynamical Systems, (2011), 1520–1532.

[3] P. A. Clarkson and M. D. Kruskal, New similarity solutions of the Boussinesq equation, J. Math. Phys. 30 (1989) 2201–2213.

[4] Q. M. Liu and A. S. Fokas, New similarity solutions of the Boussinesq equation, J. Math. Phys. 37 (1996) 3241–3850.

[5] T. Heimburg and A. D. Jackson, On soliton propagation in biomembranes and nerves, Proc. Natl. Acad. Sci. USA 102 (2005) 9790–9795.

[6] Cimpoiasu Rodica, Nerve pulse propagation in biological membranes — Solitons and other invariant solutions, International Journal of Biomathematics, (2016) Vol. 09

[7] G. B. Whitham, Variational methods and applications to water wave, Prec. Roy. Soc. London, Ser. A, (1967) Vol. 299 6-25.

[8] L.J. Broer, Approximate equations for long water waves, Appl. Sci. Res., (1975) Vol. 31, 377-395.

[9] D.J. Kaup, A higher-order water wave equation and method for solving it, Prog. Theor. Phyx., (1975) Vol.54, 396-408.

[10] E. Fan, H. Zhang, Backlund transformation and exact solutions for Whitham-Broer-Kaup equations in shallow water, Applied Mathematics and Mechanics, (1998) Vol.19, 713-716.

[11] Guiqiong Xu; Zhibin Li, Exact travelling wave solutions of the Whitham–Broer–Kaup and Broer–Kaup–Kupershmidt equations, Chaos, Solitons & Fractals, (2005), Vol. 24, 549-556