Solitonic solutions for RK-RLW equation with different order of nonlinearity

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In this poster we investigate the solutions of the important nonlinear partial differential equation **Rosenau-Kawahara-RLW.**

By coupling the generalized Rosenau-RLW equation with the generalised Rosenau-Kawahara equation the generalised Rosenau-Kawahara-RLW equation is obtained [6]:

$$u_t + u_x + u_x u^n + u_{4xt} + u_{3x} - u_{2xt} - u_{5x} = 0 \qquad (1)$$

We will study this equation in the form of $u_t - \alpha u_{2xt} + \beta u_{4xt} + \gamma u_x + \delta u^n u_x + \tau u_{3x} + \lambda u_{5x} = 0$ which multiplied with $2u$ gives:

$$u_t - \alpha u_{2xt} + \beta u_{4xt} + \gamma u_x + \delta u^n u_x + \tau u_{3x} + \lambda u_{5x} = 0 \qquad (2)$$

After integration in relation to x between the limits $x_1 < 0$ and $x_2 > 0$ large enough, so that if the solution is a soliton, its peak is placed between the two values x_1 and x_2, and $u(x_1,t) = u(x_2,t) = 0, \forall t$.

By doing the calculations we obtain (energy conservation law):

$$\frac{\partial}{\partial t}\int_{x_1}^{x_2}\left(u^2 + \alpha(u_x)^2 + \beta(u_{2x})^2\right)dx = 0 \qquad (3)$$

The expression $E = \int_{x_1}^{x_2}\left(u^2 + \alpha(u_x)^2 + \beta(u_{2x})^2\right)dx$ is constant over time.

We can reduce the number of degrees of fredom in the previous equation, by transforming the NPDE into a NODE, by using as unique variable $\xi = x - vt$:

$$(\gamma - v)u_\xi + (\alpha v + \tau)u_{3\xi} + (\lambda - \beta v)\,u_{5\xi} + \frac{\delta}{n+1}(u^{n+1})_\xi = 0 \qquad (4)$$

By integrating (4) in relation to ξ and vanishing the integration constant, we get:

$$(\gamma - v)u + (\alpha v + \tau)u_{2\xi} + (\lambda - \beta v)\,u_{4\xi} + \frac{\delta}{n+1}u^{n+1} = 0 \qquad (5)$$

We can apply the sin-cos method for the equation (5). Thus, we can express a general solution to this equation in the form:

$$u(\xi) = A\cos^\eta(B\xi) \qquad (6)$$

From the balance of the maximum powers of the cos function we find:

$$\eta - 4 = \eta(n+1) \Rightarrow \eta = -\frac{4}{n} \qquad (7)$$

We group the terms according to his powers $\cos(B\xi)$ and equaling zero coefficients. We get an algebraic system of 3 equations with unknowns A, B, v:

$$\begin{aligned}
\cos^\eta(B\xi) &: (\gamma - v)A - (\alpha v + \tau)AB^2\eta^2 + (\lambda - \beta v)\,AB^4\eta^4 = 0\\
\cos^{\eta-2}(B\xi) &: (\alpha v + \tau)AB^2\eta(\eta-1) - 2(\lambda - \beta v)\,AB^4\eta(\eta-1)\left(\eta^2 - 2\eta + 2\right) = 0\\
\cos^{\eta-4}(B\xi) &: (\lambda - \beta v)\,AB^4\eta(\eta-1)(\eta-2)(\eta-3) + \frac{\delta}{n+1}A^{n+1} = 0
\end{aligned} \qquad (8)$$

The solutions of the equation system are:

$$\begin{aligned}
B &= \pm\sqrt{\frac{(\alpha v + \tau)}{2(\lambda - \beta v)(\eta^2 - 2\eta + 2)}}\\
A &= \left[-\frac{(n+1)\eta(\eta-1)(\eta-2)(\eta-3)(\alpha v + \tau)^2}{4\delta(\lambda - \beta v)(\eta^2 - 2\eta + 2)^2}\right]^{1/n}\\
v &= \frac{-b \pm \sqrt{\Delta'}}{a},\ \text{unde}\ \Delta' = b^2 - ac,\ a = \alpha^2\eta^2(\eta^2-2)^2 - 4\beta(\eta^2 - 2\eta + 2)^2,\\
b &= \alpha\tau\eta^2(\eta^2-2)^2 + 2(\eta^2 - 2\eta + 2)^2(\gamma\beta + \lambda), c = \eta^2\tau^2(\eta^2-2)^2 - 4\gamma\lambda(\eta^2 - 2\eta + 2)^2
\end{aligned} \qquad (9)$$

Case 1

Parameters:

$$\begin{aligned}
n &= 2 \Rightarrow \eta = -2\\
\alpha &= 2, \tau = \beta = \gamma = \lambda = \delta = 1\\
v_1 &= 7\ ,\ A_1 = 3\frac{\sqrt{15}}{2}, B_1 = i\frac{\sqrt{2}}{4}\\
v_2 &= \frac{1}{3},\ A_2 = i\frac{\sqrt{15}}{2},\ B_2 = \frac{\sqrt{2}}{4}
\end{aligned}$$

Solutions:

$$\begin{aligned}
u_1(x,t) &= \frac{3}{2}\sqrt{15}\cos^{-2}\Big(i\frac{\sqrt{2}}{4}(x - 7t)\Big) = \frac{3}{2}\sqrt{15}\,\mathrm{sech}^2\left(\frac{\sqrt{2}}{4}(x - 7t)\right)\\
u_2(x,t) &= \sqrt{-\frac{15}{4}}\cos^{-2}\Big(\frac{\sqrt{2}}{4}(x - \frac{t}{3})\Big) = i\sqrt{\frac{15}{4}}\sec^{-2}\left(\frac{\sqrt{2}}{4}(x - \frac{t}{3})\right)
\end{aligned}$$

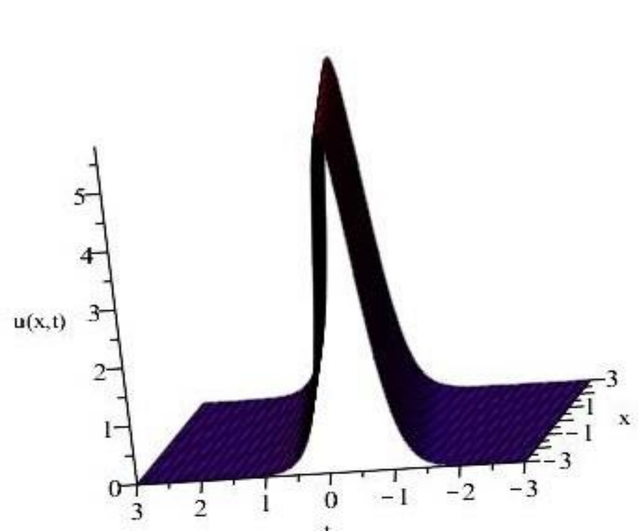

Fig. 1

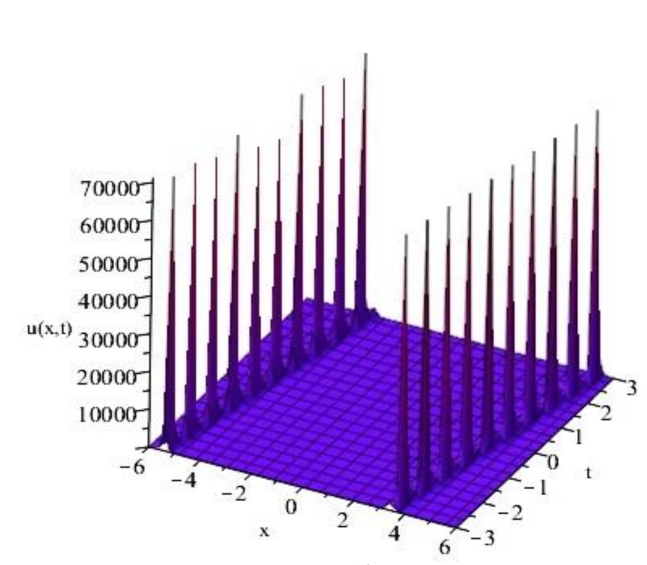

Fig. 2

Case 2

Parameters:

$$\begin{aligned}
n &= 1 \Rightarrow \eta = -4\\
\alpha &= 2, \tau = \beta = \gamma = \lambda = \delta = 1\\
v_3 &= \frac{7}{25},\ A_3 = -\frac{21}{10},\ B_3 = \frac{\sqrt{6}}{12}\\
v_4 &= 19\ ,\ A_4 = \frac{105}{2}, B_4 = i\frac{\sqrt{6}}{12}
\end{aligned}$$

Solutions:

$$\begin{aligned}
u_3(x,t) &= -\frac{21}{10}\cos^{-4}\left(\frac{\sqrt{6}}{12}(x - \frac{7}{25}t)\right)\\
u_4(x,t) &= \frac{105}{2}\cosh^{-4}\left(\frac{\sqrt{6}}{12}(x - 19t)\right)
\end{aligned}$$

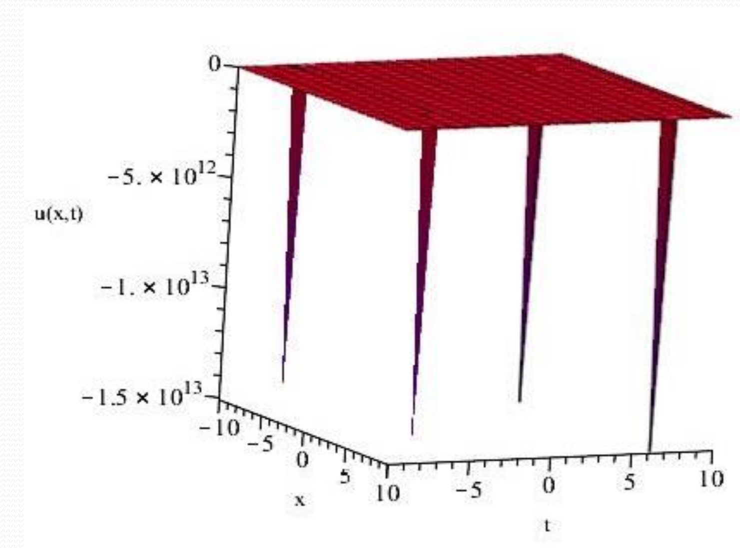

Fig. 3

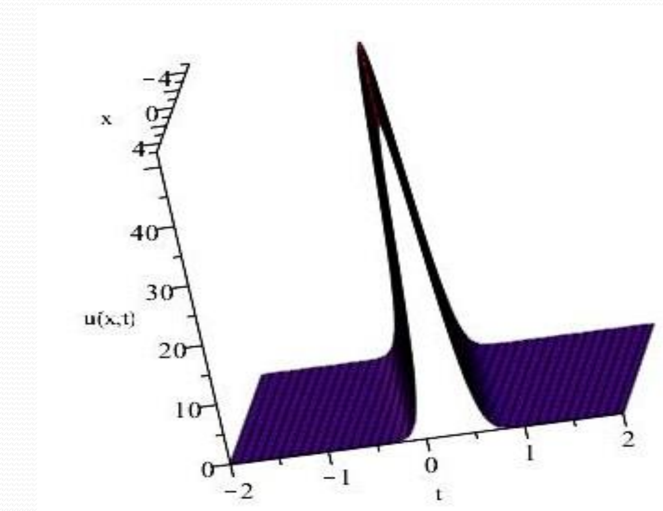

Fig. 4

Case 3

Parameters:

$$\begin{aligned}
n &= 3 \Rightarrow \eta = -4/3\\
\alpha &= 2, \tau = \beta = \gamma = \lambda = \delta = 1\\
v_5 &= \frac{19}{49}, A_5 = (-\frac{39}{7})^{1/3}, B_5 = \frac{3\sqrt{10}}{20}\\
v_6 &= \frac{13}{3}, A_6 = (\frac{91}{3})^{1/3}, B_6 = i\frac{3\sqrt{10}}{20}
\end{aligned}$$

Solutions:

$$\begin{aligned}
u_5(x,t) &= (-\frac{39}{7})^{1/3}\cos^{-4/3}\left(\frac{3\sqrt{10}}{20}(x - \frac{19}{49}t)\right)\\
u_6(x,t) &= (\frac{91}{3})^{1/3}\cosh^{-4/3}\left(\frac{3\sqrt{10}}{20}(x - \frac{13}{3}t)\right)
\end{aligned}$$

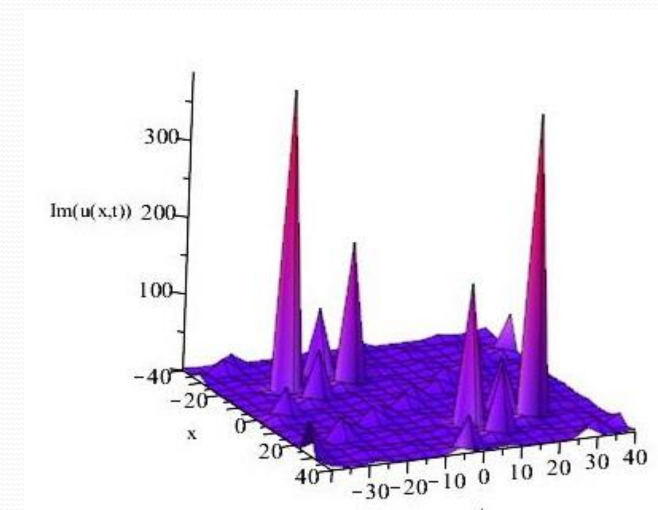

Fig. 5

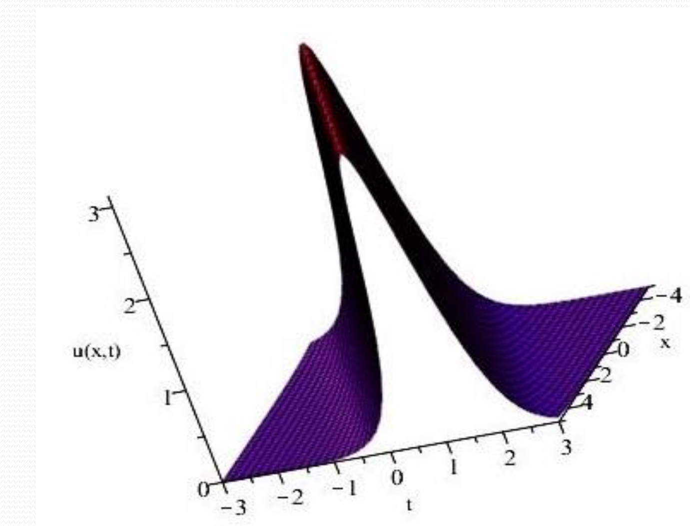

Fig. 6

Case 4

Parameters:

$$\begin{aligned}
n &= 4 \Rightarrow \eta = -1\\
\alpha &= 2, \tau = \beta = \gamma = \lambda = \delta = 1\\
v_7 &= \frac{13}{4}, A_7 = (30)^{1/4}, B_7 = \frac{i\sqrt{3}}{3}\\
v_8 &= \frac{7}{16}, A_8 = (-\frac{15}{2})^{1/4}, B_8 = \frac{\sqrt{3}}{3}
\end{aligned}$$

Solutions:

$$\begin{aligned}
u_7(x,t) &= (30)^{1/4}\cosh^{-1}\left(\frac{\sqrt{3}}{3}(x - \frac{13}{4}t)\right)\\
u_8(x,t) &= (-\frac{15}{2})^{1/4}\cos^{-1}\left(\frac{\sqrt{3}}{3}(x - \frac{7}{16}t)\right)
\end{aligned}$$

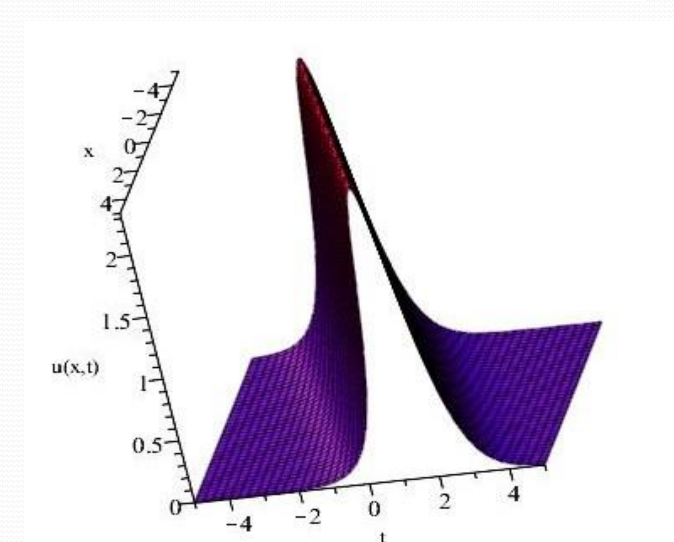

Fig. 7

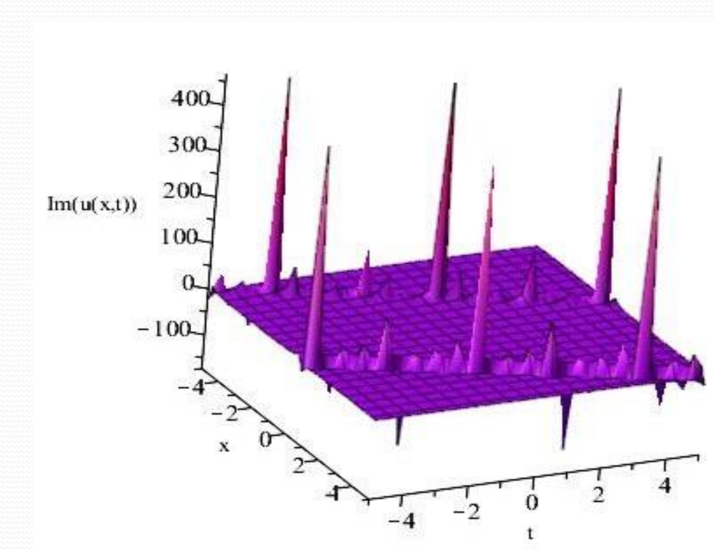

Fig. 8

References:

[1] Dongdong He, Kejia Pan, A linearly implicit conservative difference scheme for the generalized Rosenau-Kawahara-RLW equation, Applied Mathematics and Computation 271 (2015).

[2] Razborova, P., Triki, H., Biswas, A.: Perturbation of dispersive shallow water waves, Ocean Eng. 63, 1-7 (2013).

[3] P. Razborova, A. H. Kara,A. Biswas, Additional conservation laws for Rosenau--KdV--RLW equation with power law nonlinearity by Lie symmetry Nonlinear Dyn (2015) 79:743--748

[4] P. Rosenau, A quasi-continuous description of a nonlinear transmission line, Phys. Scr. 34 (1986) 827-829.

[5] X. Pan, L. Zhang, On the convergence of a conservative numerical scheme for the usual Rosenau-RLW equation, Appl. Math. Model. 36 (2012) 3371-3378.

[6] J.-M. Zuo, Y.-M. Zhang, T.-D. Zhang, F. Chang, A new conservative difference scheme for the general Rosenau-RLW equation, Boundary Value Prob. 2010 (2010) 13. Article ID 516260.

[7] A. Esfahani, Solitary wave solutions for generalized Rosenau-KdV Eq, Commun. Theor. Phys. 63 (2013) 1-7.

[8] P. Razborova, B. Ahmed, A. Biswas, Solitons, shock waves and conservation laws of Rosenau-KdV-RLW equation with power law nonlinearity, Appli. Math. Info. Sci., 8 (2014) 485-491.

[9] T. Kawahara, Oscillatory solitary waves in dispersive media, J. Phys. Soc. Japan 33, (1972) 260-26

[10] D. He, K. Pan ,A linearly implicit conservative difference scheme for the generalized Rosenau--Kawahara-RLWequation, Applied Mathematics and Computation (2015),

[11] J.-M. Zuo, Solitons and periodic solutions for the Rosenau-KdV and Rosenau-Kawahara equations, Appl. Math. Comput. 215 (2) (2009) 835-840.

[12] J. Hu, Y. Xu, B. Hu and X. Xie, Two Conservative Difference Schemes for Rosenau-Kawahara Equation, Adv. Math. Phys. 2014 (2014) 11. Article ID 217393.

[13] A. Biswas, H. Triki, and M. Labidi, Bright and dark solitons of the Rosenau-Kawahara equation with power law nonlinearity, Phys. Wave. Phenom. 19 (2011) 24-29.

[14] J.-M. Zuo, Soliton solutions of a general Rosenau-Kawahara-RLW equation, J. Math. Research, 7 (2015) 24-28.