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The Dimensions of M-theory

J.F. Vázquez-Poritz*
Physique Théorique et Mathématique
Université Libre de Bruxelles
Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium

Abstract

We review some key ideas developed during the search for a theory which unifies all fundamental phenomena. This search has culminated in M-theory, which is believed to live in eleven dimensions. How can we account for the missing seven dimensions? One possibility is that of dimensional compactification, by which dimensions are rolled up too small for us to observe. Another possibility is that all observable fields are confined to a four-dimensional hypersurface. The holographic principle together with a brane world scenario may be invoked to achieve the latter possibility.

1 The Road to M-theory

1.1 General Relativity and the Standard Model

Our current understanding of nature at the most fundamental level is contained within two theories, Einstein's theory of General Relativity and the Standard Model of particle physics. General Relativity describes gravity as a consequence of spacetime curvature. This theory correctly predicts physics on large scales, such as the bending of light by the Sun's gravitational field, the orbital motion of Mercury and the large-scale structure and dynamics of the entire universe.

On the other hand, the Standard Model agrees with experiments down to scales 100,000,000,000,000 smaller than are resolvable by the human eye. This theory unifies the electromagnetic, strong nuclear and weak nuclear forces. A quantum field theoretic description is employed, in which force-carrier particles exist in quantized packets of matter.

Gravity is normally insignificant for small-distance physics, the domain in which the Standard Model comes into play. However, nature does not completely decouple its phenomena into two separate regimes. In a strong gravitational field, such as in the vicinity of a black hole central singularity or during an early epoch of the universe, aspects of both curved spacetime and particle physics play a crucial role. In

*jvazquez@ulb.ac.be

certain limits, the semi-classical approach of quantum field theory in curved spacetime may suffice, which treats spacetime as a classical manifold. For example, for field perturbations near a black hole, one may neglect the back-reaction that the mass-energy of the fields have on the background spacetime. This enables one to calculate thermodynamic properties of the black hole, such as temperature. However, close to the spacetime singularity of the black hole, it is believed that a quantum theory of gravity is required. One might be tempted to argue that the region near a spacetime singularity is cloaked by the surface of an event horizon. That is, any information which is only deciphered by a quantum theory of gravitation might be locked away within the black hole, never making contact with the outside universe. However, as briefly mentioned above, even at the semi-classical level it is predicted that black holes radiate energy. Whether or not this energy carries information has long been a subject of debate. Regardless of the outcome, as a black hole evaporates, the surface of the event horizon gets ever closer to the central singularity, until a full quantum theory of gravity is needed in order to describe processes taking place outside of the event horizon as well. Thus, if we take General Relativistic solutions seriously, then we are forced to consider regimes which incorporate both gravity and particle physics. Many physicists now believe that the quantization of the gravitational field is linked to its unification with the fields of the Standard Model.

1.2 Early unification

The idea of unification has practically always been a theme in physics. Newtonian gravitation explained Earth's attraction and celestial mechanics as two results of the same force. During the late 19^{th} century, the Maxwellians combined electric and magnetic phenomena into a unified field description. It was found that the speed of electromagnetic waves was the same as that of light, leading to the brilliant insight that light is actually oscillations of the electromagnetic field.

These preceding examples serve to demonstrate the overall idea of unification, that of describing natural phenomena with as few free parameters as possible. In light of this (no pun intended) it may be rather disappointing, even without considerations of gravity, if the Standard Model is the end of the road. This is because it contains about twenty free parameters, whose values are specified only by experiment. One would hope that a fundamental theory of nature could be completely specified by self-consistency, and would only need experiments to boast its predictive power.

It can still be argued that the Standard Model has incredible predictive power once values to its twenty parameters have been assigned. In fact, it agrees with practically all of our observations of the physical world. This brings into question the purpose of theoretical physics. If our job as theorists is simply to create theories that predict what we observe and may lead to practical applications, then our job has been finished for quite some time now. In fact, most physicists are at a loss to find practical applications to Special or General Relativity[1].

It is our search for the underlying truth that drives us to go further. A working

[1] The Gravitational Positioning System (GPS) does use General Relativity.

model does not necessarily reflect fundamental truth. For example, Newton gave us equations with which to make predictions but he explicitly states that it is up to the reader to think of a mechanism for gravity. Albert Einstein found that mechanism to be the fabric of spacetime itself.

However, his theory of General Relativity is mutually exclusive with our present-day theories of particle physics, since the former considers spacetime as a classical manifold whereas the latter relies on quantum field theories. This incompatibility makes the idea of unification all the more tantalizing.

1.3 Relativity

At the beginning of the 20^{th} century, Einstein unified electrodynamics and kinematics via Special Relativity. Geometrically, this amounts to melding space and time into a manifold known as 'spacetime.' A generalized Pythagorean Theorem measures the coordinate-invariant spacetime distance:

$$ds^2 = -dt^2 + dx_i^2 \equiv dx_\mu^2. \tag{1.1}$$

The negative sign divides spacetime into three space-like dimensions and one time-like dimension ($3 + 1$ dimensions). Effects of this division include the length contraction and time dilation of measurements on co-moving frames. Dynamical equations for particles come about by extremizing the length of their "worldline," the distance traveled through spacetime.

Differential geometry extends this notion of invariant distance to curved manifolds by allowing the coefficients of the coordinate quadratics to vary as functions of spacetime:

$$ds^2 = g_{\mu\nu}(x^\mu)dx^\mu dx^\nu, \tag{1.2}$$

where $g_{\mu\nu}(x^\mu)$ is known as the metric and parameterizes the geometry of spacetime. General Relativity makes the conceptual leap of equating the geometry of spacetime with gravitation. Field equations describe the interplay between the dynamical geometry of spacetime and the motion of matter from an action principle. That is, particles minimize their path through the geometry of spacetime, while the geometry changes as a result of mass-energy content and motion.

1.4 Higher-dimensional General Relativity

Many mathematical generalizations of General Relativity have been explored in the hopes of including non-gravitational forces in a purely geometrical formulation. Most early attempts, including those made by Einstein himself, were somewhat of a 'top-down approach,' in which it was hoped that macroscopic physics could shed light on microscopic phenomena, via the dynamics of a classical geometric manifold. This is not surprising, considering that quantum mechanics was still new and somewhat ad hoc at the time. A strong motivation for Einstein's search of a unified field theory was, in fact, to rid our fundamental description of nature of quantum indeterminism.

There is a touch of irony, then, that quantum gravity is currently seen by many as a guiding light towards unification.

One of the first attempts at classically extending General Relativity was made by Kaluza a mere four years after the publication of Einstein's theory. Classical theory offers no constraints on the dimensionality of spacetime. While it is with absolute certainty that we observe $3+1$ dimensions, there may be additional dimensions which have yet to be detected, existing on compact submanifolds of a larger dimensional spacetime. Kaluza originally considered a fifth dimension that is "compactified" on a circle of radius L, where L is below the current observable length scales. The corresponding five-dimensional metric can be written in the form

$$ds_5^2 = e^{2\alpha\phi} ds_4^2 + e^{2\beta\phi}(dz + \mathcal{A})^2, \tag{1.3}$$

where α and β are non-zero constants which can be chosen for convenience.

The geometric degrees of freedom within ds_4^2, the observable portion of spacetime, describe four-dimensional gravity independently of the fifth dimension. In addition, $\mathcal{A} = A_\mu dx^\mu$ is an additional degree of freedom called the Kaluza-Klein vector potential, which corresponds to the electromagnetic potential in the original setup and has a field strength given by $\mathcal{F} = d\mathcal{A}$. The warping factor is given by the scalar field ϕ [2].

While this extension of Relativity does succeed in classically describing gravity and electromagnetism as the result of the curvature of a five-dimensional spacetime, on its own it was not believed to offer any predictions, which is why Einstein waited for two years before publicly supporting it[3].

During the next half-century, there was much progress in quantum physics, and the "top-down" approach to unification was abandoned by most physicists. It is rumored, however, that even on his deathbed Albert Einstein asked for a pencil and paper, in the hope that with his remaining last minutes he may stumble upon a glimmer of a unified field theory.

1.5 Quantum divergences

Quantum field theories had fantastic predictive power and successfully described electromagnetism, strong and weak nuclear forces in a unified framework known as the Standard Model. The idea of force-carrying particles was a key constituent of fundamental physics. 'Bottom-up' approaches were attempted to incorporate gravity into the Standard Model, with force-carrying particles called 'gravitons.' However, a quantum field theoretic approach to gravity has not been successful because the gravitational coupling parameter is too large for gravitational interactions to be renormalizable. That is, interactions cannot be dissected into a series of virtual processes

[2]In the original Kaluza-Klein setup, ϕ was regarded as a somewhat embarrassing extra degree of freedom and was set to zero. However, in order for the higher-dimensional equations of motion to be satisfied provided that those of the lower dimension are satisfied, ϕ must be retained. Current higher-dimensional theories refer to ϕ as the 'dilaton,' which carries information on the conformal structure of spacetime.

[3]Five-dimensional Relativity does predict gravito-electromagnetic waves, which oscillate between gravitational and electromagnetic modes.

that lead to finite results. Thus, practically every attempt to formulate a unified field theory which includes the covariance of General Relativity, or general coordinate transformation invariance, and the quantum mechanics of the Standard Model have led to inconsistencies.

1.6 Supersymmetry

At this point, it is relevant to note that almost all known fundamental theories have underlying symmetry principles. Classical mechanics has space and time translational invariances and spatial rotational invariance. Special Relativity adds space-time rotational invariance to these symmetries. General Relativity localizes the above invariances, which means that the degree of translation and rotation may vary as functions of space and time. On the other hand, electromagnetism is based on abelian gauge invariance, which is an arbitrariness in the electromagnetic field potentials corresponding to local invariance of the action with respect to wave function rotations in a complex plane. The Standard Model is based on non-abelian gauge invariance, in which the order of field transformations matters.

For quite a number of decades, quantum mechanics did not seem to be based on any symmetry principle, which may partially explain the difficulties in unifying the covariance of General Relativity with quantum physics. However, quantum mechanics introduces a new concept for particles: bosons (integer-spin) and fermions (half-integer spin). This leads to the possibility of a new symmetry, called 'supersymmetry' (SUSY), which is the invariance of the theory under interchange of bosonic and fermionic particles. The SUSY transformation acting twice over turns a particle back into itself, but it can be located at a different point in spacetime. Thus, one would expect that the corresponding super-algebra is closely related to the Poincaré (spacetime rotations plus translations) algebra. In fact, the anti-commutators of the supercharges Q which generate SUSY transformations have the structure

$$\{Q, Q\} = \Gamma^{\mu} P_{\mu}, \tag{1.4}$$

where the presence of the four-momentum operator P_{μ} implies Poincaré invariance. Local supersymmetry, for which the supersymmetric transformations depend on spacetime, is known as 'supergravity,' and automatically includes general coordinate invariance. In other words, supergravity requires Einstein's General Relativity; even though quantum mechanics is apparently incompatible with gravity, it predicts the existence of gravity!

1.7 Classical string theory

A revolutionary idea, initially manifesting itself as an unsuccessful attempt to understand hadronic interactions, is that the basic constituents of matter are 1 + 1-dimensional objects called "strings." Analogous to particles, strings obey an action principle. That is, the area of their worldsheet, the two-dimensional surface swept out by the string as it moves through spacetime, is minimized.

The vast number of particles seen in nature are hypothesized to correspond to various excitations of strings. In much the same way as a musical string on a violoncello or guitar produces notes at different frequencies, the discrete vibrations of a string correspond to a whole spectrum of particles. Thus, interactions between particles, which on the level of quantum field theory is described by the exchange of virtual particles, are described in string theory by the splitting and joining of strings. This path to unification certainly exemplifies the "bottom-up" approach, in that a drastic change to the sub-microscopic nature of matter potentially results in the unification of all forces. It seems logical that, given only one fundamental building block of matter, there can only be one type of fundamental interaction.

While the Standard Model has about twenty free parameters, which are assigned values through experiments rather than theoretical consistency, string theory has none, except perhaps for the string tension. A reasonable question to ask now is: how does the physics of gravitation and quantum field theory emerge from this rather simple notion of strings?

1.8 Quantizing the string

The first step of deriving General Relativity and particle physics from a common fundamental source may lie within the quantization of the classical string action. At a given momentum, quantized strings exist only at discrete energy levels, each level containing a finite number of string states, or particle types. There are huge energy gaps between each level, which means that the directly observable particles belong to a small subset of string vibrations. In principle, a string has harmonic frequency modes ad infinitum. However, the masses of the corresponding particles get larger, and decay to lighter particles all the quicker [1].

Most importantly, the ground energy state of the string contains a massless, spin-two particle. There are no higher spin particles, which is fortunate since their presence would ruin the consistency of the theory. The presence of a massless spin-two particle is undesirable if string theory has the limited goal of explaining hadronic interactions. This had been the initial intention. However, previous attempts at a quantum field theoretic description of gravity had shown that the force-carrier of gravity, known as the graviton, had to be a massless spin-two particle. Thus, in string theory's comeback as a potential "theory of everything," a curse turns into a blessing.

Once again, as with the case of supersymmetry and supergravity, we have the astonishing result that quantum considerations require the existence of gravity! From this vantage point, right from the start the quantum divergences of gravity are swept away by the extended string. Rather than being mutually exclusive, as it seems at first sight, quantum physics and gravitation have a symbiotic relationship. This reinforces the idea that quantum gravity may be a mandatory step towards the unification of all forces.

1.9 Supersymmetry makes a second entrance

Unfortunately, the ground state energy level also includes negative-mass particles, known as tachyons. Such particles have light speed as their limiting minimum speed, thus violating causality. Tachyonic particles generally suggest an instability, or possibly even an inconsistency, in a theory. Since tachyons have negative mass, an interaction involving finite input energy could result in particles of arbitrarily high energies together with arbitrarily many tachyons. There is no limit to the number of such processes, thus preventing a perturbative understanding of the theory.

An additional problem is that the string states only include bosonic particles. However, it is known that nature certainly contains fermions, such as electrons and quarks. Since supersymmetry is the invariance of a theory under the interchange of bosons and fermions, it may come as no surprise, post priori, that this is the key to resolving the second issue. As it turns out, the bosonic sector of the theory corresponds to the spacetime coordinates of a string, from the point of view of the conformal field theory living on the string worldvolume. This means that the additional fields are fermionic, so that the particle spectrum can potentially include all observable particles. In addition, the lowest energy level of a supersymmetric string is naturally massless, which eliminates the unwanted tachyons from the theory [1].

The inclusion of supersymmetry has some additional bonuses. Firstly, supersymmetry enforces the cancellation of zero-point energies between the bosonic and fermionic sectors. Since gravity couples to all energy, if these zero-point energies were not canceled, as in the case of non-supersymmetric particle physics, then they would have an enormous contribution to the cosmological constant. This would disagree with the observed cosmological constant being very close to zero, on the positive side, relative to the energy scales of particle physics.

Also, the weak, strong and electromagnetic couplings of the Standard Model differ by several orders of magnitude at low energies. However, at high energies, the couplings take on almost the same value– almost but not quite. It turns out that a supersymmetric extension of the Standard Model appears to render the values of the couplings identical at approximately 10^{16} GeV. This may be the manifestation of the fundamental unity of forces.

It would appear that the "bottom-up" approach to unification is winning. That is, gravitation arises from the quantization of strings. To put it another way, supergravity is the low-energy limit of string theory, and has General Relativity as its own low-energy limit.

1.10 The dimension of spacetime

String theory not only predicts the particle spectrum and interactions of nature, but the dimension of spacetime itself! In the process of quantizing the superstring, certain symmetries of the action are lost unless the number of spacetime dimensions is ten. Initially, since this is grossly inconsistent with every-day observations, this seems to indicate that we should look elsewhere for a "theory of everything." However, almost a century ago, Kaluza and Klein had explored the possibility of extra small dimensions.

At the time, this idea seemed to be an unnecessary extension of General Relativity with no additional predictions.

Thus, the "top-down" approach to unification that once motivated higher dimensional extensions of General Relativity enters the arena of fundamental physics once again, in which the gauge fields of the Standard Model are the result of ripples in a higher-dimensional spacetime.

In the original Kaluza-Klein Relativity, there was only one extra compact dimension. This didn't leave much topological freedom [4], since the extra dimension can only be curled up into a circle S^1. However, in this modern version of Kaluza-Klein Relativity, six extra compact dimensions are required. Greater number of compact dimensions brings the possibility of more complicated topologies. For example, with two extra dimensions, the compact part of the space could have the topology of a sphere, a torus, or any higher genus surface.

Higher dimensions are not simply unwanted additions that are required for theoretical consistency. They do have phenomenological implications. For instance, compactifying on a three-dimensional complex space known as Calabi-Yau manifold leads, in the low-energy limit, to a four-dimensional $\mathcal{N} = 1$ supersymmetric theory, the effective field theory that underlies many supersymmetric theories of particle phenomenology. Moreover, the geometrical and topological properties of the extra dimensions determine the number of particle generations, the particle species in each generation and the low-energy Lagrangian. Thus, determining the shape of the extra dimensions is crucial for understanding the low-energy predictions of string theory.

Unfortunately, there are thousands of Calabi-Yau manifolds from which to choose. It is hoped that the dynamics of string theory constrain the possible shapes of the compact dimensions.

1.11 M-Theory

Superstrings provided a perturbatively finite theory of gravity which, after compactification down to 3+1 dimensions, seemed potentially capable of explaining the strong, weak and electromagnetic forces of the Standard Model, including the required chiral representations of quarks and leptons. However, there appeared to be not one but five seemingly different but mathematically consistent superstring theories: the $E_8 \times E_8$ heterotic string, the $SO(32)$ heterotic string, the $SO(32)$ Type I string, and Types IIA and IIB strings. Each of these theories corresponded to a different way in which fermionic degrees of freedom could be added to the string worldsheet. Even before looking at the low-energy physics predicted by a particular compactification of string theory, it is crucial to choose *which* of the five string theories one is discussing!

Furthermore, many important questions seemed incapable of being answered within the framework of the weak-coupling perturbation expansion for which these theories were probed:

How do strings break supersymmetry?

[4] A slightly different possibility that was initially explored by Horava and Witten is that the one extra dimension can be compactified on an S^1/Z_2 orbifold, which leads an embedding of ten-dimensional $E_8 \times E_8$ heterotic string theory in eleven-dimensional M-theory [3].

How do strings choose the correct vacuum state?

How do strings explain the smallness of the cosmological constant?

How do strings supply a microscopic description of black holes?

Also, supersymmetry constrains the upper limit on the number of spacetime dimensions to be eleven. Why, then, do superstring theories stop at ten? In fact, before the "first string revolution" of the mid-1980's, many physicists sought super-unification in eleven-dimensional supergravity. Solutions to this most primitive supergravity theory include the elementary supermembrane and its dual partner, the solitonic superfivebrane. These are supersymmetric objects extended over two and five spatial dimensions, respectively. This brings to mind another question: why do superstring theories generalize zero-dimensional point particles only to one-dimensional strings, rather than p-dimensional objects?

During the "second superstring revolution" of the mid-nineties it was found that, in addition to the $1+1$-dimensional string solutions, string theory contains soliton-like Dirichlet branes [1]. These Dp-branes have $p+1$-dimensional worldvolumes, which are hyperplanes in $9+1$-dimensional spacetime on which strings are allowed to end. If a closed string collides with a D-brane, it can turn into an open string whose ends move along the D-brane. The end points of such an open string satisfy conventional free (Neumann) boundary conditions along the worldvolume of the D-brane, and fixed (Dirichlet) boundary conditions are obeyed in the $9-p$ dimensions transverse to the D-brane [1].

D-branes make it possible to probe string theories non-perturbatively, i.e., when the interactions are no longer assumed to be weak. This more complete picture makes it evident that the different string theories are actually related via a network of "dualities." T-dualities relate two different string theories by interchanging winding modes and Kaluza-Klein states, via $R \to \alpha'/R$. For example, Type IIA string theory compactified on a circle of radius R is equivalent to Type IIB string theory compactified on a circle of radius $1/R$. We have a similar relation between $E_8 \times E_8$ and $SO(32)$ heterotic string theories. While T-dualities remain manifest at weak-coupling, S-dualities are less well-established strong/weak coupling relationships. For example, the $SO(32)$ heterotic string is believed to be S-dual to the $SO(32)$ Type I string, while the Type IIB string is self-S-dual[5] [6] [1].

This led to the discovery that all five string theories are actually different sectors of an eleven-dimensional non-perturbative theory, known as M-theory. When M-theory is compactified on a circle S^1 of radius R_{11}, it leads to the Type IIA string, with string coupling constant $g_s = R_{11}^{3/2}$. Thus, the illusion that this string theory is ten-dimensional is a remnant of weak-coupling perturbative methods. Similarly, if M-theory is compactified on a line segment S^1/Z_2, then the $E_8 \times E_8$ heterotic string is recovered. As previously mentioned, a string (no pun intended) of dualities relates

[5]There is a duality of dualities, in which the T-dual of one theory is the S-dual of another.

[6]Compactification on various manifolds often leads to dualities. The heterotic string compactified on a six-dimensional torus T^6 is believed to be self-S-dual. Also, the heterotic string on T^4 is dual to the type II string on four-dimensional K3. The heterotic string on T^6 is dual to the Type II string on a Calabi-Yau manifold. The Type IIA string on a Calabi-Yau manifold is dual to the Type IIB string on the mirror Calabi-Yau manifold.

each of these string theories to all of the rest.

Just as a given string theory has a corresponding supergravity in its low-energy limit, eleven-dimensional supergravity is the low-energy limit of M-theory. Since we do not yet know what the full M-theory actually is, many different names have been attributed to the "M," including Magical, Mystery, Matrix, and Membrane! Whenever we refer to "M-theory," we mean the theory which subsumes all five string theories and whose low-energy limit is eleven-dimensional supergravity.

We now have an adequate framework with which to understand a wealth of non-perturbative phenomena. For example, electric-magnetic duality in $D = 4$ is a consequence of string-string duality in $D = 6$, which in turn is the result of membrane-fivebrane duality in $D = 11$. Furthermore, the exact electric-magnetic duality has been extended to an effective duality of non-conformal $\mathcal{N} = 2$ Seiberg-Witten theory[7], which can be derived from M-theory. In fact, it seems that all supersymmetric quantum field theories with any gauge group could have a geometrical interpretation through M-theory, as worldvolume fields propagating on a common intersection of stacks of p-branes wrapped around various cycles of compactified manifolds.

In addition, while perturbative string theory has vacuum degeneracy problems due to the billions of Calabi-Yau vacua, the non-perturbative effects of M-theory lead to smooth transitions from one Calabi-Yau manifold to another[8].

While supersymmetry ensures that the high-energy values of the Standard Model coupling constants meet at a common value, which is consistent with the idea of grand unification, the gravitational coupling constant just misses this meeting point. However, a particular compactification of M-theory envisioned by Horava and Witten, in which the Standard Model fields live on a four-dimensional spacetime while gravity propagates in five dimensions, allows the size of the fifth dimension to be chosen so that the gravitational coupling constant meets the other three at high energy. In fact, this may occur at much less energy than the originally-thought 10^{19} GeV, which leads to various interesting cosmological effects.

In fact, M-theory may resolve long-standing cosmological and quantum gravitational problems. For example, M-theory accounts for a microscopic description of black holes by supplying the necessary non-perturbative components, namely p-branes. This solves the problem of counting black hole entropy by internal degrees of freedom.

2 Holography

2.1 QCD strings

String theory began from attempts to formulate a theory of hadronic interactions. After quantum chromodynamics (QCD) entered the theoretical arena, string theory

[7] Seiberg-Witten theory has led to insights on quark confinement. However, this relies on an, at present, unphysical supersymmetric QCD.

[8] Now the question to ask is not why do we live in one topology but rather why do we live in a particular corner of the unique topology. M-theory might offer a dynamical explanation of this.

was abandoned, only to be upgraded shortly afterwards as a candidate for a theory of all fundamental interactions of the universe [2].

During the 1970's, 't Hooft proposed to generalize the $SU(3)$ gauge group of QCD to $SU(N)$ and take the large N limit while keeping $g_{YM}^2 N$ fixed. In this limit, the sum over Feynman graphs of a given topology can be regarded as the sum over world sheets of a hypothetical "QCD string." The closed string coupling constant goes as N^{-1}, so that in the large N limit we have a weakly-coupled string theory. The spectrum of these free closed strings is the same as that of glueballs in large N QCD, while open strings can describe mesons. If a method is developed to calculate this spectra and they are found to be discrete, then this would be an elegant explanation of quark confinement. After much work done in search of an exact gauge field/string duality, it was speculated that such strings may actually live in five dimensions.

In 1997, Maldacena made the AdS/CFT conjecture, in which supersymmetric conformal field theories are dual to supergravities with Anti-de Sitter spacetimes, which have constant negative curvature. In particular, "QCD strings" might actually be Type IIB superstrings living in five non-compact (AdS_5) and five compact (X^5) dimensions, where X^5 is a positively curved Einstein space [5]. Thus, string theory may provide exact information about certain gauge theories at strong coupling, a regime which was intractable by previously known perturbative methods. Additional motivation for this 'Holographic Principle' arose early on through the study of black hole thermodynamics.

2.2 Black hole thermodynamics

Since particle accelerators cannot currently probe the energy scale of string theory [9], we hope that cosmology and astrophysics will provide a testing ground. In particular, black holes provide a theoretical test that low-energy string theory agrees with the predictions of quantum field theory in curved spacetime. During the 1970's, it was found that the entropy of black holes is given by

$$S = \frac{A}{4G}, \tag{2.1}$$

where A is the surface area of the event horizon and G is Newton's gravitational constant. However, the microscopic meaning of black hole entropy, in terms of counting the degrees of freedom, was far from clear; it was believed that a black hole did not possess any degrees of freedom other than its energy, charges and angular momenta.

Understanding of the microscopics of black holes came with the proposition that certain types of black holes are actually made up of collections of D-branes. With this model, a detailed microscopic derivation of black hole entropy was provided by counting the ways in which branes could be configured to form a black hole.

The idea that black hole entropy scales like the surface area of the event horizon, rather than the volume within, contradicts our naive intuition about the extensivity

[9] Supersymmetry, which is an essential component for consistent string theories, may be tested at the LHC as soon as 2009, with the search for supersymmetric partners of known particles.

of thermodynamic entropy which we have gained from quantum field theories. This motivated 't Hooft and Susskind to conjecture that the degrees of freedom describing the system are characterized by a quantum field theory with one fewer space dimensions.

We have previously discussed how supergravity, in which all fields are the result of fluctuations in the spacetime geometry, represents a "top-down" approach to unification. On the other hand, the excitations of the string yields the particle spectrum of quantum field theory, which is a "bottom-up" approach. The duality of these descriptions is embodied within the Holographic Principle. The notion that certain gravitational descriptions are dual to quantum field theories may provide understanding of how these descriptions can be unified.

The most explicit example of holography known today, AdS/CFT, implies that the weakly-coupled gravity theory is dual to the strongly-coupled super-conformal field theory. This allows us to probe the quantum field theory side non-perturbatively via supergravity. Anti-de Sitter spacetimes arise as the near-horizon regions of certain p-branes, which indicates a dual CFT on the brane's worldvolume.

2.3 Dp-branes and Black Holes

As previously mentioned, black hole entropy scales as the surface area of the event horizon, rather than the volume within. This motivated 't Hooft and Susskind to conjecture that the degrees of freedom describing the system are characterized by a quantum field theory of one fewer spatial dimensions [6].

It had been believed that the state of a black hole was completely specified by its energy, charges and angular momenta. This left no internal degrees of freedom to be counted by the entropy, thus making the meaning of this thermodynamic quantity rather illusive.

The resolution of this puzzle came about through the study of Dp-branes, soliton-like solutions of Types IIA and IIB supergravity which carry Ramond-Ramond charge and are specified by the metric

$$ds^2 = H^{-1/2}(r)\big(-f(r)dt^2 + dx_1^2 + .. + dx_p^2\big) + H^{1/2}(r)\big(f(r)^{-1}dr^2 + r^2 d\Omega_{8-p}^2\big), \quad (2.2)$$

and a dilaton

$$e^{\phi} = H^{(3-p)/4}(r), \quad (2.3)$$

where the harmonic function is given by

$$H(r) = 1 + \frac{R^{7-p}}{r^{7-p}}. \quad (2.4)$$

Also,

$$f(r) = 1 - \frac{r_o^{7-p}}{r^{7-p}}, \quad (2.5)$$

where r_o is the nonextremality parameter. In the extremal case, when r_o vanishes, the mass saturates the lower (BPS) bound for a given choice of charges. The coordinates

t, x_i describe the brane worldvolume while r and the $8-p$-sphere coordinates describe the space transverse to the brane.

Polchinski illuminated the importance of these solutions with his discovery that Dp-branes are the fundamental objects in string theory which carry RR charges [1]. These p-brane supergravity solutions were then identified with the long-range background fields produced by stacks of parallel Dp-branes, for which the constant R is given by

$$R^{7-p} = \alpha'^{(7-p)/2} g_s N (4\pi)^{(5-p)/2} \Gamma(\frac{7-p}{2}), \tag{2.6}$$

where N is the number of coincident Dp-branes.

More complicated systems were considered, in which D-branes of different dimensionalities were intersecting and possibly wrapped on compact manifolds. It was found that such configurations could be identified with black holes and black p-brane solutions of supergravity with the appropriate charges. This provides the microscopic description of black holes that was needed to count the degrees of freedom that is parametrized by entropy.

Strominger and Vafa were the first to build this type of correspondence between black hole solutions and D-branes [8]. In particular, they began with an intersection of D1 and D5-branes in ten-dimensional type IIB theory. After compactification on a five-dimensional manifold, this brane configuration corresponds to a five-dimensional black hole carrying two separate $U(1)$ charges. This is a generalization of the four-dimensional Reissner-Nordstrom black hole.

It was now possible to understand black hole entropy in terms of the degrees of freedom living on the D-branes. Strominger and Vafa calculated the Bekenstein-Hawking entropy as a function of the charges and successfully reproduced the result provided by macroscopic entropy [8] [10].

As previously mentioned, the association of black hole entropy with area would imply that black holes have a temperature. In the classical regime this is nonsensical, since nothing can escape from inside the event horizon of a black hole. Thus, Hawking's proposal that black holes radiate energy was initially met with ridicule. However, Hawking used a semi-classical calculation, in which quantum processes occur within a fixed, classical backdrop of spacetime, allowing energy to quantum tunnel through the event horizon. Since such processes involve the annihilation of the the original particles by their anti-particle partners, this brings about the issue as to whether information is lost. It is hoped that the D-brane description of black holes can be used to resolve this question.

Black hole radiation implies that scattering processes of black holes have greybody factors. This is reflected in the absorption of particles, which can be pictured in two different ways. In the semi-classical approach, a particle is attracted by long-range gravity of the black hole, tunnels through an effective potential barrier, and is absorbed by the event horizon. In the D-brane picture, the incident particle travels

[10] The extremal Strominger-Vafa black hole preserves 1/8 of the supersymmetries present in the vacuum, which ensures that the number of states does not change as the coupling is increased to the regime in which the D-brane configuration corresponds with a black hole. This correspondence was quickly generalized to near-extremal solutions.

through flat space and decays into a number of new particles which are constrained to live on the brane intersection.

The thermodynamics and scattering processes of black holes seem to indicate that there is an underlying Holographic Principle at work– a deep equivalence between gauge theories and gravity. Since it would appear that the roots of gravity and quantum field theory are incompatible on the level of particle physics, it is quite surprising that they are, in some ways, equivalent on the deeper level of string theory. However, a set of precise examples of this notion was not found until 1997.

2.4 The AdS/CFT Correspondence

The celebrated AdS/CFT conjecture of Maldacena states, in its most familiar form, that string theory in the near-horizon geometry of a large number of N coincident D3-branes ($AdS_5 \times S^5$) is completely equivalent to the low-energy $U(N)$ $\mathcal{N}=4$ Super-Yang-Mills (SYM) gauge theory in four dimensions, which describes the excitations on the brane [4].

To be more specific, consider a stack of N parallel D3-branes in the “zero slope limit,” for which $\alpha' \to 0$ and the masses of all massive string modes go to infinity. In this limit, the gravitational coupling $\kappa \sim g_s\alpha' \to 0$, so that the bulk closed string modes decouple from the massless open string modes on the brane. In addition, all higher derivative terms of the worldvolume fields vanish in the action. Thus, the dynamics on the brane are completely described by the low-energy theory of the massless open string modes, the $U(N)$ $\mathcal{N} = 4$ Super-Yang-Mills theory in $3+1$ dimensions.

In this limit, the supergravity metric becomes that of flat spacetime. That is, the open string modes on the brane decouple from the bulk closed string modes; from the point of view of a distant observer, the brane disappears from the geometry. Closed strings propogate in the decoupled flat spacetime of the bulk while decoupled open strings obey a non-trivial field theory on the brane.

On the field theory side, the masses of the lowest energy level of strings stretched between two D-branes separated by a distance r are given by $uR^2 = r/\alpha'$. In the decoupling limit, these are important degrees of freedom. In order to keep their energies finite, we keep u fixed as we take the limit $\alpha' \to 0$.

Now we consider what happens to the supergravity metric in this limit. The metric for a stack of extremal D3-branes is

$$ds^2 = H^{-1/2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + H^{1/2}(dr^2 + r^2 d\Omega_5^2). \tag{2.7}$$

The decoupling limit $\alpha' \to 0$ with fixed u corresponds to the ”near-horizon” limit $r \ll R$, for which the constant ”1” in the harmonic function H can be neglected. The metric becomes

$$ds^2 = \alpha' R^2 \Big(u^2(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{du^2}{u^2} + d\Omega_5^2\Big), \tag{2.8}$$

which describes the space $AdS_5 \times S^5$. Note that α' becomes a constant overall factor in this limit. By the change of coordinates $z = 1/u$, this metric can be expressed as

$$ds^2 = \alpha' R^2 \Big(\frac{1}{z^2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dz^2) + d\Omega_5^2 \Big), \tag{2.9}$$

which corresponds to the conformally-flat frame in the five-dimensional supergravity after a dimensional-reduction over S^5. This property of AdS spaces is also reflected in the constancy of the dilaton field. Thus, branes whose spacetimes approach AdS in the near-horizon region are non-dilatonic, such as the D3-brane of type IIB string theory.

The AdS/CFT conjecture makes the claim that the full string theory on the near-horizon geometry of $AdS_5 \times S^5$ is dual to the four-dimensional $\mathcal{N} = 4$ Super-Yang-Mills gauge theory on the brane. Note that the string coupling constant is related to the Yang-Mills coupling constant by $g_s = g_{YM}^2$. This can be understood qualitatively as two open strings, each of which has a corresponding factor of g_{YM}, coming together to form a closed string [2].

The radius of curvature of the space in string units is $\sqrt{4\pi g_s N}$, which means that the supergravity solution can only be trusted for $1 \ll g_s N$. This corresponds to strong 't Hooft coupling in the gauge theory. In the limit $g_s N \to \infty$ and $g_s \to 0$, both string loops and stringy α' effects may be neglected. In order to have these limits simultaneously, we require that $N \to \infty$. Thus, classical supergravity on $AdS_5 \times S^5$ should be dual to the large N limit of $U(N)$, $\mathcal{N} = 4$ SYM theory in the limit of strong 't Hooft coupling [5].

A stronger form of the AdS/CFT conjecture asserts that for $N \to \infty$ and $1 \ll g_s N$ but finite, so that $g_s \to 0$ still, $1/(g_s N)$ corrections on the field theory side correspond to classical string theoretic corrections to supergravity. In the strongest version of the AdS/CFT conjecture, we may take N to be finite, and $1/N$ corrections in the field theory correspond to string loop effects.

The AdS/CFT Conjecture is extremely powerful, even in its weakest form. It provides a way to probe gauge theory at the previously inaccessible regime of strong 't Hooft coupling via classical supergravity calculations.

Since the AdS/CFT correspondence has to do with a gauge theory at strong coupling, calculations tend to be difficult on one side of the duality and relatively simple on the other side. Thus, this correspondence is both incredibly useful and rather difficult to check directly. However, the symmetries of the theory should be independent of the parameters and may be compared directly.

On the supergravity side, the bosonic symmetry group includes SO(4,2), the isometry group of AdS_5, as well as the global symmetry group SO(6), the rotational symmetry of S^5. These groups match with the bosonic global symmetry group on the field theory side, where SO(4,2) is the conformal group in 3+1 dimensions and $SO(6)$ is locally SU(4), which is the R-symmetry group of $\mathcal{N} = 4$ SYM gauge theory. In addition, both theories have 32 fermionic global symmetry generators, which combine with the bosonic symmetries to yield a supergroup SU(2,2|4) on both sides [5].

In addition, there should be an exact correspondence between operators in the field theory and particle states in $AdS_5 \times S^5$. As we shall discuss in the next chapter on absorption by branes, explicit calculations of correlation functions show complete agreement between the supergravity and field theory sides of the AdS/CFT correspondence.

3 Brane Worlds

3.1 Another Look at Higher-Dimensional Spacetime

String theories and M-theory require that our universe has more than three spatial dimensions. Studies along more phenomenological lines have recently led to new insights on how extra dimensions may manifest themselves, and how they may help solve long-standing problems such as hierarchy or the cosmological constant problem.

M-theory determines the dimensionality of spacetime to be eleven. However, it remains a matter of speculation as to the mechanism by which extra dimensions are hidden, so that spacetime is effectively four-dimensional in a low-energy regime. Until recently, the extra dimensions were usually assumed to be a size of roughly the Planck length $l_{Pl} \sim 10^{-33}$cm. With a corresponding energy scale of $M_{Pl} \sim 10^{19}$ GeV, probing extra dimensions directly appeared to be a hopeless pursuit.

In order to explain why there are three large spatial dimensions, Brandenberger and Vafa have proposed that all dimensions were initially compact and three spatial dimensions grew in size as a result of string-string annihilations– a process that is roughly analogous to breaking a rubber band that had been keeping a paper rolled up tight. On the other hand, extra dimensions may have once been large, compactified by a dynamical mechanism– such as the requirement that the effective four-dimensional mass density remains nonzero.

3.2 Sub-millimeter Extra Dimensions

Recently there has been renewed interest in the notion that our observable world is a three-dimensional brane embedded in a higher-dimensional space, whose extra dimensions may be large or even infinite. Gauge fields are naturally trapped on the brane by way of open strings whose ends are confined to the worldvolume of a D-brane. Indeed, the distances at which non-gravitational interactions cease to be four-dimensional are determined by the dynamics on the brane and may be much smaller than R, the size of extra dimensions (for simplicity, we assume that all extra dimensions are of the same size).

Gravity, on the other hand, becomes multi-dimensional at scales just below R, at which point the gravitational force is

$$F_{grav} = \frac{G_D m_1 m_2}{r^{D-2}}, \tag{3.1}$$

where D is the dimensionality of spacetime. The four-dimensional gravitational law has been verified experimentally down to distances of about 0.2 mm, which means that R can actually be as large as 0.1 mm!

This possibility provides a novel way of addressing the hierarchy problem, i.e., why the electroweak scale ($M_{EW} \sim 1$ TeV) is drastically different from the Planck scale

($M_{Pl} \sim 10^{16}$ TeV). In higher-dimensional theories, the four-dimensional Planck scale is not a fundamental parameter. Instead, the mass scale M of higher-dimensional gravity is fundamental. The four-dimensional Planck mass goes as

$$M_{Pl} \sim M(MR)^{d/2}, \tag{3.2}$$

where d is the number of extra dimensions. Thus, if the size of the extra dimensions is large relative to the fundamental length scale M^{-1}, then the Planck mass is much greater than the fundamental gravity scale.

If, motivated by unification, one supposes that the fundamental gravity scale is of the same order as the electroweak scale, $M \sim 1$ TeV, then the hierarchy between M_{Pl} and M_{EW} is entirely due to the large size of extra dimensions. Thus, the hierarchy problem now takes on a geometrical setting, i.e., why is R large?

Assuming that $M \sim 1$ TeV, then from equation (3.2) we find that

$$R \sim M^{-1}(\frac{M_{Pl}}{M})^{2/d} \sim 10^{\frac{32}{d}-17} cm. \tag{3.3}$$

For one extra dimension R is unacceptably large. However, for increased number of extra dimensions, R decreases. For $d = 2$, $R \sim 1$ mm, which has been the motivation for the recent experimental search for deviations from Newton's gravitational law at sub-millimeter distances. However, cosmology excludes a mass scale as low as $M \sim 1$ TeV for $d = 2$[11]. A more realistic value $M \sim 30$ TeV implies $R \sim 1 - 10\mu$m. While an experimental search for deviations from four-dimensional gravity is difficult in the micro-meter range, it is not impossible.

M-theory requires 7 unobserved dimensions. If these are all of the size R, then with the above assumptions an experimental search for violations in four-dimensional gravity appear to be hopeless. However, the compactification scales of extra dimensions are not guaranteed to be of the same order, which means that some large extra dimensions may be observable in this manner.

3.3 Warped Geometry

We saw above how the accuracy in our observations of a four-dimensional law of gravity impose an upper limit on the size of an extra dimension. However, we shall soon discuss how Randall and Sundrum have found that, in the case of a certain warped higher-dimensional spacetimes, an extra dimension can be very large, or even infinite, without destroying the four-dimensional gravitation at low energy.

When considering distance scales much larger than the brane thickness, we can model the brane as a delta-function source of the gravitational field. The brane is characterized by the energy density per unit three-volume σ, which is also known

[11] Kaluza-Klein graviton modes may be produced at high temperatures, such as during the Big Bang nucleosynthesis. Thus, having an upper bound on the mass density of KK gravitons yields an lower bound on M in terms of d and the maximum temperature that ever ocurred in the universe.

as the brane tension. We shall focus on the case of one extra dimension. The five-dimensional gravitational action in the presence of the brane is

$$S_{grav} = -\frac{1}{16\pi G^{(5)}} \int d^4x dz \sqrt{g^{(5)}} R^{(5)} - \Lambda \int d^4x dz \sqrt{g^{(5)}} - \sigma \int d^4x \sqrt{g^{(4)}}, \quad (3.4)$$

where Λ is the five-dimensional cosmological constant, and the superscripts denote the dimensionality of the space for which Newton's gravitational constant, the metric and the curvature applies. The existence of a four-dimensionally flat solution requires fine-tuning between Λ and σ:

$$\Lambda = -\frac{4\pi}{3} G^{(5)} \sigma^2. \quad (3.5)$$

This is similar to fine-tuning the four-dimensional consmological constant to zero. If equation (3.5) does not hold, then the intrinsic geometry on the brane is AdS, a case which will be discussed shortly. With (3.5) satisfied, the metric solution is

$$ds^2 = e^{-2k|z|}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + dz^2. \quad (3.6)$$

This spacetime is Anti-de Sitter with Z_2 symmetry and of radius k^{-1}.

The graviton obeys a the wave equation for a massless, minimally-coupled scalar propagating in this spacetime:

$$\partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \Phi(x_\mu, z) = 0. \quad (3.7)$$

Consider a wave function of the form

$$\Phi(x_\mu, z) = e^{x_\mu p^\mu} h(z). \quad (3.8)$$

The mass m of the graviton is $m^2 = -p^2$. The massless graviton wave function is

$$h_0(z) = e^{-kz}, \quad (3.9)$$

and is, therefore, normalizable with its maximum at $z = 0$. That is, the massless graviton is localized at $z = 0$. The higher modes of the graviton are given by

$$h_m(z) = J_2(\frac{m}{k} e^{kz}) + A N_2(\frac{m}{k} e^{kz}), \quad (3.10)$$

where J_2 and N_2 are Bessel functions. We solve for A by satisfying the boundary condition imposed by the delta-function source:

$$h_m(z) = N_1(\frac{m}{k}) J_2(\frac{m}{k} e^{kz}) - J_1(\frac{m}{k}) N_2(\frac{m}{k} e^{kz}). \quad (3.11)$$

Thus, these massive graviton modes propagate in the extra dimension, providing a high-energy correction to the four-dimensional gravitational law, which we will see in more detail for a specific model.

3.4 A Higher-Dimensional Solution to the Hierarchy Problem

One approach which uses a warped higher-dimensional spacetime compactifies the extra dimension over S^1/Z_2 by imposing a brane at each end of the dimension. The branes have tensions of opposite sign. Randall and Sundrum initially considered a brane world model in which our observable universe resides on the negative tension brane at $z = 0$ (RS1 scenario) [9].

Newton's gravitational constant in four and five dimensions are related by

$$G_{(4)} = G_{(5)} \frac{k}{e^{2kz_c} - 1}, \tag{3.12}$$

where z_c is the length of the extra dimension. Thus, for relatively large z_c, the gravitational interactions of matter living on the negative tension brane are weak.

If one takes the five-dimensional gravity scale and the Anti-de Sitter radius k to be of the order of the weak scale, $M_{EW} \sim 1$ TeV, then (3.12) implies that the effective four-dimensional Planck mass is of the order

$$M_{Pl} \sim e^{kz_c} M_{EW}. \tag{3.13}$$

Thus, for z_c about 37 times larger than the Anti-de Sitter radius k^{-1}, the value of M_{Pl}/M_{EW} is of the right order of magnitude to solve the hierarchy problem.

3.5 Solving the Cosmological Constant Problem?

In the context of the brane world, the cosmological constant problem amounts to the question of why the vacuum energy density has almost no effect on the curvature induced on our brane. It may be plausible that perhaps the vacuum energy density induces a non-trivial warp factor on the higher-dimensional geometry, while the four-dimensional Poincaré invariance is maintained. The suggestion has been put forth that a hypothetical bulk scalar field conformally coupled to brane world matter may play an important role. That is, a nonzero vacuum energy may be compensated by a shift in this scalar field.

Unfortunately, this solution involves a naked singularity at finite proper distance to the brane, and it has been argued that a possible resolution of this singularity reintroduces the cosmological constant problem. For example, a second brane can be introduced, so that the space between the branes is completely non-singular. However, in order to be a solution to the Einstein equations, the tension of the second brane must be fine-tuned, and this fine-tuning is no more desireable than that of the cosmological constant in four-dimensional theories. Nevertheless, this idea may point the way to a new solution.

3.6 An Alternative to Compactification

The massless graviton wave function remains normalizable as $z_c \to \infty$. Thus, even if there exists a single brane of positive tension in an infinite extra dimension, gravity as well as gauge fields are localized on the brane (RS2 scenario). For the author, this is the most intriguing aspect of brane worlds, and why simple models that may not be readily embedded in fundamental string/M-theories are still of interest. For the first time in 70 years, this offers an alternative to the Kaluza-Klein compactified extra dimensions.

The wave functions of the Kaluza-Klein gravitons are normalized by

$$\int dz e^{2k|z|} h_m(z) h_{m'}(z) = \delta(m - m'). \tag{3.14}$$

The measure $dze^{2k|z|}$ is due to the warping factor in the geometry. Using the asymptotics of the Bessel functions, the normalized KK graviton modes are found to be

$$h_m(z) = \sqrt{\frac{m}{k}} \frac{J_1(\frac{m}{k})N_2(\frac{m}{k}e^{kz}) - N_1(\frac{m}{k})J_2(\frac{m}{k}e^{kz})}{\sqrt{[J_1(\frac{m}{k})]^2 + [N_1(\frac{m}{k})]^2}}. \tag{3.15}$$

At large z, these wave functions oscillate:

$$h_m(z) \sim \sin(\frac{m}{k}e^{kz}), \tag{3.16}$$

and they are suppressed at $z = 0$:

$$h_m(0) \sim \sqrt{\frac{m}{k}}. \tag{3.17}$$

These KK modes correspond to gravitons escaping into the extra dimension or coming in towards the brane. The coupling of matter living on the brane to these modes is fairly weak at small m, and so they have a low production rate at low energy.

Consider the Yukawa-type contribution of KK graviton exchange to the gravitational potential between two unit point masses living on the braneL

$$V_{KK}(r) = -G_{(5)} \int_0^\infty dm |h_m(0)|^2 \frac{e^{-mr}}{r} = -\frac{G_{(4)}}{r}(1 + \frac{const}{k^2r^2}). \tag{3.18}$$

Thus, the four-dimensional gravitational potential, including the graviton zero mode, is given by

$$V(r) = -\frac{G_{(4)}}{r}(1 + \frac{const}{k^2r^2}). \tag{3.19}$$

In contrast to compact extra dimensions, for which corrections are suppressed exponentially at large distances, the correction to Newton's law has power law behavior at large r. However, at distances greater than the Anti-de Sitter radius k^{-1}, this correction is negligible.

Duff showed, in his PhD thesis, that one-loop quantum corrections to the graviton propogator lead to a Newtonian potential of the form (3.19). Since one path to this potential is the result of a five-dimensional classical calculation and the other results from a four-dimensional quantum calculation, they seem to be completely unrelated–at first sight. However, the AdS/CFT correspondence can be invoked to show that these are two dual ways of describing the same physics. In particular, the gravitational modes propagating in the extra dimension are dual to the quantum fields which live on the brane world.

Thus, we have seen that holography together with a brane world scenario can be invoked to account for unseen dimensions that are not necessarily small.

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