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A note on the BRST cohomology of the extended antifield formalism

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G. Barnich*
Physique Théorique et Mathématique,
Université Libre de Bruxelles,
Campus Plaine C.P. 231, B–1050 Bruxelles, Belgium

Abstract

The relevance of the BRST cohomology of the extended antifield formalism is briefly discussed along with standard homological tools needed for its computation.

*Collaborateur scientifique du Fonds National Belge de la Recherche Scientifique.

The lectures given at the school *Q.F.T., Supersymmetry and Superstrings*, in Călimăneşti, Romania in April 1998 were intitled *Classical and quantum aspects of the Batalin-Vilkovisky formalism*. They covered the following material:

- Lesson 1: Algebraic structure of gauge symmetries

  Finite dimensional toy model. Noether identities. Koszul-Tate resolution of on-shell functions. Gauge symmetries. Longitudinal differential. BRST differential. Antibracket. Master equation. Homological perturbation theory.

- Lesson 2: Locality in field theory

  Jet-spaces. Local functionals and local functions. Euler-Lagrange derivatives. Algebraic Poincaré lemma. Local antibracket. Master equation and BRST differential.

- Lesson 3: Consistency conditions on anomalies. Non renormalization theorems.

  Non minimal sector. Gauge fixing. Generating functionals. Statement of quantum action principles. Consistency conditions on anomalies. Lie-Massey brackets. Higher order cohomological restrictions. Power counting in the antifield formalism. Beta functions. First non renormalization theorem. Local version of Callan-Symanzik equation. Second non renormalization theorem. Chern-Simons theory.

Useful review references connected to the material covered here are [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Details on specific topics, reflecting the point of view of the author, can be found in [11, 12, 13, 14, 15, 16, 17, 18] and in the literature cited therein.

The purpose of this note is to discuss briefly the BRST cohomology of the extended antifield formalism, to give some details on exact couples and spectral sequences, and to apply these concepts in the problem at hand.

1 Introduction

1.1 Classical theory

The Batalin-Vilkovisky formalism [19, 20] allows one to formulate the BRST differential [21, 22, 23, 24] controling the gauge symmetries under renormalization for generic gauge theories. The formalism can be extended so as to include (non linear) global symmetries (see [12, 13] and references therein), which is achieved by coupling the BRST cohomology classes in negative ghost numbers with constant ghosts. A further extension including the BRST cohomology classes in all the ghost numbers can be constructed [16]. Some features of this extension are :

- it allows one to take into account in a systematic way all higher order cohomological constraints due to the antibracket maps [14],
- it is the appropriate formalism to prove stability independently of power counting restrictions, also called renormalizability in the modern sense [25], in the case of generic gauge theories,
- an appropriate BRST differential can be constructed on the classical and the quantum level, even in the case of anomalous theories [17].

Let us briefly summarize the results of [16] needed in the following. The extended formalism is obtained by first computing a basis for the local BRST cohomology classes. This basis contains as a subset those classes that can be obtained from the solution S of the master equation by differentiation of S with respect to so-called essential coupling constants. The additional classes completing the basis are then coupled with the help of new independent coupling constants. This action can then be extended by terms of higher orders in the new couplings in such a way that, if we denote by ξ^A all the couplings corresponding to the independent BRST cohomology classes, the corresponding action S satisfies the extended master equation

$$\frac{1}{2}(S,S) + \Delta_c S = 0. \tag{1.1}$$

The BRST differential associated to the solution of the extended master equation is

$$\bar{s} = (S,\cdot) + \Delta_c^L, \tag{1.2}$$

where $\Delta_c = \frac{\partial^R \cdot}{\partial \xi^A} f^A$, while $\Delta_c^L = (-)^A f^A \frac{\partial^L}{\partial \xi^A}$, with f^A depending (at least quadratically) on the couplings ξ alone. Both antiderivations satisfy $(\Delta_c)^2 = 0 = (\Delta_c^L)^2$. Since there is no dependence on the fields and the antifields, $\Delta_c^L(A, B) = (\Delta_c^L A, B) + (-)^{A+1}(A, \Delta_c^L B)$, with the appropriate version holding for the right derivation Δ_c. The local BRST cohomology classes contain the generators of all the generalized non trivial symmetries of the theory in negative ghost number, the generalized observables in ghost number zero, and the anomalies (and anomalies for anomalies) in positive ghost number. This is the reason why the extended master equation encodes the invariance of the original action under all the non trivial gauge and glocal symmetries, their commutator algebra as well as the antibracket algebra of all the local BRST cohomology classes.

The cohomology of $\bar{s}$ in the space F of ξ dependent local functionals in the fields, the antifields and their derivatives is isomorphic to the cohomology of

$$s_{\Delta_c} = [\Delta_c, \cdot] \tag{1.3}$$

in the space G of graded right derivations $\lambda = \frac{\partial^R \cdot}{\partial \xi^A} \lambda^A$, with λ^A a function of ξ alone, $[\cdot, \cdot]$ being the graded commutator for graded right derivations,

$$H(\bar{s}, F) \simeq H(s_{\Delta_c}, G). \tag{1.4}$$

If μ is a s_{Δ_c} cocycle, the corresponding $\bar{s}$ cocycle is given by $\mu S = \frac{\partial^R S}{\partial \xi^B} \mu^B$.

1.2 Quantum theory

In the standard version of the BRST-Zinn-Justin-Batalin-Vilkovisky set-up, there are two main issues to be considered: stability and anomalies.

The problem of stability is the question if to every local BRST cohomology class in ghost number 0, there corresponds an independent coupling of the action. The extended formalism solves this problem trivially by construction, since all cohomology classes have been coupled with independent couplings. The non trivial part of the formalism is the proof of the existence of the extended master equation and the associated differential, which allows to control the symmetries after the extension. Of course, it will be often convenient in practice not to couple all the local BRST cohomology classes but only a subset needed to guarantee that the theory is stable.

In the standard set-up, the question of anomalies is mostly reduced to the question of the local BRST cohomology in ghost number 1 and to a discussion of the coefficients of these cohomology classes. In the presence of anomalies, there is no differential on the quantum level associated to the anomalously broken Zinn-Justin equation for the effective action. In the extended formalism however, because all the local BRST cohomology classes in positive ghost numbers have been coupled to the solution of the master equation, such a differential exists [16]. Indeed, the quantum action principle [26, 27, 28] applied to (1.1) gives

$$\frac{1}{2}(\Gamma,\Gamma)+\Delta_c\Gamma=\hbar\mathcal{A}\circ\Gamma, \tag{1.5}$$

where Γ is the renormalized generating functional for 1PI vertices associated to the solution S of the extended master equation and the local functional $\mathcal{A}$ is an element of F in ghost number 1. Using the result (1.4) on the cohomology of $\bar{s}$, one can show [16, 17] that, through the addition of local counterterms, (1.5) can be written as

$$\frac{1}{2}(\Gamma^\infty,\Gamma^\infty)+\Delta^\infty\Gamma^\infty=0, \tag{1.6}$$

where Γ^∞ is associated to the action $S^\infty=S-\Sigma_{k=1}\hbar^k\Sigma_k$ containing local finite BRST breaking counterterms Σ_k and $\Delta^\infty=\Delta_c+\hbar\Delta_1+\hbar^2\Delta_2+\dots$ satisfies $(\Delta^\infty)^2=0$. The associated quantum BRST differential is

$$s^q=(\Gamma^\infty,\cdot)+(\Delta^\infty)^L. \tag{1.7}$$

In the limit $\hbar$ going to zero, we recover both the classical extended master equation (1.1) and the classical differential $\bar{s}$.

In the extended antifield formalism, the anomalous Zinn-Justin equation can thus be written as a functional differential equation for the renormalized effective action. The derivations $\Delta_1,\Delta_2,\dots$ are guaranteed to exist due to the quantum action principles. They satisfy a priori cohomological restrictions due to the fact that the differential Δ^∞ is a formal deformation with deformation parameter $\hbar$ of the differential Δ_c.

For instance, the derivation Δ_1 is a cocycle of s_{Δ_c} in ghost number 1, because $[\Delta_c,\Delta_1]=0$. This cocyle can be assumed to be non trivial, because otherwise, Δ_1 could have been absorbed by the counterterm Σ_1. Hence, non trivial anomalies, which correspond in this formalism to non trivial deformations of Δ_c, are controled by $H^1(s_{\Delta_c}, G)$.

In the standard way, once care has been taken of the trivial anomalies through the counterterms Σ_k, the remaining infinite and finite counterterms are required to belong to $H^0(\bar{s}, F) \simeq H^0(s_{\Delta_c}, G)$ in order to preserve (1.6) to that order.

It is thus of interest to compute the cohomology of s_{Δ_c}.

2 BRST cohomology in the extended antifield formalism

Let $\lambda = \frac{\partial^R \cdot}{\partial \xi^A} \lambda^A$ be a right derivation. We assume that the λ^A are formal power series in ξ^A. In the following, we provide this space with an obvious filtration. It will however not have finite length, and for particular theories, better filtrations have to be found in order to do a complete computation. Since the techniques will be similar, it is nevertheless useful to show how they work on this example.

2.1 Grading and filtration on the space of right derivations

Let $N_\xi = \frac{\partial^R}{\partial \xi^A} \xi^A$ be the operator counting the number of ξ's. A general right derivation admits the following decomposition according to the eigenvalues of N_ξ: $\lambda = \lambda_{-1} + \lambda_0 + \lambda_1 + \ldots$, where $[\lambda_p, N_\xi] = p\lambda_p$. Hence, G is a graded space, $G = \oplus_{p=-1} G^p$. (It is actually a bigraded space, the other grading, for which s_{Δ_c} is homogeneous of degree 1 being the ghost number.)

The graded right commutator satisfies $[[\lambda_m, \mu_n], N_\xi] = (m+n)[\lambda_m, \mu_n]$. The decomposition of Δ_c starts at eigenvalue 1: $\Delta_c = \Delta_{c1} + \Delta_{c2} + \ldots$; the corresponding decomposition of s_{Δ_c} being $s_{\Delta_c} = [\Delta_{c1}, \cdot] + [\Delta_{c1}, \cdot] = \ldots \equiv s_1 + s_2 + \ldots$. It follows that the cocycle condition $s_{\Delta_c}\lambda = 0$ decomposes as

$$\begin{aligned} s_1\lambda_{-1} &= 0, \\ s_1\lambda_0 + s_2\lambda_{-1} &= 0, \\ s_1\lambda_1 + s_2\lambda_0 + s_3\lambda_{-1} &= 0, \\ &\vdots, \end{aligned} \tag{2.1}$$

while the coboundary condition $\lambda = s_{\Delta_c}\mu$ decomposes as

$$\begin{aligned} \lambda_{-1} &= 0, \\ \lambda_0 &= s_1\mu_{-1}, \\ \lambda_1 &= s_1\mu_0 + s_2\mu_{-1}, \\ \lambda_2 &= s_1\mu_1 + s_2\mu_0 + s_3\mu_{-1}, \\ &\vdots, \end{aligned} \tag{2.2}$$

In order to construct the spectral sequence associated to this problem, we follow [29].

Consider the spaces K_p of derivations having N_ξ degree greater than p, i.e., $\lambda \in K_p$ if $\lambda = \lambda_p + \lambda_{p+1} + \ldots$. The space of all right derivations is $G = K_{-1}$, $K_{p+1} \subset K_p$ and $s_{\Delta_c}K_p \subset K_p$. The sequence of spaces K_p is a decreasing filtration of G, with $K_p/K_{p+1} \simeq G^p$.

We have the short exact sequence[1]:

$$0 \longrightarrow \oplus_{p=-1}K_{p+1} \xrightarrow{i} \oplus_{p=-1}K_p \xrightarrow{j} \oplus_{p=-1}K_p/K_{p+1} \longrightarrow 0, \tag{2.3}$$

where $\oplus_{p=-1}K_p/K_{p+1} \simeq \oplus_{p=-1}G^p$. The following diagram is exact at each corner:

$$\begin{array}{ccc} H(s_{\Delta_c}, \oplus_{p=-1}K_{p+1}) & \xrightarrow{i_0} & H(s_{\Delta_c}, \oplus_{p=-1}K_p) \\ & k_0 \nwarrow \swarrow j_0 & \\ & E_0, & \end{array} \tag{2.4}$$

where $E_0 = \oplus_{p=-1}K_p/K_{p+1} \simeq \oplus_{p=-1}G^p$. In this diagram, $H(s_{\Delta_c}, K_p)$ is defined by the cocycle condition $s_{\Delta_c}(\lambda_p+\lambda_{p+1}+\ldots) = 0$, and the coboundary condition $\lambda_p+\lambda_{p+1}+\ldots = s_{\Delta_c}(\mu_p+\mu_{p+1}+\ldots$. The maps i_0 and j_0 are induced by i and j, $i_0[\lambda_{p+1}+\lambda_{p+2}+\ldots] = [\lambda_{p+1}+\lambda_{p+2}+\ldots]$ and $j_0[\lambda_p+\lambda_{p+1}+\ldots] = [j(\lambda_p+\lambda_{p+1}+\ldots)] = [\lambda_p]$. They are well defined, because i_0 maps cocycles to cocycles and coboundaries to coboundaries, while $j(s_{\Delta_c}(\mu_p+\mu_{p+1}\ldots)) \in K_{p+1}$. The map k_0 is defined by $k_0[\lambda_p] = [s_{\Delta_c}\lambda_p]$. It does not depend on the choice of representative for $[\lambda_p] \in K_p/K_{p+1}$, because $[s_{\Delta_c}(\lambda_{p+1}+\ldots)] = 0 \in H(s_{\Delta_c}, K_{p+1})$.

Let us check explicitly that this diagram is exact:

[1] A diagram is said to be exact if the image of a map is equal to the kernel of the next map.

- ker j_0 is given by elements $[\lambda_p + \lambda_{p+1} + ...] \in H(s_{\Delta_c}, K_p)$ such that $s_{\Delta_c}(\lambda_p + \lambda_{p+1} + ...) = 0$ and $\lambda_p = 0$. This is the same than $i_0 H(s, K_{p+1})$, which is given by $[\lambda_{p+1} + \lambda_{p+2} + \ldots]$, with $s_{\Delta_c}(\lambda_{p+1} + \lambda_{p+2} + ...) = 0$, the equivalence relation being the equivalence relation in $H(s, K_p)$ by definition of i_0.

- ker k_0 is given by elements $[\lambda_p]$ such that $[s_{\Delta_c}\lambda_p] = 0 \in H(s_{\Delta_c}, K_{p+1})$, i.e. such that $s_{\Delta_c}\lambda_p = s_{\Delta_c}(\mu_{p+1} + \mu_{p+2} + ...)$. By the identification $\lambda_{p+1} = -\mu_{p+1}, \lambda_{p+2} = -\mu_{p+2}, \ldots$, this is indeed the same than $j_0 H(s_{\Delta_c}, K_p)$ given by $[\lambda_p]$ with $s_{\Delta_c}(\lambda_p + \lambda_{p+1} + \ldots) = 0$.

- ker i_0 is given by elements $[\lambda_{p+1} + \lambda_{p+2} + ...]$ such that $s_{\Delta_c}(\lambda_{p+1} + \lambda_{p+2} + ...) = 0$ and $\lambda_{p+1} + \lambda_{p+2} + ... = s_{\Delta_c}(\mu_p + \mu_{p+1} + ...)$, while $k_0[\mu_p]$ is given by $[\lambda_{p+1} + \lambda_{p+2} + ...]$ of the form $[s_{\Delta_c}\mu_p]$ so that $\lambda_{p+1} + \lambda_{p+2} + ... = s_{\Delta_c}\mu_p + s_{\Delta_c}(\mu_{p+1} + \ldots)$, which is indeed the same.

2.2 Exact couples and associated spectral sequence

To every exact couple (A_0, B_0), i.e., exact diagram of the form

$$\begin{array}{ccc} A_0 & \xrightarrow{i_0} & A_0 \\ k_0 \nwarrow & & \swarrow j_0 \\ & B_0, & \end{array} \tag{2.5}$$

one can associated a derived exact couple

$$\begin{array}{ccc} A_1 & \xrightarrow{i_1} & A_1 \\ k_1 \nwarrow & & \swarrow j_1 \\ & B_1. & \end{array} \tag{2.6}$$

In this diagram, the spaces and maps are defined as follows: $A_1 = i_0 A_0$; $B_1 = H(d_0, B_0)$, where $d_0 = j_0 \circ k_0$ ($d_0^2 = 0$ because $k_0 \circ j_0 = 0$); for $a_1 = i_0 a_0$, $i_1 a_1 = i_1(i_0 a_0) = i_0^2 a_0$; $j_1 a_1 = [j_0 a_0]$ (this map is well defined: $j_0 a_0$ is a cocycle, because $k_0 \circ j_0 = 0$, furthermore the map does not depend on the representative choosen for a_0, because if $i_0 a_0 = 0$, $a_0 = k_0 b_0$ for some b_0 and $j_1 a_1 = [j_0 \circ k_0 b_0] = 0$); $k_1[b_0] = k_0 b_0$ ($k_0 b_0 = i_0 a_0$ for some b_0 because $d_0 b_0 = j_0(k_0 b_0) = 0$ implies $k_0 b_0 = i_0 a_0$, furthermore $k_0 d_0 c_0 = 0$ because $k_0 \circ j_0 = 0$).

Let us also check explicitly exactness of this diagram:

- ker j_1 is given by elements $a_1 = i_0 a_0$ such that $[j_0 a_0] = 0$, i.e., $j_0 a_0 = j_0 k_0 b_0$ and then $a_0 - k_0 b_0 = i_0 c_0$, implying that $a_1 = i_0^2 c_0$. $i_1 A_1$ is given by elements $a_1 = i_1 c_1 = i_0^2 c_0$. It follows that ker $j_1 \subset i_1 A_1$, while the inverse inclusion follows from $j_0 \circ i_0 = 0$.

- ker k_1 is given by elements $[b_0]$ such that $k_0 b_0 = i_0 a_0 = 0$, i.e., such that $b_0 = j_0 c_0$, for some c_0, while im j_1 is given by elements $[b_0]$ such that $[b_0] = [j_0 e_0]$, i.e $b_0 = j_0(e_0 + k_0 f_0)$. It follows that ker $k_1 =$ im j_1.

- ker i_1 is given by elements $a_1 = i_0 a_0$ such that $i_0(i_0 a_0) = 0$, i.e., $i_0 a_0 = k_0 b_0$ (which implies in particular $d_0 b_0 = 0$). im k_1 is given by elements $a_1 = i_0 a_0 = k_0 b_0$ for some b_0 with $d_0 b_0 = 0$, so both spaces are indeed the same.

Clearly, this construction can be iterated by taking as the starting exact couple the derived couple. We thus get a sequence of exact couples

$$\begin{array}{ccc} A_r & \xrightarrow{i_r} & A_r \\ k_r \nwarrow & & \swarrow j_r \\ & B_r. & \end{array} \tag{2.7}$$

and the associate spectral sequence (B_r, d_r), for $r = 0, 1, \ldots$, i.e., spaces B_r and differentials d_r satisfying $B_{r+1} = H(d_r, B_r)$.

2.3 Spectral sequence associated to the BRST cohomology of the extended antifield formalism

Let us now apply the general theory to the case of the exact couple (2.4) and give explicitly the differentials d_r and the spaces B_r (called E_r) in this case for $r = 0, 1, 2, 3$.

We have $E_0 = \oplus_{p=-1} K_p / K_{p+1} \simeq \oplus_{p=-1} G^p$. The differential d_0 is defined by $d_0[\lambda_p]_0 = j_0[s_{\Delta_c}\lambda_p]$, where $[s_{\Delta_c}\lambda_p] \in H(s_{\Delta_c}, K_{p+1})$. It follows that $d_0[\lambda_p]_0 = [s_1 \lambda_p]$. This means that E_1^p is defined by elements $[[\lambda_p]_0]_1$ with the cocycle condition

$$s_1 \lambda_p = 0 \tag{2.8}$$

and the coboundary condition

$$\lambda_p = s_1 \mu_{p-1}. \tag{2.9}$$

Because $s_1 = \frac{\partial^R}{\partial \xi^A} f^A_{BC} \xi^B \xi^C$, and $s_1^2 = 0$ implies that the f^A_{BC} are the structure constants of a graded Lie algebra, this group is just a graded version of standard Lie algebra (Chevalley-Eilenberg) cohomology with representation space the adjoint representation.

Take now $[[\lambda_p]_0]_1 \in E_1^p$. The differential $d_1[[\lambda_p]_0]_1 = j_1 k_1 [[\lambda_p]_0]_1 = j_1 k_0 [\lambda_p]_0 = j_1[s_{\Delta_c}\lambda_p] = [j_0 i_0^{-1}[s_{\Delta_c}\lambda_p]]_1$. This means that $[s_{\Delta_c}\lambda_p]$ has to be considered as an element of $H(s_{\Delta_c}, K_{p+2})$ so that $d_1[[\lambda_p]_0]_1 = [[s_2\lambda_p]_0]_1$. Hence E_2^p is defined by elements $[[[\lambda_p]_0]_1]_2$ with the cocyle condition

$$s_2\lambda_p + s_1\lambda_{p+1} = 0, \tag{2.10}$$

$$s_1\lambda_p = 0, \tag{2.11}$$

and the coboundary condition

$$\lambda_p = s_2\mu_{p-2} + s_1\mu_{p-1}, \tag{2.12}$$

$$0 = s_1\mu_{p-2}. \tag{2.13}$$

We thus find that $E_2^p = H^p(s_2, H(s_1))$.

The differential d_2 in E_2^p is defined by $d_2[[[\lambda_p]_0]_1]_2 = j_2 k_2 [[[\lambda_p]_0]_1]_2 = j_2 k_1 [[\lambda_p]_0]_1 = j_2 k_0 [\lambda_p]_0 = [j_1 i_1^{-1} k_0 [\lambda_p]_0]_2 = [[j_0 i_0^{-1} i_1^{-1} k_0 [\lambda_p]_0]_1]_2$. In order to make sure that $k_0[\lambda_p]_0$ belongs to $i_1 i_0 H(s_{\Delta_c}, K_{p+1})$ we use $\lambda_p + \lambda_{p+1}$ as a representative for $[\lambda_p]_0$. It follows that $d_2[[[\lambda_p]_0]_1]_2 = [[[s_3\lambda_p + s_2\lambda_{p+1}]_0]_1]_2$. The cocycle condition for an element $[[[[\lambda_p]_0]_1]_2]_3 \in E_3^p$ is then given by

$$s_3\lambda_p + s_2\lambda_{p+1} = s_2\mu_{p+1} + s_1\mu_{p+2}, \tag{2.14}$$

$$s_2\lambda_p + s_1\lambda_{p+1} = 0, \tag{2.15}$$

$$s_1\lambda_p = 0, \tag{2.16}$$

with $s_1\mu_{p+2} = 0$. The redefinition $\lambda_{p+1} \to \lambda_{p+1} - \mu_{p+1}$ and $\lambda_{p+2} = -\mu_{p+2}$, then gives as cocycle condition

$$s_3\lambda_p + s_2\lambda_{p+1} + s_1\lambda_{p+2} = 0, \tag{2.17}$$

$$s_2\lambda_p + s_1\lambda_{p+1} = 0, \tag{2.18}$$

$$s_1\lambda_p = 0. \tag{2.19}$$

The coboundary condition is $[[[\lambda_p]_0]_1]_2 = d_3[[[\mu_{p-3}+\mu_{p-2}]_0]_1]_2$, where $s_1\mu_{p-3} = 0, s_2\mu_{p-3} + s_2\mu_{p-2} = 0$, hence $[[\lambda_p]_0]_1 = [[s_3\mu_{p-3} + s_2\mu_{p-2}]_0]_1 + d_2[[\sigma_{p-2}]_0]_1$, with $s_1\sigma_{p-2} = 0$ which gives

$$\lambda_p = s_3\mu_{p-3} + s_2\mu_{p-2} + s_2\sigma_{p-2} + s_1\rho_{p-1}, \tag{2.20}$$
$$0 = s_2\mu_{p-3} + s_1\mu_{p-2}, \tag{2.21}$$
$$0 = s_1\mu_{p-3},\ 0 = s_1\sigma_{p-2}. \tag{2.22}$$

The redefinition $\mu_{p-2} \rightarrow \mu_{p-2}+\sigma_{p-2}$ and $\rho_{p-1} = \mu_{p-1}$, then gives the coboundary condition

$$\lambda_p = s_3\mu_{p-3} + s_2\mu_{p-2} + s_1\mu_{p-1}, \tag{2.23}$$
$$0 = s_2\mu_{p-3} + s_1\mu_{p-2}, \tag{2.24}$$
$$0 = s_1\mu_{p-3}. \tag{2.25}$$

This construction can be continued in the same way for higher r's.

The original problem was the computation of $H(s_{\Delta_c}, G) = H(s_{\Delta_c}, K_{-1})$. From exactness of the couples (2.7), it follows that

$$\begin{aligned} H(s_{\Delta_c}, K_{-1}) &\simeq j_0 E_0^{-1} \oplus \ker\, j_0 \\ &\simeq \ker\, k_0(\subset E_0^{-1}) \oplus i_0 H(s_{\Delta_c}, K_0) \\ &\simeq \ker\, k_0(\subset E_0^{-1}) \oplus \ker\, k_1(\subset E_1^{0}) \oplus i_1 i_0 H(s_{\Delta_c}, K_1) \qquad (2.26)\\ &\vdots \qquad (2.27)\\ &\simeq \oplus_{r=0}^{R} \ker\, k_r(\subset E_r^{r-1}) \oplus i_R \dots i_0 H(s_{\Delta_c}, K_R). \qquad (2.28)\end{aligned}$$

Furthermore, $E_0 \simeq E_1 \oplus F_0 \oplus d_0F_0$ and $E_0^{-1} \simeq E_1^{-1} \oplus F_0^{-1}$. F_0 does not belong to ker k_0 because $d_0F_0 \neq 0$. Thus ker $k_0(\subset E_0^{-1})) \simeq$ ker $k_1(\subset E_1^{-1})$. Similarily, $E_1 \simeq E_2 \oplus F_1 \oplus d_1F_1$ and $(d_1F_1)^{-1} = (d_1F_1)^0 = 0$. Again, $d_1[F_1]_1 \neq 0$ implies that F_1 does not belong to ker k_1. This means that ker $k_1(\subset E_1^{-1}) \simeq$ ker $k_2(\subset E_2^{-1})$ and ker $k_1(\subset E_1^0) \simeq$ ker $k_2(\subset E_2^0)$. Going on in the same way, we conclude that ker $k_r(\subset E_r^{r-1})) \simeq$ ker $k_R(\subset E_R^{r-1}))$. We thus get

$$H(s_{\Delta_c}, K_{-1}) \simeq \oplus_{r=0}^{R} \ker\, k_R(\subset E_R^{r-1}) \oplus i_R \dots i_0 H(s_{\Delta_c}, K_R). \tag{2.29}$$

This construction is most useful if it would stop at some point. Indeed, suppose that $K_R = 0$. Because $k_R[\dots[\lambda_p]_0\dots]_R$ belongs to $i_R \dots i_0 H(s_{\Delta_c}, K_{p+R+1}) = 0$, it follows that

$$H(s_{\Delta_c}, K_{-1}) \simeq \oplus_{r=0}^{R-1} E_R^{r-1}. \tag{2.30}$$

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