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Effects of the shear fluctuations on the plasma stability and transport

F.Spineanu[1], M.Vlad[1], J.D.Reuss[2]
[1]National Institute for Laser, Plasma and Radiation Physics,
P.O.Box MG-36, Magurele, Bucharest, Romania
[2]Association Euratom-C.E.A. sur la Fusion, CEA/DSM/DRFC,
C.E.A.Cadarache, F-13108 Saint-Paul-lez-Durance, France

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1 Introduction

The transport intermittency is a topic of current interest in both the experimental and theoretical studies of the tokamak plasma [1]. An intermittent rate of transport cannot be directly derived in the standard, diffusive, approach to the energy and particle transport. The most common ideas in this field associates the (anomalous) transport to the instabilities which have evolved into a statistically stationary state of turbulence. The regime of transport is then stationary and no fluctuations of the rates can be accomodated in this model.

There are however recent proposal for the source of intermittency, mainly related to the model of avalanches as in sand-pile-like systems. Irregular bursts of the transport rates are also observed numerically in a model of the ion-temperature gradient driven turbulence with a boundary condition imposed as a heat flux from the core region toward the confinement region [2].

Random destabilization of plasma modes. Stationarity obtained from

- saturation of turbulence at equal rates of growth and dissipation (in a spectral cascade); **or, alternatively**

- random sequence of rise and decay of plasma instabilities [4]; short bursts of transport, at random; self-suppression of transport: the other attractors, like in many dynamical systems described by differential equations with different behaviours according to the initialization in the basin of a particular attractor.

Competition of transport mechanisms [3]. Mixing of the properties of the transport mechanisms.

Numerical simulation with fluctuating diffusion coefficient [4]. Statistical properties of the outgoing fluxes of particles and energy.

2 Shear length and the transport rate

If the transport is determined by the drift modes, then the radial extension of these modes gives a measure of the effectiveness of the elementary diffusion event. But the radial extension is dependent on the radial position of the resonance of the mode parallel velocity with the ion thermal velocity. This general connection is more complicated in the case of toroidally induced drift branch [5].

It is believed that the transport rate is strongly dependent on the shear parameter (see [6]).

The diffusion coefficient is, in the simplest (mixing length) approximation,

$$\chi = \frac{L_r^2}{\tau_c} \tag{1}$$

where L_r is the characteristic radial length of the mode and τ_c is the correlation time. In general L_r is connected to the linear extension of the mode. This is very often considered to be of the order of ρ_s.

When we take into account the toroidal geometry, the perturbed electrostatic potential is written as a superposition of poloidal harmonics coupled by the toroidicity effect . This comes from the presence of the drift velocities in the particle trajectories, giving a shift ω_D in the resonance of the propagator. But ω_D is more than a simple shift in frequency. It has a non-trivial dependence on the parallel and perpendicular particle velocities, which complicates the integration over the velocity space. In addition it contains *spatial* variables, since the drifts are due to the gradient of the magnetic field and to the curvature. The dependence of ω_D on the poloidal angle θ imposes a radical change in the linear treatment of the mode stability. This comes from the combination which arises from the poloidal harmonic expansion $\exp(im\theta)$ and the trigonometric functions present in ω_D. The simple presence of the function $\cos\theta$ generates, for every harmonic $m\theta$ terms of interaction with neighbouring harmonics $(m+1)\,\theta$ and respectively $(m-1)\,\theta$.

This has nothing to do with the nonlinear or turbulent interaction between the modes due to the nonlinearity. Now, the particularity of this situation consists of the following: the harmonics $(m \pm 1)\,\theta$ are located on *different* magnetic surfaces, i.e. they are associated with different radial coordinates. This makes the linear problem nonlocal in x (or r) and two-dimensional. The solution is the *ballooning representation* of the modes.

If the overlap of the neighbouring harmonics is substantial, the harmonics are **strongly correlated** and the harmonics have the tendency of oscillation at the same frequency ω. But this is in contradiction with the fact that each harmonic rotates on its resonant magnetic surface with the local diamagnetic velocity. The result of these opposite effects is a finite radial correlation length L_r.

If the differences between the different values of the diamagnetic velocities decreases, the number of harmonics which can be coupled increases and the radial correlation length L_r also increases.

Experimental data. The ion thermal transport rate decreases with increasing the shear (in the outer part of the discharge, where $q > 1$).

In order to determine L_r one must solve the **two-dimensional** eigenmode equation for ITG mode. Result: strong dependence of the radial correlation length on the shear.

3 Models

3.1 Electron drift instability

3.1.1 Electrostatic drift waves in the toroidal geometry

The electron-ion collision frequency exceeds the inverse transit time

$$\nu_{ei} > \omega_{T_e} = \frac{v_{the}}{Rq}$$

then the equations are: **the continuity equation**

$$\frac{\partial n}{\partial t} + n\nabla_{\|} v_{\| e} + \boldsymbol{\nabla} \cdot (n\mathbf{v}_{\perp e}) = 0$$

and the **parallel equation of motion**

$$\frac{dv_{\| e}}{dt} + \nu_{ei} v_{\| e} = v_{the}^2 \nabla_{\|} \left(\frac{|e|\,\varphi}{T_e} - n \right)$$

Here the perpendicular velocity is

$$\begin{aligned}\mathbf{v}_{\perp e} &= \mathbf{v}_{De} + \mathbf{v}_{\mathbf{E}} \\ &= \frac{-\boldsymbol{\nabla}\varphi \times \widehat{\mathbf{n}}}{B_0} - \frac{T_e}{|e|\, B_0}\widehat{\mathbf{n}} \times \boldsymbol{\nabla} \ln B\end{aligned}$$

composed of the $\mathbf{E} \times \mathbf{B}$ convection in the field of the wave and the electron magnetic drift velocity.

The second equation (**equation of parallel momentum conservation**) expresses the balance of the parallel electric field and the parallel gradient of the pressure, with a contribution from electro-ion collisions (parallel viscosity). This momentum balance allows us to obtain the parallel electron velocity

$$v_{\|e} = \frac{v_{the}^2}{\nu_{ei}} \nabla_{\|} h(r, \theta, \varphi, t)$$

where h is the non-adiabatic part of the electron density fluctuation. The equation for h is the equation of continuity with this replacement

$$\frac{\partial h}{\partial t} - \frac{v_{the}^2}{\nu_{ei}} \nabla_{\|}^2 h + \mathbf{v}_{De} \cdot \boldsymbol{\nabla} h - \frac{-\boldsymbol{\nabla}\varphi \times \widehat{\mathbf{n}}}{B_0} \cdot \boldsymbol{\nabla} h = (\omega - \omega_{*e}) \frac{i\,|e|\,\varphi}{T_e}$$

3.2 ITG

3.2.1 The ion mode related to the ion-drift motions ($\boldsymbol{\nabla} B$ and curvature) ω_{Di}

Toroidicity induced modes and linear coupling of modes: $(m-1, m, m+1)$.

3.2.2 The ion mode related to the toroidicity

The ion polarization drift is important when the response of the density is close to adiabaticity, since then (quasi-tridimensionality) the density has aBoltzmann type profile along filed lines. the main nonlinearity, the convection of the fluctuating filed by itself is suppressed and higher order nonlinearity are considered. Ballonning representation is the geometric method required to correctly introduce the dependence of the fluctuation on the magnetic field.

Fluid treatment. Hydrodynamic equations for electrostatic slab model (Lee and Diamond 1986)

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla} \cdot (n_i \mathbf{v}_{\perp i}) + \nabla_{\|} \left(n_i \widetilde{v}_{\|i}\right) = 0$$

$$m_i n_i \left(\frac{\partial \widetilde{v}_{\| i}}{\partial t} + (\mathbf{v}_E \cdot \boldsymbol{\nabla}) \widetilde{v}_{\| i} \right) = -e n_i \boldsymbol{\nabla}_{\|} \Phi - \boldsymbol{\nabla}_{\|} P_i + \mu_i \boldsymbol{\nabla}_{\|}^2 \widetilde{v}_{\| i}$$

$$\frac{\partial P_i}{\partial t} + (\mathbf{v}_E \cdot \boldsymbol{\nabla}) P_i + \Gamma P_i \left(n_i \widetilde{v}_{\| i} \right) = 0$$

3.2.3 Other form of the equations

A similar form of the equation for the ITG eigenmode is obtained in the ballonning representation in toroidal geometry. In this case the asymptotic form of the function $f(\eta)$ which is introduced by the balooning representation is

$$f_+ (\eta) = A_+ \exp \left(\frac{i q k_\perp \rho \widehat{s} \omega L_n}{2 \varepsilon_n c_s} \eta^2 \right)$$

with $\widehat{s} = r q'/q$ and $\varepsilon_n = L_n / R$. This is the condition of outward energy propagation (radiation) identical to the asymptotic condition imposed for the slab drift mode. The shear damping of these modes is effective except for strong toroidal localization in the bad curvature region of the torus. The solution is

$$f_n (\eta) = H_n (\sigma_n \eta) \exp \left(-\frac{1}{2} \sigma_n \eta^2 \right)$$

where

$$\sigma_n = \frac{q \omega L_n}{\varepsilon_n c_s} \left[\frac{2 \varepsilon_n k_\perp \rho c_s}{\omega L_n} \left(\frac{1}{2} - \widehat{s} \right) - k_\perp^2 \rho^2 \widehat{s}^2 \right]^{1/2} .$$

3.3 Two-dimensional structure of the ITG mode

The sensitivity of the radial extension to the shear is well edscribed in Ref. [6].

The electrostatic potential is a superposition of poloidal harmonics:

$$\phi = \sum_p \phi_p (x) \exp \left[i n \varphi - i (m_0 + p) \theta - i \omega t \right] \tag{2}$$

where $m_0 = q(r_0)\, n$.

In the ballooning mode representation it is assumed that the neighbouring poloidal *eigenmodes* ϕ_p have similar shape which are simply shifted by a radial distance

$$\frac{1}{n q'} \tag{3}$$

which corresponds to the distance between two consecutive resonant surfaces. (This should reflect the presence of $\cos\theta$ and $\sin\theta$ in the expression of the drift frequency ω_D).

The *global mode structure* is given by a superposition of the harmonics (several harmonics, labeled by the integer number p) with similar shape modulated by a **slowly varying envelope function**

$$\phi_p\left(x\right) = A\left(x\right)\,\phi_0\left(x - p\,\frac{1}{nq'}\right) \tag{4}$$

The radial extent of the mode is dictated by the *envelope function.* The *eikonal representation*

$$A\left(x\right) = \exp\left(i\int k_x\left(x'\right)dx'\right) = \exp\left(\int inq'\theta_k\left(x'\right)dx'\right) \tag{5}$$

where the radial wavenumber k_x has been normalized by nq'.

The solution of the global *eigenmode* equation will be obtained in two steps:

- determination of the *eigenmode* structure along the field line, using the ballooning formalism, and finding an explicit expression for the function ϕ_0.
- calculation of the envelope function $A\left(x\right)$ i.e. of the global *eigenmode.*

Consider the Fourier transform of the function ϕ_0

$$\phi_0\left(x\right) = \int \widehat{\phi}\left(\theta\right)\,\exp\left(-i\theta nq'x\right)\,d\theta \tag{6}$$

which is inserted in the cuasineutrality condition

$$\begin{aligned} 0 \quad = \quad & \frac{\partial^2\widehat{\phi}}{\partial\theta^2} + \left(\frac{\omega}{\omega_{ti}}\right)^2\left\{\frac{\frac{1}{\tau}+\frac{\omega_{*i}}{\omega}}{1-\frac{\omega_{pi}}{\omega}} + + \left(k_\theta\rho_i\right)^2\left[1+\widehat{s}^2\left(\theta-\theta_k\right)^2\right] - \right. \\ & \left. -\frac{\omega_D}{\omega}\left[\cos\theta + \widehat{s}\left(\theta-\theta_k\right)\sin\theta\right]\,\widehat{\phi}_0\right\} \end{aligned} \tag{7}$$

The local dispersion relation is obtained after imposing the boundary condition

$$\widehat{\phi}_0 \text{ vanishes exponentially } \left|\theta\right| \rightarrow \infty \tag{8}$$

the general form of the local dispersion relation

$$F\left(\omega, x, \theta_k\right)\,A\left(x\right) = 0 \tag{9}$$

This equation is the result of the ballooning formalism applied in the "first order". Order means here to consider θ_k as a constant parameter (actually it is taken equal to zero). The next order is obtained taking

$$\theta_k = -i \frac{1}{nq'} \frac{d}{dx} \tag{10}$$

This is in contrast to the "local approximation" where the parameter θ_k and the variable x are simply set to zero.

To find the explicit form of the enveloppe $A(x)$ by solving the equation $F = 0$, one should apply the WKBJ method. far from the turning points it is possible to employ the *eikonal representation* for $A(x)$

$$A(x) = \exp\left(i \int k_x(x')\, dx'\right) = \exp\left(\int inq'\theta_k(x')\, dx'\right) \tag{11}$$

where the parameter θ_k is expressed as a function of x from the local dispersion equation (9). The condition for **turning points** is

$$\left.\left(\frac{\partial F}{\partial \theta_k}\right)\right|_{\theta_k=\theta_T} = 0 \tag{12}$$

and the radial position of the turning points x_T is found by replacing θ_T into the local dispersion equation (9). Close to the turning point we have an analytical solution obtained by expanding F

$$\left[\left.\left(\frac{\partial F}{\partial x}\right)\right|_{x=x_T}(x - x_T) + \frac{1}{2}\left.\left(\frac{\partial^2 F}{\partial \theta_k^2}\right)\right|_{\theta_k=\theta_T}(\theta_k - \theta_T)^2\right] A(x) = 0 \tag{13}$$

With the substitution of

$$\theta_k = -i \frac{1}{nq'} \frac{d}{dx} \tag{14}$$

this equation becomes an Airy equation which can be solved. Connection formulas are then used to obtain the **global eigenmode**. With boundary conditions

$$|A| \rightarrow 0 \text{ for } |x| \rightarrow \infty \tag{15}$$

we obtain the condition of "quantification", i.e. the eigenvalues

$$\oint nq'\, x(\theta_k)\; d\theta_k = 2\pi\,(l + \beta) \tag{16}$$

where $x(\theta_k)$ is obtained from the dispersion relation (9) and β a constant.

3.3.1 General approach to solve the radial equation (i.e. determine $A(x)$)

It is assumed that the general form of the dispersion relation is

$$F = a(\omega) + f(x) + b(\omega)\cos\theta_k \tag{17}$$

Here the function $f(x)$ is associated with the radial (x) variation of the diamagnetic velocity, i.e. of the temperature. The term $\cos\theta_k$ is due to the curvature.

There are two cases. **If in the domain there is a point where** $(\partial F/\partial x) = 0$. Then one makes expansions around this point. In particular, the function

$$f(x) \approx \frac{x^2}{2L^2} \tag{18}$$

The point which gives the zero of $(\partial F/\partial x)$ is taken the origin and one expand around $x = 0$ and $\theta_k = 0$. The dispersion relation becomes

$$a(\omega) + b(\omega) = \frac{[-b(\omega)]^{1/2}}{nq'L}(l + 1/2) \tag{19}$$

where from it is found

$$x_{T\pm} = \pm L\left[\frac{[-b(\omega)]^{1/2}}{nq'L}(l + 1/2)\right]^{1/2} \tag{20}$$

The correlation length is

$$L_r = \mathrm{Re}\,(x_{T+} - x_{T-}) \tag{21}$$

If there is no point where $(\partial F/\partial x) = 0$.

$$x_T = \pm Lb(\omega) - \frac{l}{nq'} \tag{22}$$

3.4 The bidimensional problem for the ITG in the case with small shear

The variational method to determine the eigenmodes and eigenvalues.

The trial functions.

Dependence of the type

$$\exp(-c/q\widehat{s}) \tag{23}$$

which is very sensitive to the fluctuations in shear.

Consider the functional

$$H = \int \psi L \widehat{\phi}_0 dx \tag{24}$$

where L is the oparator appearing in the dispersion relation. The conjugated function ψ is finally $\widehat{\phi}_0$ which gives the following form of the functional

$$\begin{aligned} H &= \int_{-\infty}^{\infty} dx \left[-\left(\frac{\partial \widehat{\phi}_0}{\partial \theta} \right)^2 + \left(\frac{\frac{1}{\tau} + \frac{\omega_{*i}}{\omega}}{1 - \frac{\omega_{pi}}{\omega}} \left(\frac{\omega}{\omega_{ti}} \right)^2 + \right. \right. \\ & \quad + \left(\frac{\omega}{\omega_{ti}} \right)^2 (k_\theta \rho_i)^2 \left[1 + \widehat{s}^2 (\theta - \theta_k)^2 \right] - \\ & \quad \left. \left. - \frac{\omega \omega_D}{\omega_{ti}^2} \left[\cos\theta + \widehat{s} (\theta - \theta_k) \sin\theta \right] \right) \widehat{\phi}_0^2 \right] \end{aligned} \tag{25}$$

Now, for the toroidal branch there are "trial functions"

$$\widehat{\phi}_0 = \left[\cos\left(\frac{\theta}{2} \right) - \frac{2\sigma + \varepsilon/2}{\lambda - \varepsilon/s} (\theta - \theta_k) \sin\left(\frac{\theta}{2} \right) \right] \exp\left(-\sigma (\theta - \theta_k)^2 \right) \tag{26}$$

For example, for the case where there is a point $(\partial F / \partial r) = 0$ the mode is localized between the turning points

$$x_T \approx 3^{1/4} \left(\frac{L_0}{nq'} \right)^{1/2} \left(1 - \frac{2\varepsilon}{3\widehat{s}} \right)^{1/4} \sqrt{2l+1} \exp\left(\frac{1}{8\varepsilon_0} \right) \tag{27}$$

The fluctuations of the correlation length.

3.5 Drift waves and turbulence

About ions. The following model has been proposed by Carreras:

$$\frac{\partial \widetilde{n}_i}{\partial t} + \widetilde{V}_x \frac{dn_0}{dx} + \widetilde{\mathbf{V}} \cdot \boldsymbol{\nabla} \widetilde{n}_i = -n_0 \left(\boldsymbol{\nabla}_\perp \cdot \widetilde{\mathbf{V}}_\perp + \boldsymbol{\nabla}_\parallel \cdot \widetilde{\mathbf{V}}_\parallel \right)$$

where the perpendicular ion velocity is due to the $\mathbf{E} \times \mathbf{B}$ motion and polarization drift.

In these works the variation of the drift waves along the magnetic filed is neglected

$$\boldsymbol{\nabla}_\parallel = 0$$

and the model becomes a cuasi-two-dimensional one.

The model equation becomes

$$\frac{\partial}{\partial t}\left(1-\rho_s^2\boldsymbol{\nabla}_\perp^2\right)n+V_{*n}\frac{\partial n}{\partial y}+$$
$$+D_0\frac{\partial^2 n}{\partial y^2}\text{ (the drive, effective } i\delta\text{ , or } ik_y^2D_0)$$
$$-L_nD_0\left[\boldsymbol{\nabla}_\perp\left(\frac{\partial n}{\partial y}\right)\times\mathbf{e}_z\right]\cdot\boldsymbol{\nabla}_\perp n\text{ (the }\mathbf{E}\times\mathbf{B}\text{ convection of the nonadiabatic term)}$$
$$+\rho_sc_s\left(\boldsymbol{\nabla}_\perp n\times\mathbf{e}_z\right)\cdot\boldsymbol{\nabla}_\perp\left(\rho_s^2\boldsymbol{\nabla}_\perp^2\right)n\text{ (the polarization nonlinearity)}$$

$$=0$$

where the normalized ion density is

$$n\equiv\frac{\widetilde{n}}{n_0}$$

the ion diamagnetic drift velocity is

$$V_{*n}=\frac{c_s\rho_s}{L_n}$$

and the notation is introduced

$$D_0=\alpha\sqrt{\varepsilon}\frac{\left(\rho_sc_s\right)^2}{L_TL_n\nu_{eff}}$$

Without the non-adiabatic electrons (the third: $D_0\frac{\partial^2 n}{\partial y^2}$ and fourth: $-L_nD_0\left[\boldsymbol{\nabla}_\perp\left(\frac{\partial n}{\partial y}\right)\times\mathbf{e}_z\right]\cdot\boldsymbol{\nabla}_\perp n$ terms) the equation reduces to the original Hasegawa-Mima equation. An energy sink can be modelled by adding a hyperviscosity term in the model equation. This leads to a finite band of unstable drift modes with a high k cutoff. Let's note $\mathbf{k}_\perp=\mathbf{k}$.

In Fourier space

$$i\frac{\partial}{\partial t}n_\mathbf{k}-\frac{\omega_{*\mathbf{k}}+ik_y^2D_0}{1+k^2\rho_s^2}n_\mathbf{k}+\frac{i}{1+k^2\rho_s^2}\left(N_\mathbf{k}^{\mathbf{E}\times\mathbf{B}}+N_\mathbf{k}^{POL}\right)=0$$

where the nonlinearities are

$$N_\mathbf{k}^{\mathbf{E}\times\mathbf{B}}=-i\frac{1}{2}L_nD_0\sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}}\left[\left(\mathbf{k}\times\mathbf{k}'\right)\cdot\mathbf{e}_z\right]\left(k_y''-k_y'\right)n_{\mathbf{k}'}n_{\mathbf{k}''}$$

and

$$N_{\mathbf{k}}^{POL} = \frac{1}{2}\rho_s c_s \sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}} [(\mathbf{k}\times\mathbf{k}')\cdot\mathbf{e}_z]\ \rho_s^2\ \left(k''^2 - k'^2\right)\ n_{\mathbf{k}'}\, n_{\mathbf{k}''}$$

The linear dispersion relation $i\frac{\partial}{\partial t} = \omega_{\mathbf{k}}$

$$\omega_{\mathbf{k}} = \omega_{\mathbf{k}}^{(0)} + i\gamma_{\mathbf{k}}^{(0)} = \frac{\omega_{*\mathbf{k}}}{1+k^2\rho_s^2} + i\frac{k_y^2 D_0}{1+k^2\rho_s^2}$$

which means that the term

$$\frac{k_y^2 D_0}{1+k^2\rho_s^2} \text{ is the drive.}$$

For comparison, Hasegawa-Mima equation, starting from the polarization drift of the ions

$$\frac{\partial}{\partial t}\left(1-\rho_s^2\nabla_\perp^2\right)n + V_{*n}\frac{\partial n}{\partial y} + \rho_s c_s\left(\nabla_\perp n\times\mathbf{e}_z\right)\cdot\nabla_\perp\left(\rho_s^2\nabla_\perp^2\right)n = 0$$

where we can take the density to be adiabatic, which gives an equation for the potential φ. The problem is two-dimensional.

To examine the conserved quantities we ignore the **drive** and the **sink** (damping). If only the **polarization nonlinearity**

$$N_{\mathbf{k}}^{POL} = \rho_s c_s\left(\nabla_\perp n\times\mathbf{e}_z\right)\cdot\nabla_\perp\left(\rho_s^2\,\nabla_\perp^2 n\right)$$

is retained **the system has two conserved quantities** :

$$\begin{aligned}
\text{the energy } E &= \frac{1}{2}\int dV\ \left(|n|^2 + \rho_s^2\,|\nabla_\perp n|^2\right) \\
&= \frac{1}{2}\sum_{\mathbf{k}}\left(1+k^2\rho_s^2\right)|n_{\mathbf{k}}|^2 \\
\text{the enstrophy } \Omega &= \frac{1}{2}\int dV\ \left(\left|\rho_s^2\nabla_\perp^2 n\right|^2 + \rho_s^2\,|\nabla_\perp n|^2\right) \\
&= \frac{1}{2}\sum_{\mathbf{k}} k^2\rho_s^2\ \left(1+k^2\rho_s^2\right)|n_{\mathbf{k}}|^2
\end{aligned}$$

The statistical mechanics prediction for the density fluctuation spectrum in an equilibrium state

$$\left\langle |n_{\mathbf{k}}|^2\right\rangle = \frac{1}{\left(1+k^2\rho_s^2\right)\left(a+b\,k^2\rho_s^2\right)}$$

where a and b are Lagrange multipliers.

The isotropic energy spectrum is

$$E_{\mathbf{k}} = \pi \, k\rho_s \left(1 + k^2\rho_s^2\right) \left\langle |n_{\mathbf{k}}|^2 \right\rangle = \frac{\pi \, k\rho_s}{a + b \, k^2 \rho_s^2}$$

and the system pushes the energy to large scales.

The isotropic spectrum of the enstrophy is

$$\Omega_{\mathbf{k}} = k^2\rho_s^2 \, E_{\mathbf{k}} = \frac{\pi \, k^3 \rho_s^3}{a + b \, k^2 \rho_s^2}$$

and the system pushes enstrophy to large scales. This is the **dual cascade**. The energy going to the large scales is the **inverse cascade**.

When there is also the $\mathbf{E} \times \mathbf{B}$ nonlinearity

$$N^{\mathbf{E}\times\mathbf{B}} = -L_n D_0 \left[\boldsymbol{\nabla}_{\perp} \left(\frac{\partial n}{\partial y} \right) \times \mathbf{e}_z \right] \cdot \boldsymbol{\nabla}_{\perp} n$$

the system has only one conserved quantity, the energy. The equilibrium density fluctuation spectrum is

$$\left\langle |n_{\mathbf{k}}|^2 \right\rangle = \frac{c}{1 + k^2 \rho_s^2}$$

When the electron's motion along the field lines is much faster the parallel phase velocity of the drift waves, the electron response is adiabatic and the $\mathbf{E} \times \mathbf{B}$ nonlinearity vanishes. The dynamics is dominated by the ion polarization.

A change of the magnetic shear can transiently change the phase velocity and the electrons aquire a distribution which is not Boltzmannian in the parallel potential. The $\mathbf{E} \times \mathbf{B}$ nonlinearity can have a contribution.

4 Numerical simulations with local shear fluctuation

The fluctuations of the output energy flow at the tokamak plasma border have been observed experimentally and in the frame of the diffusive model of the transport flux they are associated to the change of the transport regimes and the competition of the instabilities. A self-consistent model of this random change of the transport regimes being difficult to build, it appears useful to study this problem by numerical simulations.

We have introduced a fluctuating diffusion coefficient in the balance equations and studied the statistical properties of the fluctuating plasma parameters. The fluctuations of the electron temperature and of the densities of impurities induce the fluctuations of the resistivity (calculated in the neoclassical model) and of the current density. The volume integral of the current density is less sensitive but the fluctuations are still present and as a consequence we observe random changes of the parameter q and of $\widehat{s}$.

The code solves the balance equations for energy, density and fields [4]. The variables are: the electron and ion temperatures T_e, T_i; the current density j, the poloidal magnetic field B_θ, the toroidal electric field E_φ, the electron density n_e, the radial pinch particle velocity V_r. The following equations are discretized on a one-dimensional space mesh (on the small radius) and evolved in time by a semi-implicit scheme.

$$\frac{3}{2}n_e\frac{\partial T_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\chi_e\frac{\partial T_e}{\partial r} + nVT_e\right)\right) + Ej - \frac{3}{2}n_e\frac{T_e - T_i}{\tau_{ei}} - P_{rad} - P_{ion} + P^e_{add} \tag{28}$$

$$\frac{3}{2}n_i\frac{\partial T_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\chi_i\frac{\partial T_i}{\partial r} + nVT_i\right)\right) + \frac{3}{2}n_e\frac{T_e - T_i}{\tau_{ei}} - P_{cx} + P^i_{add} \tag{29}$$

$$j = \frac{1}{\mu_0}\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\theta\right) \tag{30}$$

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial E}{\partial r} \tag{31}$$

$$E = \eta j \tag{32}$$

$$\frac{\partial n_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial n_e}{\partial r} + rn_eV\right) + S_{ion} \tag{33}$$

$$V = V_{pinch} = \mathrm{const}\frac{E}{B_\theta}\left(\frac{r}{a}\right) \tag{34}$$

The neutral's density $n_0(r,t)$ provides the indirect control of the electron density through the ionization term and also participates in the energy balance by ionization and charge-exchange. The radial profile is prescribed and the time variation of the boundary value of the density is programmed. Although it provides stationary profiles of the density on the current plateau, this simple model does not allow much freedom in the choice of density regimes.

The impurities are considered in corona model for both light (Carbon, Oxygen) and heavy (Iron, Wolfram, Molibden) atoms. The radial profiles of the total densities of these impurity atoms are prescribed. For Carbon and Oxygen, the densities on the various ionization levels are calculated directly with corona tables and for heavy ions using the polynomial fit of the global effects (Z_{eff} , radiation, etc.). The resistivity is neoclassical and the particle pinch velocity has neoclassical form with an empirical value of the constant coefficient [7].

The *electron thermal diffusion coefficient* is modelled as:

$$\chi_e = \chi_{MM} + \chi_{RLW} + \widetilde{\chi} \tag{35}$$

where χ_{MM} (Merejkhin-Mukhovatov) $= 10^{17}\frac{\sqrt{T_e}}{q n_e R}\left(\frac{r}{R}\right)^{7/4}$ [7] and χ_{RLW} is Rebut-Lallia-Watkins model. A particular advantage of using the semi-empirical coefficient χ_{MM} is that the numerical scheme is robust and allows the study of the fluctuating plasma dynamics with not a very sophisticated transport code. The sawtooth is simulated with a fast and high increase in the heat diffusion, which ensures the propagation of the heat from the central region. To avoid the periodic discontinuities generated by the saw-teeth, currently the function χ_e is corrected in the central region according to the empirical prescription which requires to obtain the same radial profile of the temperature as given on the average by the saw-teeth.

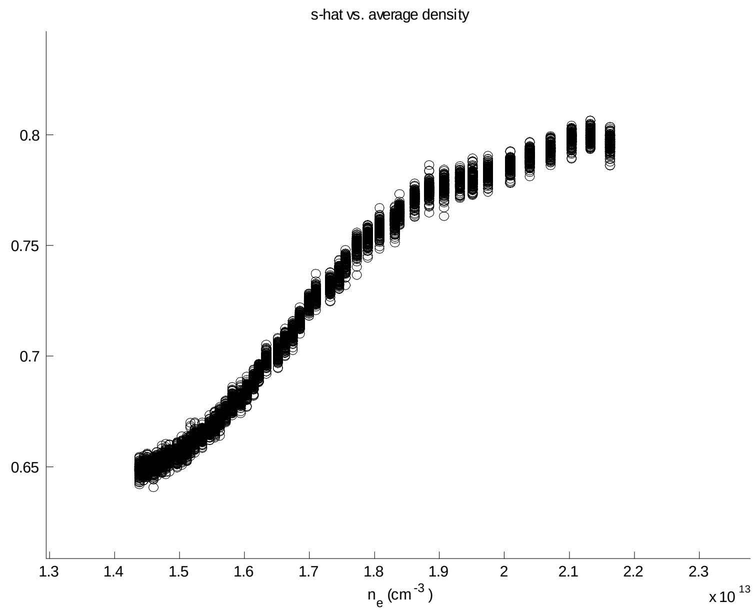

Figure 1: Dependence of the shear parameter on the average density

The simulations have been performed in ohmic regimes with the parameters of the Tore-Supra tokamak. In the absence of $\widetilde{\chi}$, a standard simulation with total plasma current $I_p \sim 1.4\, MA$ reaches a stationary plateau characterized by $T_e(r=0) \sim 1.4\, KeV$, $\overline{n} \sim 3.5 \times 10^{13}\,(cm^{-3})$, $\tau_e \sim 50\, ms$. In the

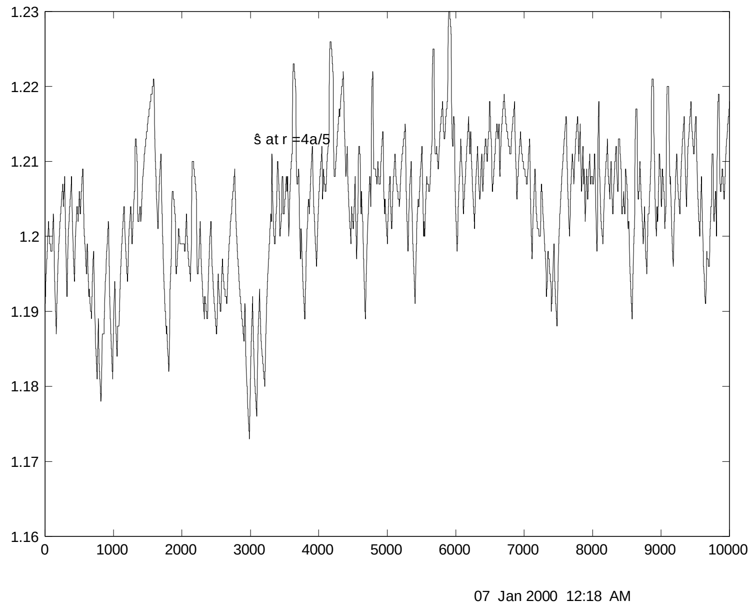

Figure 2: Shear parameter at 4a/5

discussion below, the changes in the magnitudes of the parameters induced by $\widetilde{\chi}$ will be compared to this reference discharge.

Fluctuations of the diffusion coefficient $\widetilde{\chi}$. The limitation of our numerical scheme puts severe bounds on any attempt to realize the transport almost exclusively during intermittent events, i.e. with vanishing background diffusion. Actually, this situation does not seem physically reasonable. In our simulations, on the background diffusion given by $\chi_0 = \chi_{MM} + \chi_{RLW}$ we superpose an intermittent quantity representing $\widetilde{\chi}$. The time series of the intervals where $\widetilde{\chi}$ is switched on is dichotomic (a random discrete series of 0 or 1) and is noted T_{dih} . It is constructed with the following statistical properties:

- Poisson distribution of the density of events over a fixed number of time intervals during the discharge evolution; the duration of events is random with uniform distribution of starting and ending times, however respecting the condition of nonoverlapping for the effective number of events in every interval. This will induce significant self-correlation which replace, in our simulation, those of a more physical origin.

- Random choice (with uniform distribution) of the position of the maximum of $\widetilde{\chi}$ over the radial region where $\widetilde{\chi} \neq 0$ (distance $\widetilde{r}$ to the plasma edge). This region corresponds approximatly to the confinement region, and is limitted by constant values (r_1, r_2). The function $\widetilde{\chi}$ decays on both sides of the maximum as the th function with widths representing a fraction of the corresponding intervals, $|\widetilde{r} - r_{1,2}|$.

- Random amplitude: Gaussian distribution of the fraction of increase above the local diffusion coefficient.

A particular realization of $\widetilde{\chi}$ is obtained using random number generators with these distributions. The code runs on workstation (where the simulations are prepared) and on the massively parallel Cray T3E where the statistical ensemble of realisations of fluctuating diffusive process $\widetilde{\chi}$ are generated and analysed with standard statistical methods.

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