A Study on Geometry inside a Proton

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Abstract

Reconciliation of the theory of General Relativity with the laws of Quantum phenomena is an unsolved problem till now. The proton is a baryon formed by three quarks confined by the gluon field. Again, it may be comparable with a Gaussian Wave Packet. However, in this work, a trial has been made to introduce the general relativistic view of gravity in the system of a proton.

Key words: Gaussian wave packet, energy density inside proton, space time geometry inside a proton.

1. Introduction

This work is a brief study on Space Time Geometry inside a Proton [1] and on pressure and energy density distribution inside a proton [2]. The forms of those two distributions, pressure and energy density inside a proton considered in [1] have been revised in next one [2]. These previous works [1,2] are, initiated being assumed that a proton is comparable with Gaussian wave packet [3] which may also be presented by a Gaussian Wave Form [3]

$$\psi(r) = (2\pi R_P^2/3)^{-3/4} e^{-(3/4R_P^2)\, r^2} \tag{1}$$

where, R_P is rms radius of the localized proton probability distribution. The energy density distribution inside a proton discussed in [1] is $\rho(r) = \rho_0 e^{-2B\, r^2}$. But, revised form of it with known constants is as given below [2]

$$\rho(r) = 1.709 \times e^{-4.687\, r^2} \quad \text{or,} \quad \rho(r) = \rho_0 e^{-(3/2R_P^2)\, r^2} \tag{2}$$

where, $\rho_0 = 1.709$ GeV/fm^3, $R_P = \sqrt{0.32}$ fm and $\int_0^\infty 4\,\pi\, r^2\, \rho(r)\, dr = 0.938$.
This result shows a good agreement with total rest mass of a proton.

It is pointed out that the above consideration apparently enters in collision with the proton structure predicted by quantum chromodynamic (QCD). The proton is a baryon formed by three quarks confined by the gluon field. Therefore, the idyllic image of the quantum mechanical wave packet is far from the proton structure we know from the experimental data we have.

Next, pressure inside a proton considered in [1] is $p(r) = p_0(1 - \alpha\, r^\beta) e^{\eta r^2}$ But, more suitable and revised result of the same given in [2] is $p(r) = 7.2(1 - \frac{r}{0.6}) e^{-5r}$. This may be read in general form as

$$p(r) = p_0\left(1 - \frac{r}{r_0}\right) e^{-\frac{3}{r_0} r} \tag{3}$$

where, $r_0 = 0.6\,fm$. If, one chooses $p_0 \approx 0.76$ GeV/fm^3 (i.e. 1.2×10^{35} pascal) then it can be shown that both pressure and shear force inside a proton comply with the diagrams (holographic QCD) of same given in [4] Therefore, one may revise the work 'space time geometry inside a proton' [1] and, in this work, we have tried to do it.

2. Pressure and Shear Force Distribution inside a Proton

Space time geometry or gravity in a system is directly linked with energy momentum tensor (EMT) in the system. The total quark + gluon stress tensor (T^{ij}) can be decomposed in a traceless part associated with shear forces $s(r)$ and a trace associated with the pressure $p(r)$. For the spherical symmetry of proton, we have tress tensor [5].

$$T^{ij} = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij} p(r)$$

where, the shear forces $s(r)$ and pressure$p(r)$ can be computed as follows [6]

$$s(r) = -\frac{1}{2} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} D(r) \qquad \ldots(4A) \qquad , \qquad p(r) = \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} D(r) \qquad \ldots(4B)$$

The $D(r)$ is a form factor contributed by quark and gluons. Now, we obtain from (4B),

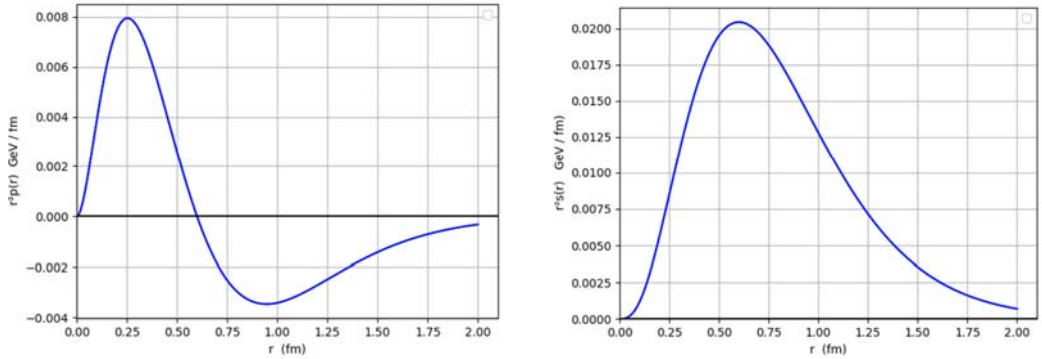
$$D(r) = \int \left(\frac{3}{r^2} \int r^2 p(r)\, dr\right) dr$$

Therefore, with the help of (3) and $D(r)$ shown above, we obtain from (4A)

$$s(r) = -\frac{1}{2} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} D(r) = -\frac{1}{2} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \left[\int \left(\frac{3}{r^2} \int r^2 p(r)\, dr\right) dr\right]$$
$$= \frac{9}{2r^3} \int r^2 p(r)\, dr - \frac{3}{2} p(r)$$

This leads to read $$s(r) = 3 \times 0.76 \times \frac{r}{2r_0} e^{-\frac{3}{r_0} r} \approx 1.9\, r\, e^{-5r} \qquad (5)$$

where, we choose $p_0 \approx 0.74$ GeV/fm^3 and $r_0 = 0.6\,fm$ in (3). As a result, equation (3) and equation (5) leads to the diagrams given in Figure-1.



(a) (b)

Figure-1: (a) The pressure distribution inside the proton [obtained from (3)] and (b) The shear force distribution inside a proton [obtained from (5)].

Again, diagram of the same obtained from reference [4] is given in figure-2. It is interesting to say that the result representing solide blue lines (by holographic QCD) in figure-2 complies with the result of figure-1.

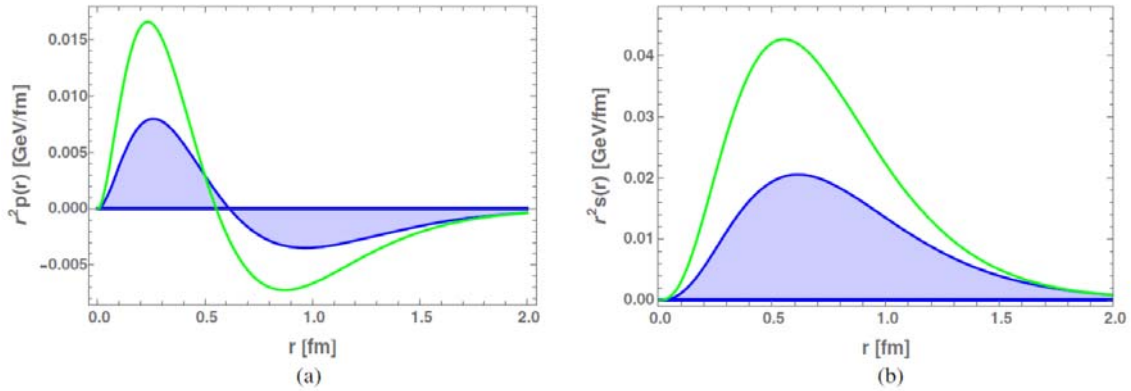


Figure-2: (a) The pressure distribution inside the proton [solid-blue curve (holographic QCD)] and [solid-green curve (lattice QCD)]. (b) The shear force distribution inside a proton [solid-blue curve (holographic QCD)], and [solid-green curve (lattice QCD)].[4]

The data collected from figure-2 [4] are as shown in Table-1 and that exactly obtained from equations (3) and (5) are in Table-2. It is seen that the results in both Table are almost same.

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Table-1: Data on radial pressure and shear force distribution inside a proton. Data collected from Figure-2 by tracing the scanned diagram of Figure-2 (Solid blue curve) onto a graph paper.

r (fm)	0	0.10	0.25	0.40	0.60	0.80	0.94	1.30	1.50
$r^2\ p(r)$ (GeV/fm)	0	0.00379	0.00795	0.00596	0.0	-0.00291	-0.00343	-0.00234	-0.00144
$r^2\ s(r)$ (GeV/fm)	0	0.00105	0.00877	0.01645	0.02030	0.01785	0.01435	0.00631	0.00350

Table-2: Exact data on radial pressure and shear force distribution inside a proton obtained from equation (3) and (5) respectively.

r (fm)	0	0.10	0.25	0.40	0.60	0.80	0.94	1.30	1.50
$r^2\ p(r)$ (GeV/fm)	0	0.00384	0.00794	.00548	0.0	-0.00297	-0.00346	-0.00225	-0.00142
$r^2\ s(r)$ (GeV/fm)	0	0.00115	0.00850	0.01645	0.02043	0.01782	0.01435	0.00627	0.00354

However, energy density distribution given in (2) hold gaussian exponential function but pressure distribution given (3) is a simple exponential function. As a result, energy density vanishes more earlier than that of pressure with increase of r. But, expectation is that if one exists, other must exist, however small, it may be.

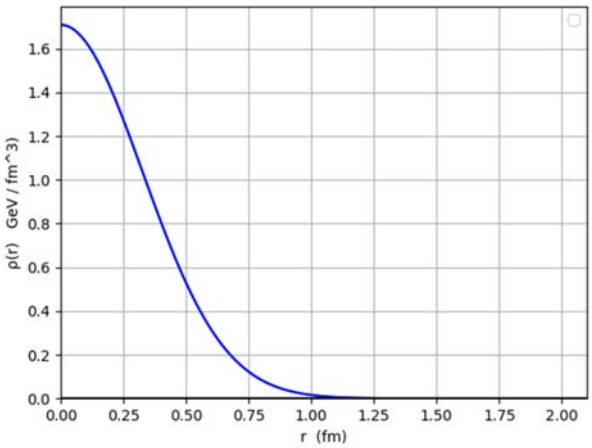

Figure-3: Energy density distribution inside a proton obtained from equation (2).

It is seen that a function of the form $r^2p(r) = p_0r^2(1-\frac{r}{r_0})e^{-\frac{3}{r_0}r}$ satisfies the von Laue stability condition [5]: $\int_0^\infty r^2p(r)dr = 0$ for any real values of p_0 and r_0 . Therefore, we choose it to find out pressure distribution inside a proton in first order approximation. Again, equation (1) is assumed as a gaussian wave form of proton [3] in our first order approximation. We extend it to find out energy density distribution inside a proton in the same approximation. Very interestingly the results show good agreement with our expectation but, a little bit addition is needed for fine adjustment with actual values when r is large comparatively.

Therefore, to make the result correct with second order approximation, one may assume, energy density and pressure distribution inside a proton respectively are

$$\tilde{\rho}(r) = \rho(r) + \phi(r) \quad \ldots(6), \qquad \tilde{p}(r) = p(r) + \varphi(r) \quad \ldots(7)$$

The exact form of $\varphi(r)$ and $\phi(r)$ are to be found out where, functions $\phi(r) = 0$ and $\varphi(r) = 0$ at $r = \infty$. When r is large then, $e^{-5\,r} \approx e^{-\,4.687\,r^2} = 0$, as a result

$$\tilde{\rho}(r) = \rho(r) + \phi(r) \approx \phi(r) \text{ and } \tilde{p}(r) = p(r) + \varphi(r) \approx \varphi(r) \quad (8)$$

This work is completely based on the assumption that proton (or neutron) is not only a particle but also, there exist a quantum field widespread from its center to infinity. Its density falls down sharply as shown in Figure-3 but, a weak part of it remains extant up to a far distance. For a multiparticle system (i.e. extended body), said weak parts collectively appear as a kind of weak field called gravitational field. So, gravity is a quantum field that may be explained by General relativity. Here, $\varphi(r)$ and $\phi(r)$ may be considered as pressure and energy density of gravitational field surrounding a proton.

3. Geometry inside a Proton

The results in previous section imply that Proton is like an incompressible perfect fluid droplet, distribution of equivalent mass inside it is considered as linearly static, it shows spherical symmetry and widespread from $r = 0$ *to* $r = \infty$. So, one can solve Einstein's static field equation inside proton. This topic may be initiated by representing the Einstein's static field equation [7]

$$G_a^b + kT_a^b = 0\ , \quad (k = 8\pi) \tag{9}$$

Here, we work in a natural system of units where speed of light and Newton's gravitational constant are defined to be equal to one. Now, the general form of line element obtained from the above equation is

$$ds^2 = e^{\lambda}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 - e^{\nu}d\tau^2 \tag{10}$$

where, λ and ν are functions of r only and those will be zero at $r = \infty$.

Distribution of equivalent mass inside it is considered as linearly static and it shows spherical symmetry. So, the field equation given in (9) and metric in (10) lead to the relations [7]

$$r^{-2} - r^{-2}e^{-\lambda}(1 + r\nu') + kT_1^1 = 0 \tag{11}$$

$$e^{-\lambda}\left(-\frac{1}{2}\nu'' - \frac{1}{4}\nu'^2 - \frac{1}{2}r^{-1}\nu' + \frac{1}{2}r^{-1}\ \lambda' + \frac{1}{4}\ \lambda'\nu'\right)\delta_B^A + kT_B^A = 0 \tag{12}$$

$$r^{-2} - r^{-2}e^{-\lambda}(1 - r\lambda') + kT_4^4 = 0 \tag{13}$$

where, $\nu' = \frac{d\nu}{dr}$, $\lambda' = \frac{d\lambda}{dr}$. A and B take the values 2 and 3. Now, the new interior Schwarzschild solution of variable density inside a spherical mass of radius , derived by (11) , (12) and (13) given in [7] is

$$ds^2 = \left(1 - \frac{2\mu}{r}\right)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 - \left(1 - \frac{2m}{a}\right)exp\left(\int_a^r \frac{2\mu}{r^2(1 - 2\mu/r)}dr\right)d\tau^2 \tag{14}$$

… (14)

where, $\mu(r) = \int_0^r 4\,\pi\,r^2\,\tilde{\rho}(r)dr$ and $\mu(a) = m$ is total proper mass or equivalent energy present inside the sphere of radius a. In solving the above equation non-vanishing component of energy momentum tensor are taken as $T_1^1 = T_2^2 = T_3^3 = \frac{\mu\tilde{\rho}(r)}{2r(1-2\mu/r)}$ and $T_4^4 = -\tilde{\rho}(r)$. This implies that

$$\tilde{p}(r) = \frac{\mu\tilde{\rho}(r)}{2r(1-2\mu/r)} \tag{15}$$

where, energy momentum tensor is taken in the form $T_k^l = g_k^l\tilde{p} - (\tilde{p} + \tilde{\rho})g_\alpha^k\frac{dx^\alpha}{ds}\frac{dx^l}{ds}$.

If, we assume $a = \infty$ then, we obtain from (14)

$$e^{\nu} = \left(1 - \frac{2m}{a}\right)exp\left(\int_\infty^r \frac{2\mu}{r^2\left(1-\frac{2\mu}{r}\right)}dr\right) = exp\left(\int_\infty^r \frac{2\mu}{r^2(1-2\mu/r)}dr\right) \tag{16}$$

Use of the above relation in (14) leads to read

$$ds^2 = \left(1 - \frac{2\mu}{r}\right)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 - exp\left(\int_\infty^r \frac{2\mu}{r^2(1-2\mu/r)}dr\right)d\tau^2 \tag{17}$$

In the system of proton $\phi(r)$ and $\varphi(r)$ in (6) and (7) respectively are very small valued function and

$$\mu = \int_0^r 4\,\pi\,r^2\,\tilde{\rho}(r)dr \approx \int_0^r 4\,\pi\,r^2\,\rho(r)\,dr.$$

However, equation (17) may be considered as the space time geometry inside a proton.

With the help of (11), (12) and (13) we get

$$\frac{d\tilde{p}(r)}{\tilde{p}(r)+\tilde{\rho}(r)} = -\frac{d\nu}{2} \tag{18}$$

where, $\tilde{p}(r)$ is given in (15) and $dv = \frac{2\mu}{r^2(1-2\mu/r)}$ as shown in (17). When r is large then, μ ≈ m therefore, equations (15), (18) and (8) imply that

$$\frac{d\varphi(r)}{\varphi(r)+\phi(r)} = -\frac{\mathrm{m}}{r^2(1-2m/r)}dr \qquad \text{and} \qquad \varphi(r) = \frac{m\phi(r)}{2r(1-2m/r)} \tag{19}$$

Now, solution of (19) leads to the results as stated below

$$\phi(r) = kr^{-1}(1-2m/r)^{\frac{1}{2}} \quad \text{and} \quad \varphi(r) = \frac{1}{2}kmr^{-2}(1-2m/r)^{-\frac{1}{2}} \tag{20}$$

where speed of light and Newton's gravitational constant are defined to be equal to one. In the above equations, k is a constant in a condition that r is large or $\mu \approx m$. This alludes that k may be a variable

near the core of a proton. So, one may choose, $k = k(\mu)$ where, $k(m)$ must be a constant.

However, energy density and pressure in gravitational field surrounding a proton would be as given in equation (20). If we use normal unit system then m will be replaced by $\frac{mG}{c^2}$. The exact value of k along with its unite will satisfy the equation (20) quantitively and dimensionally.

4. Conclusion

This paper apparently enters in collision with the proton structure predicted by QCD. The idyllic image of the quantum mechanical wave packet is far from the proton structure we know from the experimental data we have. But, it is the fact that the equation (2) shows a good agreement with the result on central mass density, rms radius and total mass of a proton. Therefore, a proton is comparable with the quantum mechanical wave packet.

This work is an effort in reconciliation of the theory of General Relativity with the laws of Quantum phenomena. It is considered that proton (or neutron) is not only a particle but also a system, there exist a quantum field widespread from its center to infinity. In this system the general relativistic view of gravity may be depicted by the geometry given in equation (17).

Again, a massive object is a collective form of huge number of atomic nuclei (i.e. proton and neutron) and electrons. Therefore, it alludes that the gravitational field surrounding an object is the contribution of all atomic nuclei or atomic particles of the object. In this work, the said contribution of a proton has been tried to be shown by the relation given in (17). So, this effort may be a step to relate the concept of General Relativistic gravity with the system of microworld.

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