Electromagnetic Transitions Probability for ^{250}Cf Isotope

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Abstract

The interacting boson model was used to calculate the electric transitions probability for Cf (A=250) isotope, the electric transitions probability B(E2)↓ and B(E2)↑ was calculated, and there is good agreement between the resulting values theoretically comparison with available experimental data, in addition, reduced matrix elements $\langle L_f \| \hat{T}^{(l)} \| L_i \rangle$ is found for each B(E2) values, B(E2) ratios were calculated, some the other related quantities were calculated such as intrinsic quadrupole moment Q_0 and deformation parameter β.

Keywords Reduced transitions probability; reduced matrix elements; interacting boson model; intrinsic quadrupole moment; deformation parameter.

1. Introduction

Comprehending the nuclear structure of nuclei is one of the focal problems in nuclear physics [1]. Many models have been found to study intrinsic structure in the heavy and medium nuclei [2], while the light nuclei, much controversy about their nature still exists today [1]. The interacting boson model (IBM) is one of the models used to study nuclear properties in the heavy and medium nuclei of the low-lying collective states [3]. IBM-1 is the simplest version of it, which does not distinguish between protons bosons and neutrons bosons [4], so it is very suitable for describing even-even nuclei [2,3].

IBM-1 can study many nuclear properties and their related quantities, such as calculating energy levels, energy transitions, energy ratios, electromagnetic transitions, electric quadrupole moment, potential energy surface, and other properties that give an integrated theoretical picture of the nuclei [2-6].

Some properties of $^{250}_{98}Cf_{152}$ isotope were calculated using IBM-1, as it was found that it belongs to transition region SU(3)-U(5), and its properties lie between two dynamical symmetries but are closer to the rotational region than to the vibrational region [7].

In this study, the electrical transition probability B(E2) of the $^{250}_{98}Cf_{152}$ isotope was calculated, values of reduced matrix elements, B(E2) ratios, intrinsic quadrupole moment, and deformation parameter.

1 General Basics

1.1 Electromagnetic Transitions

Electromagnetic transitions, besides reduced matrix elements, can be well described using IBM-1 [2-4], where the transition operators are specified by terms of the boson operators, the transition operators are supposed to contain only one-body terms [3].

The electromagnetic transition operator T^l_m in IBM-1 takes the form [3,8]:

$$T^l_m = \gamma_0 \delta_{l0} \delta_{m0} [s^\dagger \times \tilde{s}]^0_0 + \alpha_2 \delta_{l2} \left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]^2_m + \beta_L \left[d^\dagger \times \tilde{d}\right]^L_m \quad (1)$$

Where $\gamma_0, \alpha_2, \beta_L (L = 0,1,2,3,4)$ are parameters that define the different terms in the conformable transition operator [3].

Equation 1 represents both the electric transition operator $\hat{T}^{(EL)}$ and the magnetic transition operator $\hat{T}^{(ML)}$ [3,8]. Furthermore, equation 1 gives transition operators for (E0, M1, E2, M3, E4, ….) with suitable values of the conformable parameters [9].

The cases of the electric monopole, quadrupole, and hexadecapole transitions, respectively, are indicated in the below equations [3,8,9]:

$$T^{E0}_0 = \beta_0 \left[d^\dagger \times \tilde{d}\right]^0_0 + \gamma_0 [s^\dagger \times \tilde{s}]^0_0 \quad (2)$$

$$T^{E2}_m = \alpha_2 \left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]^2_m + \beta_2 \left[d^\dagger \times \tilde{d}\right]^2_m = e_B \hat{Q} \quad (3)$$

$$T^{E4}_m = \beta_4 \left[d^\dagger \times \tilde{d}\right]^4_m \quad (4)$$

The case of the magnetic dipole and octupole transitions, respectively, are indicated in the below equations [3,8,9]:

$$T^{M1}_m = \beta_1 \left[d^\dagger \times \tilde{d}\right]^1_m \quad (5)$$

$$T^{M3}_m = \beta_3 \left[d^\dagger \times \tilde{d}\right]^3_m \quad (6)$$

Equation 3 can be written in another form as follows [3,8]:

$$T^{E2}_m = E2SD \left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]^2_m + \frac{1}{\sqrt{5}} E2DD \left[d^\dagger \times \tilde{d}\right]^2_m = e_B \hat{Q} \quad (7)$$

In equations 3 and 7, $\alpha_2 = e_B = E2SD$ (effective charge of s and d bosons), $\beta_2 = \chi\alpha_2 = -\frac{\sqrt{7}}{2}\alpha_2 = \frac{1}{\sqrt{5}} E2DD$ (effective charge of d boson), and $\hat{Q}$ is the quadrupole operator [8].

In addition to calculating electromagnetic transitions, one can be calculated the reduced matrix elements [2-4,8]. Where it is obtained from electromagnetic transitions probability values [3], using the compatible transition operator of the reduced matrix element, which lies between the initial state and the final state [2-4], as the following [3,9]:

$$B(l; L_i \to L_f) = \frac{1}{(2L_i+1)} \left|\langle L_f \| \hat{T}^{(l)} \| L_i \rangle\right|^2 \quad (8)$$

Where $l = EL \; or \; ML$.

B(E2) term includes both electric transitions probability B(E2)↓ and B(E2)↑ measured in square electron barn unit e^2b^2. The relationship between B(E2)↓ and B(E2)↑ is the following [10]:

$$B(E2) \downarrow \times \, 5 = \mathrm{B(E2)} \uparrow \text{ or vice versa } \mathrm{B(E2)} \uparrow \div \, 5 = B(E2) \downarrow \quad (9)$$

Available experimental B(E2) values in Weisskopf units (W.u.), then converted to square electron barn units e^2b^2 as follows [11]:

$$B(EL)(e^2b^2) = B(EL)(W.u.) \times 5.94 \times 10^{-6} \times A^{4/3} \quad (10)$$

Here, e is the electron charge, and b (1 barn=10^{-28} square meters) is a unit of area.

Each dynamical symmetry has a B(E2) transition probability of its own depending on the bosons number (N) for the isotope [2,4].

1.2 B(E2) Ratios

Branching ratios (B(E2) ratios) are important quantities that explain the difference between different dynamical symmetries [8], where it represents the ratio of the reduced transition probability of the electric quadrupole transitions for three cases, as the following [3,8]:

$$R = \frac{B(E2; 4_1^+ \to 2_1^+)}{B(E2; 2_1^+ \to 0_1^+)}, R' = \frac{B(E2; 2_2^+ \to 2_1^+)}{B(E2; 2_1^+ \to 0_1^+)}, R'' = \frac{B(E2; 0_2^+ \to 2_1^+)}{B(E2; 2_1^+ \to 0_1^+)} \quad (11)$$

Equation 11 using to calculate a B(E2) ratio of the transition regions as well as the three limits from dynamical symmetries [8,9].

1.3 The Electric Quadrupole Moments

The electric quadrupole moments (Q_L) represent the deviation measure for the spherical symmetry of the distribution of nuclear charge density within the nucleus [8,9,12,13]. It is an important quantity to determine the shape of the nucleus, as the following [4,8]:

The nucleus shape is spherical ($Q = 0$).
The nucleus shape is deformed oblate ($Q < 0$).
The nucleus shape is deformed prolate ($Q > 0$).
Electric quadrupole moments (Q_L) are defined by [8]:

$$Q_L = \sqrt{\frac{16\pi}{5}} \sqrt{\frac{L(2L-1)}{(2L+1)(L+1)(2L+3)}} \sqrt{B(E2; L_i \to L_f)} \quad (12)$$

L: the angular momentum.

Then, by definition, the quadrupole moment $Q_{2_1^+}$ and the intrinsic quadrupole moment Q_0 are, respectively [8,9,12]:

$$Q_{2_1^+} = \sqrt{\frac{16\pi}{5}} \begin{bmatrix} L & 2 & L \\ -L & 0 & L \end{bmatrix} \langle L_f \| \hat{T}^{(E2)} \| L_i \rangle \quad (13)$$

$$Q_0 = \sqrt{\frac{16\pi}{5} \times B(E2) \uparrow / e^2} \quad (14)$$

1.4 Related Quantities

From other related quantities with electric transition probability B(E2) that give an exact description of the shape of nuclei, the deformation parameter (β) was calculated using the following equation [12]:

$$\beta = \sqrt{B(E2) \uparrow / e^2}\left(4\pi / \left(3ZeR_0{}^2\right)\right) \quad (15)$$

Where Z is the atomic number, R_0 is the average radius of the nucleus given by [8,12]:

$$R_0{}^2 = 0.0144A^{2/3}b = \left(1.2 \times 10^{-13}A^{1/3}cm\right)^2 = \left(1.2A^{1/3}fm\right)^2 \quad (16)$$

Where A is the mass number of nuclei.

The relationship between β and $B(E2) \uparrow$ in equation 15 gives an important physical indication, so it's rather useful, but as evidence for the existence of quadrupole collective effects in nuclei, the single-particle model predicted that the (β) value is less helpful because it includes effects, which vary with the size of the nucleus (lower values for heavy nuclei). Therefore, it is preferable to calculate the ratio $\beta/\beta_{(sp)}$, as an indication of the collective quadrupole motion in the nuclei, by using [12]:

$$\beta_{(sp)} = 1.59/Z \quad (17)$$

As a result of using the quantity $\beta_{(sp)}$ is followed by using the value $B(E2) \uparrow_{(sp)}$ in equation 15. This $B(E2) \uparrow_{(sp)}$ value in the single-particle model is given by [12]:

$$B(E2) \uparrow_{(sp)} = 2.97 \times 10^{-5}A^{4/3}\, e^2b^2 \quad (18)$$

On the other hand, the energy-weighted sum-rule (EWSR) strength, gives the amount of total transition strength expected in a given particular nucleus. These are as follows [14]:

$$S(I) = \sum E \times B(E2) \uparrow = 30e^2(\hbar^2/8\pi m)AR_0{}^2 \quad (19)$$

Where m is the nucleon mass and $(3/5)R_0{}^2$ is used for the single-particle, mean-square radius.

The sum-rule isoscalar $E2$ strength, which is part of the full sum is given as follows [15]:

$$S(II) = S(I)(Z/A)^2 \quad (20)$$

Also, the other important quantity τ mean a lifetime of the state in ps units, extracting B(E2)↑ from a lifetime measurement (or vice versa), mean lifetime τ relates with B(E2)↑ through the following equation [12]:

$$\tau = 40.81 \times 10^{13} E^{-5} [B(E2) \uparrow / e^2 b^2]^{-1} (1+\alpha)^{-1} \quad (21)$$

$$\tau(1+\alpha) = \tau_\gamma = 40.81 \times 10^{13} E^{-5} [B(E2) \uparrow / e^2 b^2]^{-1} \quad (22)$$

Where E is the energy of 2_1^+ level in keV units, the theoretical α value calculated for a specific E value is considered exact, where it represents the total internal conversion coefficient ($\alpha > 0.001$).

2 Results and Discussion

2.1 Electric Transitions Probability

Electric transition probability B(E2) is an important property for studying nuclei's nuclear structure, as it gives an exact description of the nucleus of any isotope. B(E2) values were calculated theoretically using (IBST.FOR) program according to equation 7.

The values of B(E2) generated by the program, represented by (IBMT.OUT) dependent on the input data (BE2.DAT), represented by three parameters (E2SD, E2DD, and SO6), as shown in Table 1.

Table 1 The values of the parameters of the B(E2) and $\langle L_f || \hat{T}^{(E2)} || L_i \rangle$ for Cf (A=250) isotope by using (IBST.FOR) program

Isotope	N_π	N_ν	N	$\alpha_2\ (eb)$	$\beta_2\ (eb)$	$E2SD(eb)$	$E2DD(eb)$	SO(6)
$^{250}_{98}Cf_{152}$	8	13	21	1.78506	-2.36140	1.78506	-5.28027	0.0000

($\boldsymbol{N_\pi, N_\nu,}$ **N**) proton bosons, neutron bosons, and the total bosons number, respectively.

After input data treatment in the program, the output data is B(E2), then the resulting values for B(E2) from the program were compared with available experimental data, as shown in Table 2.

Table 2 Theoretical values of reduced matrix elements $\langle L_f || \hat{T}^{(E2)} || L_i \rangle$ and probabilities of electric transitions B(E2) compared with the available experimental data for the Cf (A=250) isotope

Isotope		**Spin sequences**	**B(E2)↓ $(eb)^2$**		**B(E2)↑ $(eb)^2$**		$\langle L_f \vert\vert \hat{T}^{(E2)} \vert\vert L_i \rangle$ ***eb***
		$L_f^+ - L_i^+$	**IBM-1**	**EXP.[16,17]**	**IBM-1**	**EXP.[16,17]**	
$^{250}_{98}Cf_{152}$	SU(3)-SU(5)	$2_1^+ - 0_1^+$	3.1806	3.18067	15.9030	15.90335	3.9878
		$0_2^+ - 2_1^+$	0.3583	----	1.7915	----	2.9929
		$0_3^+ - 2_1^+$	0.96956	----	4.8478	----	4.9233
		$2_2^+ - 0_1^+$	0.01984	0.02152	0.09923	0.10758	9.9612

$2_2^+ - 2_1^+$	0.02269	0.03461	0.11345	0.17306	7.5317
$2_2^+ - 4_1^+$	0.00225	0.00197	0.01123	0.00987	10.0525
$0_2^+ - 2_2^+$	2.23556	----	11.1778	----	7.4759
$0_3^+ - 2_2^+$	0.97452	----	4.8726	----	4.9359
$2_3^+ - 0_1^+$	0.4222	----	2.111	----	1.4529
$2_3^+ - 0_2^+$	0 .3663	----	1.8315	----	1.3533
$0_3^+ - 2_3^+$	1.1710	----	5.8550	----	5.4106
$2_4^+ - 0_1^+$	0.2639	----	1.3195	----	1.1487
$2_4^+ - 0_2^+$	14.7460	----	73.7300	----	8.5866
$2_4^+ - 0_3^+$	0.5815	----	2.9075	----	1.7051
$2_3^+ - 2_1^+$	2.7266	----	13.633	----	8.2562
$2_3^+ - 2_2^+$	2.7219	----	13.6095	----	8.2491
$2_4^+ - 2_1^+$	0.2708	----	1.3540	----	2.6019
$2_4^+ - 2_2^+$	0.1890	----	0.9450	----	2.1737
$2_4^+ - 2_3^+$	0.0000	----	0.0000	----	0.0000
$4_1^+ - 2_1^+$	2.2691	----	11.3455	----	7.5317
$2_3^+ - 4_1^+$	0.5481	----	2.7405	----	4.9663
$4_2^+ - 2_1^+$	12.6965	----	63.4825	----	17.8161
$4_2^+ - 2_2^+$	0.5077	----	2.5385	----	3.5626
$2_3^+ - 4_2^+$	3.06692	----	15.3346	----	11.7478
$4_3^+ - 2_1^+$	0.2708	----	1.3540	----	2.6019
$4_3^+ - 2_2^+$	0.3360	----	1.6800	----	2.8983
$4_3^+ - 2_3^+$	0.0000	----	0.0000	----	0.0000

Table 2 includes both electric transitions probability B(E2)↓ and B(E2)↑ measured in square electron barn unit, and comparison with available experimental data, there is a good agreement in some values and dissimilarity with others, in addition, the reduced matrix elements $\langle \boldsymbol{L_f} || \widehat{\boldsymbol{T}}^{(\boldsymbol{E2})} || \boldsymbol{L_i} \rangle$ corresponding to each B(E2) are found theoretically using equation 8, and their values depend on values of B(E2), and measured using electron barn unit.

Figs. 1 and 2 show the relationship between spin sequences $(L_f^+ - L_i^+)$ and electric transitions probability B(E2)↓ and B(E2)↑, respectively. It can be seen from the Fig. 1 that the relationship between $(L_f^+ - L_i^+)$ and B(E2)↓ is a fluctuating relationship, since B(E2)↓ according to equation 8 is inversely proportional to $(2L_i^+ + 1)$, as a result of this and from equation 9, B(E2)↑ is in the same way, which is explained by Fig. 2.

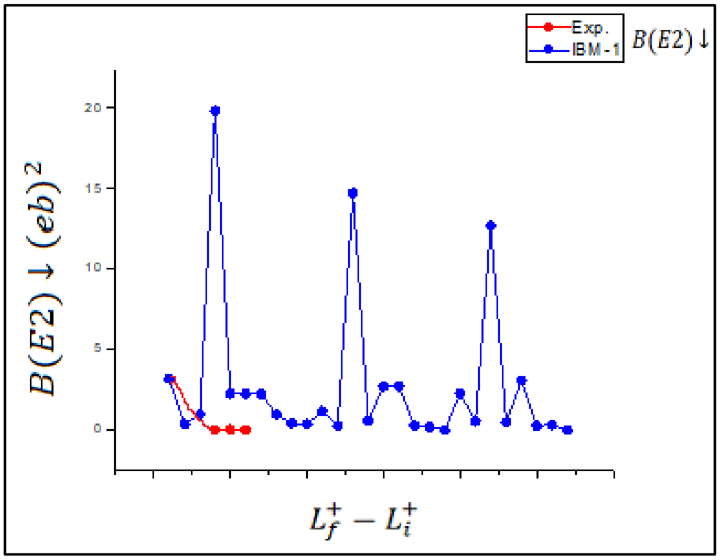

Fig. 1 Relationship between angular momentum and reduced electric transitions B(E2)↓ (experimental [17] and IBM-1 calculated values) in $^{250}_{98}Cf_{152}$ isotope

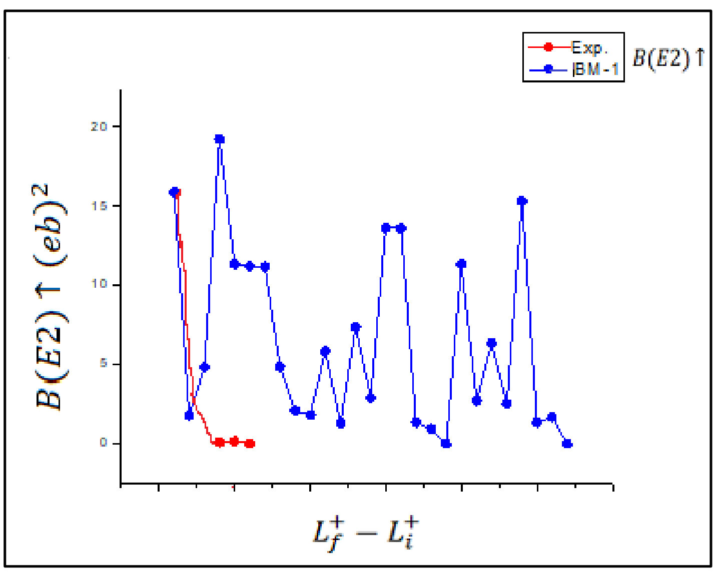

Fig. 2 Relationship between angular momentum and reduced electric transitions B(E2)↑ (experimental [17] and IBM-1 calculated values) in $^{250}_{98}Cf_{152}$ isotope

Fig. 3 as in Figs. 2 and 3 explains the relationship between $(L_f^+ - L_i^+)$ and $\langle L_f || \hat{T}^{(E2)} || L_i \rangle$ theoretically and depending on the values of the B(E2)↑ corresponding to each level of transition as well as the initial transition level, where it is directly proportional to B(E2)↑ and inversely proportional to $(2L_i^+ + 1)$.

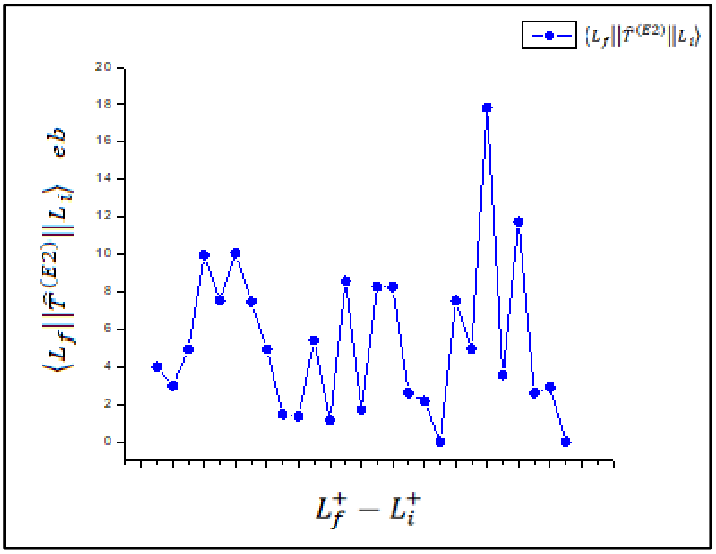

Fig. 3 Relationship between angular momentum and reduced matrix elements in ${}^{250}_{98}Cf_{152}$ isotope

The experimental data do not contain all B(E2) values, but rather contain a few values that are adopted to calculate the parameters entered into the program, hence, the calculation of B(E2) values theoretically. Available experimental B(E2) values in Weisskopf units (W.u.), then converted to square electron barn units $(eb)^2$ using equation 10.

Table 3 represents some available experimental data for probabilities of electric transitions B(E2) in [W.u.] for the Cf (A=250) isotope, it was found that there is some difference between the theoretical values and the available experimental values. The reason for this difference is attributed to the fact that all measured properties of isotopes theoretically depend on the total number of bosons ($\mathbf{N} = \boldsymbol{N_\pi} + \boldsymbol{N_\nu}$), and since proton bosons ($\boldsymbol{N_\pi}$) is constant for any element, then neutron bosons ($\boldsymbol{N_\nu}$) is the primary and variable factor in each isotope, and therefore most of the properties of isotopes depend on it primarily in addition to other factors, as the increase the number of neutrons bosons affects the accuracy of theoretically adopted results from experimental measurements.

Table 3 The available experimental data for probabilities of electric transitions B(E2) in [W.u.] compared with theoretical values for the Cf (A=250) isotope

Spin sequences $\mathbf{L_f^+ - L_i^+}$	**B(E2)↓(W.u.)** **EXP.[12,17]**	**B(E2)↓(W.u.)** **Theo.**
$2_1^+ - 0_1^+$	340	340
$2_2^+ - 4_1^+$	0.211	0.2121
$2_2^+ - 2_1^+$	3.7	2.42
$2_2^+ - 0_1^+$	2.3	2.40

2.2 B(E2) Ratios

B(E2) ratios were calculated using equation 11, as shown in Table 4.

Table 4 Standard B(E2) ratios for dynamical symmetries [8] and for Cf (A=250) isotope calculated using IBMT

Isotope		$^{250}_{98}Cf_{152}$	N	21
B(E2) Ratio	**SU(5)**	**SU(3)**	**O(6)**	**IBMT**
R	< 2	$\frac{10}{7}$	$\frac{10}{7}$	0.7134
R′	< 2	0	$\frac{10}{7}$	0.7134
R″	< 2	0	0	0.1126

Table 4 indicates the resulting B(E2) ratios (R, R', R'') theoretically, and compared to their ideal values for the three limits SU(5), SU(3), and O(6), respectively, as shown that Fig. 4, the isotope under study lies between SU(5) and SU(3) dynamical symmetries. From B(E2) ratios, it is noticed that the isotope approaches more to dynamical symmetry SU(3) than to SU(5), and this is confirmed by the reference [7].

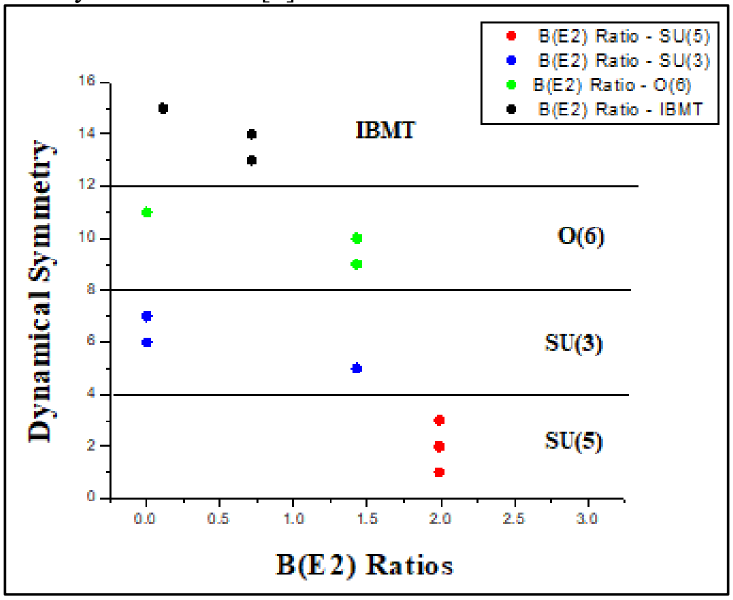

Fig. 4 Comparison between B(E2) ratios for the three dynamical symmetries and calculated with IBMT for $^{250}_{98}Cf_{152}$ isotope

2.3 Other Related Quantities

Quadrupole moment has been calculated theoretically using IBM-1, as a result of the calculation of reduced electric quadrupole transition probability B(E2), it was calculated for the second level at the same band as shown in Table 5.

Table 5 Theoretical values of Q(E2) for the Cf (A=250) isotope

Quadrupole moment	Values (b)
$Q(E2; 2_1^+ - 2_1^+)$	-15.1667
$Q(E2; 2_2^+ - 2_2^+)$	5.6195
$Q(E2; 2_3^+ - 2_3^+)$	-16.863

From important quantities for studying the nuclear structure of the isotopes, and determining the shape of the isotopes are the following quantities in a Table 6.

Table 6 Adopted value B(E2)↑ and related quantities for the Cf (A=250) isotope

Isotope			$^{250}_{98}Cf_{152}$		
Quantity	E (level) (KeV)	$\tau\ (ps)$	β	$\beta/\beta_{(sp)}$	EWSR (I)(%)
Exp. [12]	42.722	145	0.299	18.400	0.97
IBM-1	42.721	150	0.298	18.367	0.969
Quantity	B(E2)↑ $(eb)^2$	τ_γ	$B(E2)\uparrow_{(sp)}$	$Q_0(b)$	EWSR (II)(%)
Exp. [12]	15.9033	181395	0.0467	12.70	6.3
IBM-1	15.9030	187650	0.0467	12.64	6.299

Intrinsic quadrupole moment and deformation parameter were calculated using equations 14 and 15, respectively.

In the last column of Table 6, the quantity $E \times B(E2)\uparrow$ for just the first 2^+ state was expressed as a percentage of $S(I)$ and $S(II)$.

3 Conclusions

Interacting bosons model can be used to study many nuclear properties and thus know the shape of the nucleus or the amount of deformation in it. Electric transitions probability B(E2)↓ and B(E2)↑ of $^{250}_{98}Cf_{152}$ isotope was studied and compared with the available experimental results. There is a good agreement in some values and differences in other values, many unknown values have been predicted experimentally, and the reduced matrix elements closely related to the electric transitions probability were calculated, in addition to that, the branching ratios of $^{250}_{98}Cf_{152}$ isotope were calculated, which play an important role in determining the shape of the nucleus. Another important quantity related to the electric transitions probability is the electric quadrupole moment, which was found using IBMT resulting and the intrinsic quadrupole moment, which were calculated theoretically and compared with the available experimental value, the deformation parameter, mean lifetime τ and other quantities were also calculated, which gave results that are completely identical to the available experimental results. All these characteristics prominent the shape of the nucleus of the isotope accurately and confirm the shape of the dynamic symmetry of the isotope.

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Declarations

Conflict of interests none.

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