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A geometrical approach to the consubstantiality of the gravitational and electromagnetic interactions

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Abstract

In this article, we establish that the gravitational and electromagnetic fields have a consubstantial nature and obtain expressions for the description of the electromagnetic-gravitational field in Y^4 space. The applied approach can be described as follows. First, we develop Eddington's ideas to obtain an analog of Maxwell's theory in Y^4, interpreting the symmetric terms as metric and the antisymmetric part as an electromagnetic field. Then, the antisymmetric part is studied as an electromagnetic field; finally, we introduce the Weyl theory to determine the geometrical structure of the world by applying the Weyl assumption about partition between geometry and electricity.

Keywords: Electromagnetism, Weyl's theory, Geometry, Eddington's theory
AMS Subject Classification: 83C50, 83C20, 83D05, 83D99.

1 Introduction

After the theory of general relativity was developed between 1907 and 1920, a problem arose of unification of the gravitational and electromagnetic fields. There are several different approaches to the creation of unified field theory; the most recent is string theory that considers high-dimensional spaces with subsequent reduction (convolution) to the observable phenomenal space. The initial approach was made by the founding fathers of relativity theory: A. Einstein [10, 11, 12, 13, 14], A. Eddington [9], and H. Weyl [36, 37, 38]. Its main idea is to construct a four-dimensional continuum so that the mathematical description of all fundamental forces can be coherently obtained from the variational principle; this theory must contain gravitational and Maxwell's field equations in a natural way [5, 6, 41, 42, 43, 44, 45, 46].

In [41, 42, 43, 44, 45, 46] we studied the geometrical structure of the Y^n space and some of its applications to physical problems. In the present article, we consider the amalgamation of the ideas of A. Einstein, A. Eddington, and H. Weyl into one comprehensive theory based on the geometrical structure of space with torsion, where the connection is defined by

$$\Gamma^p_{kl} = \frac{1}{2} g^{pi} \left(g_{ik,l} + g_{li,k} - g_{kl,i} + g_{km} S^m_{li} + g_{lm} S^m_{ki} \right) + \frac{1}{2} S^p_{kl}.$$

Let us recall that the classical Maxwell equations in four-dimensional representation describe the electromagnetic phenomena of the physical world. The Maxwell electromagnetic field can be described in terms of a four-vector potential $A_i = \left(\phi,\ \tilde{A}_1,\ \tilde{A}_2,\ \tilde{A}_3 \right)$ in

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the form

$$F_{ik} = A_{i;k} - A_{k;i} = A_{i,k} - A_{k,i} + S^{p}_{ki} A_p.$$

Maxwell's equations are

$$F_{ik,l} + F_{li,k} + F_{kl,i} = 0$$

and

$$F^{ik}{}_{;k} = \frac{4\pi}{c} J^i,$$

where $J^i = (\rho,\ J^1,\ J^2,\ J^3)$ is the current vector. The electromagnetic field in vacuum can be expressed as

$$F^{ik}{}_{;k} = 0.$$

The main idea of this article is to combine different relativistic methods into one coherent consistent model of the electromagnetogravitational theory in Riemannian space with torsion, which provides the simplest possible description of gravitational and electromagnetic fields and reveals their consubstantiality.

2 The field equations in Y^n-space

Let us consider a four-dimensional continuum with the structure of a Y^n manifold.

In this space, the infinitesimal parallel transportation of a vector A^i is defined according to the formula

$$A^i{}_{;k} = A^i{}_{,k} + \Gamma^i_{jk} A^j, \tag{1}$$

where we denote $\frac{\partial A^i}{\partial x^k} = A^i{}_{,k}$, and it can be shown that the connection can be calculated by using the metric and torsion tensors [33] as

$$\Gamma^p_{kl} = \frac{1}{2} g^{pi} \left(g_{ik,l} + g_{li,k} - g_{kl,i} + g_{km} S^m_{li} + g_{lm} S^m_{ki} \right) + \frac{1}{2} S^p_{kl}. \tag{2}$$

The tensor R^p_{ikl} is the curvature tensor of Y^n and can be written as [33]

$$R_{ik} = R^p_{pik} = \Gamma^p_{ip,k} - \Gamma^p_{ik,p} + \Gamma^p_{qk} \Gamma^q_{ip} - \Gamma^p_{qp} \Gamma^q_{ik},$$

and

$$R_{ik} = R^p_{pik} = \Gamma^p_{ip,k} - \Gamma^p_{ik,p} + \Gamma^p_{qk} \Gamma^q_{ip} - \Gamma^p_{qp} \Gamma^q_{ik}, \tag{3}$$

where the connection can be written in the form $\Gamma^p_{kl} = \mathrm{P}^p_{kl} + L^p_{kl}$ or $\Gamma^p_{kl} - \mathrm{P}^p_{kl} = L^p_{kl}$.

From the definitions of covariant derivative and curvature tensor, we have the equation

$$S^i_{jk;p;q} - S^i_{jk;q;p} = R^t_{qpj} S^i_{tk} + R^t_{qpk} S^i_{jt} - R^i_{qpt} S^t_{jk} + S^t_{qp} S^i_{jk;t},$$

or

$$S^i_{jk;p;q} - S^i_{jk;q;p} - S^t_{qp} S^i_{jk;t} = R^t_{qpj} S^i_{tk} + R^t_{qpk} S^i_{jt} - R^i_{qpt} S^t_{jk}.$$

3 The Eddington theory

In Eddington's approach, we assume that the connection of the space does not depend on the space metric. We compose the scalar density as

$$W_g = \sqrt{-g}\, g^{ik} R_{ik}$$

and apply the variational principle of least action, postulating that all variations of the functional

$$\int g^{ik} R_{ik} \sqrt{-g}\, dV$$

with respect to the connection vanish, provided the integral is not varied at the boundaries.

By standard calculations, we have

$$\delta \int g^{ik} R_{ik} \sqrt{-g}\, dV = \int g^{ik}\, \delta \left(\Gamma^p_{ip,k} - \Gamma^p_{ik,p} + \Gamma^p_{qk}\Gamma^q_{ip} - \Gamma^p_{qp}\Gamma^q_{ik} \right) \sqrt{-g}\, dV.$$

The variation with respect to the connection Γ^m_{nt} gives

$$\int \left(g^{nt}{}_{,m} - g^{nk}{}_{,k}\delta^t_m + g^{it}\Gamma^n_{im} + g^{nk}\Gamma^t_{mk} - g^{nt}\Gamma^p_{mp} - g^{ik}\Gamma^n_{ik}\delta^t_m \right) \sqrt{-g}\, \delta(\Gamma^m_{nt})\, dV = 0,$$

and consequently

$$g^{nt}{}_{,m} - g^{nk}{}_{,k}\delta^t_m + g^{it}\Gamma^n_{im} + g^{nk}\Gamma^t_{mk} - g^{nt}\Gamma^p_{mp} - g^{ik}\Gamma^n_{ik}\delta^t_m = 0.$$

By contracting this equation over indices t and m, we obtain

$$g^{nm}{}_{,m} - 4g^{nk}{}_{,k} + g^{im}\Gamma^n_{im} + g^{nk}\Gamma^m_{mk} - g^{nm}\Gamma^p_{mp} - 4g^{ik}\Gamma^n_{ik} = 0,$$

and

$$3\left(g^{nm}{}_{,m} + g^{im}\Gamma^n_{im} \right) + g^{nm} S^p_{mp} = 0. \tag{4}$$

We contract the previous equation with the tensor g_{nt} and obtain

$$g_{nt} g^{nt}{}_{,m} - g_{nm} g^{nk}{}_{,k} + \Gamma^n_{nm} - 3\Gamma^n_{mn} - g_{nm} g^{ik}\Gamma^n_{ik} = 0, \tag{5}$$

or

$$2g^{pm}\Big((\ln(\sqrt{g}))_{,m} + \Gamma^n_{mn} \Big) + g^{pm}{}_{,m} + g^{pm} S^n_{mn} + g^{im}\Gamma^p_{im} = 0.$$

We introduce the notation and obtain

$$\begin{aligned} \gamma^n &= \frac{1}{3} g^{nm} S^p_{mp} = -\left(g^{nm}{}_{,m} + g^{im}\Gamma^n_{im} \right) \\ &= 2g^{nm}\Big((\ln(\sqrt{g}))_{,m} + \Gamma^p_{mp} \Big) + g^{nm} S^p_{mp} = -g^{nm}\Big((\ln(\sqrt{g}))_{,m} + \Gamma^p_{mp} \Big). \end{aligned} \tag{6}$$

3.1 Field equations [41].

Let us contract the equation

$$S^i_{jk;p;q} - S^i_{jk;q;p} = R^t_{qpj} S^i_{tk} + R^t_{qpk} S^i_{jt} - R^i_{qpt} S^t_{jk} + S^t_{qp} S^i_{jk;t}, \tag{7}$$

by the indices k, p and multiply by g^{js}; we have

$$\begin{aligned} &\left(g^{kp} g^{js} S^i_{sk;p} - g^{kp} g^{js} S^i_{qp} S^q_{sk} \right)_{;i} - g^{kp} g^{js} S^i_{sk;i;p} - g^{kp} g^{js} S^i_{pq;i} S^q_{sk} \\ &= g^{kp} g^{js} R^t_{ips} S^i_{tk} + g^{kp} g^{js} R^t_{ipk} S^i_{st} - g^{kp} g^{js} R^i_{ipt} S^t_{sk}. \end{aligned} \tag{8}$$

Let us denote

$$H^{ji} = g^{kp} g^{js} S^i_{sk;p} - g^{kp} g^{js} S^i_{qp} S^q_{sk}, \tag{9}$$

and

$$F^{jp} = g^{kp} g^{js} S^i_{sk;i}. \tag{10}$$

We introduce an asymmetric tensor (asymmetric in any pair of indices)

$$C^{ijk} = g^{pj} g^{qk} S^i_{pq} + g^{pk} g^{qi} S^j_{pq} + g^{pi} g^{qj} S^k_{pq}. \tag{11}$$

We have

$$H^{ji}_{;i} - F^{ji}_{;i} - g^{kp} g^{js} S^q_{sk} F_{pq} = g^{kp} g^{js} R^t_{ips} S^i_{tk} + g^{kp} g^{js} R^t_{ipk} S^i_{st} - g^{kp} g^{js} R^i_{ipt} S^t_{sk}, \tag{12}$$

where $F_{pq} = S^i_{pq;i}$.

We obtain

$$H^{jk} - H^{kj} = C^{ikj}_{;i} + F^{jk} + g^{kp} g^{qs} S^t_{pq} S^j_{ts} - g^{jp} g^{qs} S^t_{pq} S^k_{ts}, \tag{13}$$

and we have

$$g^{kp} g^{qs} S^t_{pq} S^j_{ts} - g^{jp} g^{qs} S^t_{pq} S^k_{ts} = \frac{1}{2}\left(C^{jpq} S^k_{pq} - C^{kpq} S^j_{pq}\right), \tag{14}$$

hence

$$H^{jk} - H^{kj} = C^{ikj}_{;i} + F^{jk} + \frac{1}{2}\left(C^{jpq} S^k_{pq} - C^{kpq} S^j_{pq}\right). \tag{15}$$

It is easy to show that

$$C^{ikj}_{;i} = -C^{ijk}_{;i} = -\left(C^{ijk}_{,i} + \Gamma^j_{pi} C^{ipk} + \Gamma^k_{pi} C^{ijp} + \Gamma^i_{pi} C^{pkj}\right). \tag{16}$$

We conclude

$$\Gamma^j_{pi} C^{ipk} = \Gamma^j_{ip} C^{ipk} = \frac{1}{2}\left(\Gamma^j_{ip} C^{ipk} + \Gamma^j_{pi} C^{pik}\right) = \frac{1}{2} S^j_{ip} C^{pki} = -\frac{1}{2} S^j_{pi} C^{kpi},$$

and

$$\Gamma^k_{pi} C^{ijp} = \Gamma^k_{ip} C^{pji} = \frac{1}{2} C^{jpi}\left(\Gamma^k_{ip} - \Gamma^k_{pi}\right) = \frac{1}{2} S^k_{ip} C^{jpi} = \frac{1}{2} S^k_{pq} C^{jqp}.$$

Then we have

$$C^{ikj}_{;i} = -C^{ijk}_{;i} = -C^{ijk}_{,i} - \frac{1}{2} S^j_{pq} C^{kpq} + \frac{1}{2} S^k_{pq} C^{jpq} - \Gamma^q_{pq} C^{pkj}, \tag{17}$$

and we conclude

$$H^{jk} - H^{kj} = -C^{ijk}_{,i} - \Gamma^q_{pq} C^{pkj} + F^{jk}. \tag{18}$$

Since $\Gamma^p_{pl} = \frac{1}{2} g_{ip,l} g^{ip} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}}{\partial x^l}$, $\Gamma^p_{lp} = \Gamma^p_{pl} + S^p_{lp}$, and $\Gamma^p_{pl} = \frac{1}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial x^l} + (\ln \psi)_{,l} = (\ln(\psi\sqrt{-g}))_{,l}$, we obtain

$$H^{jk} - H^{kj} - F^{jk} = -C^{ijk}_{,i} - \left(\ln\left(\psi\sqrt{-g}\right)\right)_{,i} C^{ikj}. \tag{19}$$

Finally, we have obtained the equality

$$\left(\psi\sqrt{-g}\left(H^{jk} - H^{kj} - F^{jk}\right)\right)_{,k} = 0, \tag{20}$$

where $S^p_{ip} = \varphi_i = 3 g_{in} \gamma^n = (\ln \psi)_{,i}$.

Since

$$\gamma^n = \frac{1}{3} g^{nm} S^p_{mp} = -g^{nm}\left((\ln(\sqrt{g}))_{,m} + \Gamma^p_{mp}\right), \tag{21}$$

and recalling that $\Gamma^p_{pl} = \frac{1}{2} g_{ip,l} g^{ip} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}}{\partial x^l}$, $\Gamma^p_{lp} = \Gamma^p_{pl} + S^p_{lp}$, and $\Gamma^p_{lp} = \frac{1}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial x^l} + (\ln \psi)_{,l} = (\ln(\psi\sqrt{-g}))_{,l}$, we obtain

$$\gamma^n = \frac{1}{3} g^{nm} S^p_{mp} = -g^{nm}\left((\ln(\sqrt{g}))_{,m} + \left(\ln\left(\psi\sqrt{-g}\right)\right)_{,m}\right). \tag{22}$$

The difference $L^p_{kl} - \frac{1}{2} S^p_{kl}$ is a symmetric tensor and, following Eddington's idea, we can obtain

$$L^p_{kl} - \frac{1}{2} S^p_{kl} = \frac{1}{6} \delta^p_k J_l + \frac{1}{6} \delta^p_l J_k - \frac{1}{2} g_{kl} g^{pq} J_q, \tag{23}$$

where we hypothesize $\gamma^n = J^n$.

4 The Weyl theory

The contracted Riemann tensor can be represented as the sum of a symmetric and an antisymmetric tensor:

$$R_{ik} = s_{ik} + a_{ik}, \tag{24}$$

where the tensors s_{ik} and a_{ik} are functions of the connection.

The symmetric tensor s_{ik} corresponds to the gravitational field (it can be understood as a metric tensor), and the antisymmetric tensor a_{ik} corresponds to the electromagnetic field.

From these tensors and densities, we can compose the scalar density $\wp$ and, applying the variational principle to the Hamiltonian integral and taking into consideration that the variations of the field vanish on the boundary of the variational domain, we obtain

$$\int \left(\frac{\partial \wp}{\partial s_{ik}} \delta s_{ik} + \frac{\partial \wp}{\partial a_{ik}} \delta a_{ik} \right) dV = 0. \tag{25}$$

Let f^{ik} be the density of the electromagnetic field and let us denote

$$\frac{\partial \wp}{\partial s_{ik}} = g^{ik}, \qquad \frac{\partial \wp}{\partial a_{ik}} = f^{ik}, \tag{26}$$

the tensor densities g^{ik}, f^{ik} are the densities of the gravitational and electromagnetic fields.

By the variational principle, we obtain

$$2g^{nt}{}_{;m} - g^{nk}{}_{;k}\delta^t_m - g^{tk}{}_{,k}\delta^n_m - \frac{\partial f^{nk}}{\partial x_k}\delta^t_m - \frac{\partial f^{tk}}{\partial x_k}\delta^n_m = 0. \tag{27}$$

Next, we use a duality argument and obtain

$$\int \left(\frac{\partial \wp^*}{\partial s_{ik}} \delta g^{ik} + \frac{\partial \wp^*}{\partial a_{ik}} \delta f^{ik} \right) dV = 0, \tag{28}$$

and

$$\frac{\partial \wp^*}{\partial g^{ik}} = s_{ik}, \qquad \frac{\partial \wp^*}{\partial f^{ik}} = a_{ik}. \tag{29}$$

The vector-valued function $\frac{\partial f^{nk}}{\partial x_k}$ is the electric current density, and $g_{ik} = s_{ik}$.

Conditions (26) mean that the expression in brackets under the integral in (27) is an exact differential in the variables s_{ik} and a_{ik}; similarly, (29) states that the expression under the integral in (28) is an exact differential in the arguments g^{ik} and f^{ik}.

Conclusions

An amalgamation of these three aspects of the description of natural phenomena gives us the possibility to obtain a comprehensive relativistic theory, which provides mathematical apparatuses for the representation of physical understanding of physical phenomena of the electromagnetic and gravitational fields from a single viewpoint and gives a possibility to derive the electromagnetic and gravitational field equations as its special cases.

References

[1] I. Agricola and T. Friedrich, *A note on flat metric connections with antisymmetric torsion*, Differential Geometry and its Applications, vol. 2, pp. 480–487, 2010.

[2] K. Bredies, *Symmetric tensor fields of bounded deformation*, Ann. Mat. Pura Appl. vol. 192, no. 5, pp. 815–851, 2013.

[3] E. Cartan, *Les espaces de Finsler*, Exposés de Géométrie, vol. 2, Paris, 1934.

[4] A. Chamblin, H. S. Reall, H. Shinkai, and F. Shiromizu, *Charged brane-world black holes*, Phys. Rev. D 63, 064015 (2001).

[5] E. Cartan and J. Schouten, *On Riemannian geometries admitting absolute parallelism*, Nederlandse Akademie van Wetenschappen. Proceedings. Series A, vol. 29, pp. 933–946, 1926.

[6] E. Cartan and J. Schouten, *On the geometry of the group manifold of simple and semisimple groups*, Nederlandse Akademie van Wetenschappen. Series A, vol. 29, pp. 803–815, 1926.

[7] J. D. Clayton, *On anholonomic deformation, geometry, and differentiation*, Mathematics and Mechanics of Solids, vol. 17, no. 7, pp. 702–735, 2012.

[8] E. N. Dzhafarov and H. Colonius, *Multidimensional technician scaling: Basics*, Journal of Mathematical Psychology 45 (2001), no. 5, 670–719.

[9] A. Eddington, *The Mathematical Theory of Relativity*, Cambridge University Press, 1952.

[10] A. Einstein, *Auf die Riemann-Metrik und den Fern-Parallelismus gegrundete einheitliche Feldtheorie*, Math. Annal. 102, 685 (1930). For an English translation, see A. Unzicker and T. Case, *Unified Field Theory based on Riemannian Metrics and distant Parallelism*, (arXiv:physics/0503046v1).

[11] A. Einstein, *The Meaning of Relativity*, Princeton Univ. Press, Princeton, 1921.

[12] A. Einstein, *Relativity: The Special and General Theory*, New York: H. Holt and Company, 1920.

[13] A. Einstein, *Geometry with Maintaining the Notion of Distant Parallelism*, Riemannian Session Report of the Prussian Academy of Sciences, pp. 217–221, June 7th, 1928.

[14] A. Einstein, *Théorie unitaire de champ physique*, Ann. Inst. H. Poincaré, 11 pp. 1–24, 1930.

[15] R. Essig, E. Kuffik, S. D. McDermott, T. Volansky, and K. M. Zurek, *Constraining light dark matter with diffuse X-ray and gamma-ray observations*, Journal of High Energy Physics 11 (Nov. 2013) 193.

[16] J. Jost, *Riemannian Geometry and Geometric Analysis*, Springer-Verlag, Berlin, 2005.

[17] F. Klein, *Vergleichende Betrachtungen über neuere geometrische Forschungen*, Gesammelte Abh. Vol. 1, Springer, pp. 460–497, 1921.

[18] L. Kozma and L. Tamassy, *Finsler geometry without line elements faced to applications*, Reports on Mathematical Physics 51, 2003.

[19] A. Helmi, S. D. White, and V. Springel, *The phase-space structure of a dark-matter halo: Implications for dark-matter direct detection experiments*, Phys. Rev. D 66 (Sept. 2002) 063502.

[20] M. Kuhlen, A. Pillepich, J. Guedes and P. Madau, *The Distribution of Dark Matter in the Milky Way's Disk*, Astrophys. J. 784 (Apr. 2014) 161.

[21] T. Lacroix, M. Stref and J. Lavalle, *Anatomy of Eddington-like inversion methods in the context of dark matter searches*, JCAP 1809 (2018) 040.

[22] T. Lacroix, M. Boudaud, M. Stref, and J. Lavalle, *Robust cosmic-ray constraints on p-wave annihilating MeV dark matter*, Phys. Rev. D 99 (2019) 061302.

[23] H. Liu, T. R. Slatyer and J. Zavala, *Contributions to cosmic reionization from dark matter annihilation and decay*, Phys. Rev. D 94 (Sept. 2016) 063507.

[24] J. S. Long and J. C. Price, *Current short-range tests of the gravitational inverse square law*, Comptes Rendus Physique 4, 337–346, 2003.

[25] A. Massari, E. Izaguirre, R. Essig, A. Albert, E. Bloom, and G. A. Gomez-Vargas, *Strong optimized conservative fermi-lat constraints on dark matter models from the inclusive photon spectrum*, Phys. Rev. D 91 (Apr. 2015) 083539.

[26] S. Manoff, *Frames of reference in spaces with affine connections and metrics*, Class. Quantum Grav., 2001.

[27] B. Riemann, *Mathematische Werke*, Leipzig, Uber die Hypothesen, Welche der Geometrie zu Grunde liegen, pp. 272–287, 1892.

[28] D. Sean, *Multivariate calculus, and geometry*, 3rd ed., Springer Undergraduate Mathematics Series, Berlin, Springer, 259 p., 2014.

[29] P. Steinmann, H. Altenbach, and V. A. Eremeyev, *On the roots of continuum mechanics in differential geometry a review*, in *Generalized Continua from the Theory to Engineering Applications*, vol. 541 of CISM International Centre for Mechanical Sciences, pp. 1–64, Springer, Udine, Italy, 2013.

[30] J. Saczuk, K. Hackl, and H. Stumpf, *Rate theory of nonlocal gradient damage-gradient viscoelasticity*, International Journal of Plasticity, vol. 19, no. 5, pp. 675–706, 2003.

[31] T. Sauer, *Field equations in teleparallel space-time: Einstein's Fernparallelismus approach toward unified field theory*, Hist. Math. 33, p. 399, 2006.

[32] Tabassum Naz Sindhu, Zawar Hussain & Muhammad Aslam, *Parameter and reliability estimation of inverted Maxwell mixture model*, Journal of Statistics and Management Systems, 22:3, 459–493, DOI: 10.1080/09720510.2018.1552412, 2019.

[33] J. G. Vargas, *Differential geometry for physicists and mathematicians. Moving frames and differential forms: From Euclid past Riemann*, 2014.

[34] F. Wiseman, Phys. Rev. D, Vol. 65, 2002.

[35] R. Weitzenbock, *Invariantentheorie*, P. Noordhoff, Groningen, 1923.

[36] H. Weyl, *Gravitation und Elektrizität*, Sitzungsber. Akad. Wiss. Berlin, p. 465, 1918.

[37] H. Weyl, *Space, Time, Matter*, 4th ed., Dover, 1952 (original: *Raum Zeit Materie*, Berlin: Springer, 1921).

[38] H. Weyl, *Elektron und Gravitation I*, Zeitschrift Physik, 56, pp. 330–352, 1929.

[39] R. Wojtak, E. L. Lokas, G. A. Mamon, S. Gottlober, A. Klypin, and Y. Hofman, *The distribution function of dark matter in massive haloes*, MNRAS 388 (Aug. 2008) 815–828.

[40] A. Yavari and A. Goriely, *The geometry of discombinations and its applications to semi-inverse problems in elasticity*, Proceedings of the Royal Society of London A, vol. 470, article 0403, 2014.

[41] M. I. Yaremenko, *Derivation of Field Equations in Space with the Geometric Structure Generated by Metric and Torsion*, Journal of Gravity, Volume 2014, 13 p., 2014.

[42] M. I. Yaremenko, *Field Equations in Space* Y^n *without symmetry conditions in the General Case*, Global Journal of Physics, Vol. 2, 11 July 08, pp. 89–106, 2015.

[43] M. I. Yaremenko, *The Geometry of* Y^n *Space and Its Application to Theory of General Relativity, Derivation of Field Equations*, Journal of Advances in Physics, Vol. 12, 12, pp. 4291–4306, 2016.

[44] M. I. Yaremenko, *Geometry of* Y^n *space generated by metric and torsion, the field equations*, Lambert Academic Publishing, Germany, 2017.

[45] M. I. Yaremenko, *Einstein-Maxwell Theory without Symmetry and Examples of Geodesics in Space with Torsion*, Canadian Journal of Physics, `https://doi.org/10.1139/cjp-2017-0896`, 2017.

[46] M. I. Yaremenko, *Maxwell's Equations as the Special Case of General Theory of Relativity in* Y^n*-Space*, Journal of Advanced Mathematics and Applications, Volume 6, Number 1, February 2017, pp. 87–92.

[47] M. I. Yaremenko, *Einstein-Maxwell Theory without Symmetry and Examples of Geodesics in Space with Torsion*, Journal of Progressive Research in Modern Physics and Chemistry, Volume 2, Number 1, 2017, pp. 61–77.

[48] Y. Zhao, X.-J. Bi, H.-Y. Jia, P.-F. Yin and F.-R. Zhu, *Constraint on the velocity-dependent dark matter annihilation cross-section from fermi-lat observations of dwarf galaxies*, Phys. Rev. D 93 (Apr. 2016) 083513.

[49] Q. Zhu, F. Marinacci, M. Maji, Y. Li, V. Springel and L. Hernquist, *Baryonic impact on the dark matter distribution in Milky Way-sized galaxies and their satellites*, MNRAS 458 (May 2016) 1559–1580.