Physics AUC, vol. 34, 235-244 (2024) PHYSICS AUC

Interaction vertices in a model involving two Majorana spinors, a scalar field, and an Abelian vector field

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Abstract

The investigation of interaction vertices that can be added to a free, massless theory involving two Majorana spinors, a scalar field, and an Abelian vector field is done. This is performed within the framework of the antifield-antibracket formalism by deformation of the solution to the classical master equation combined with specific techniques of BRST cohomology.

1 Introduction

The Standard Model, the current paradigm of the fundamental interactions (except the gravitational one), is designed with initial massless quanta, irrespective of their bosonic or fermionic nature. Then, the mass comes into play via the Higgs boson due to 'breaking' some internal higher symmetry of the initial theory. The experimental confirmation of the Standard Model predictions, including the existence of the Higgs boson, has sparked the interest of the scientific community in considering mass-generation schemes in field theory [1, 2] that encompass the outputs of the standard Higgs mechanism [3, 4, 5, 6]. In all these procedures, the gauge fields acquire mass in the context of their interactions with some scalar matter fields. Whereas the standard procedure [3, 4, 5, 6] assumes a scalar field potential that possesses a degenerate global minimum, the new procedures [1, 2] do not ask for this assumption but only use the interaction as a sufficient context for gauge fields to get masses. The original Higgs mechanism, devised to provide masses for gauge fields, also impacts the leptonic mass spectrum in the electroweak sector, initially trivial, by imposing a Yukawa-type interaction vertex between spinors and scalar fields just from the outset. Given these broad advantages of the Higgs mechanism, it is natural to investigate if the new procedures [1, 2] can be adapted to encompass the standard mass-generation mechanism also in the fermionic field sector. In this paper, we fructify the idea [1] by showing how a single Abelian 1-form gains mass in the presence of its interactions with a matter field spectrum consisting of a real scalar and two Majorana spinor fields.

The present paper is devoted to the analysis of consistent interactions that can be added to a massless free field theory consisting of a real scalar field, two Majorana spinor

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fields, and an Abelian 1 form. The investigation is performed using the deformation theory, implemented in the antifield-antibracket formalism as a deformation problem for the solution to the classical master equation [12, 13], and solved with the help of specific local BRST cohomology techniques [14, 15, 16]. This procedure, supplemented with some reasonable hypotheses, standard in field theory, leads to some quadratic, derivative-free interaction vertices naturally interpreted as mass terms for various field spectrum components. In this approach, the derived interacting models depend on some free real parameters, which verify a set of consistency equations with a twofold 'dichotomy' in the behavior of its solutions: i) the 1-form and the scalar field cannot be simultaneously massive, and ii) the 1-form and the spinor fields are simultaneously massive unless there are cross-couplings between them.

The paper is organized into six sections as follows. Section 2 is dedicated to the constuction of antifield-antibracket BRST symmetry associated to a massless free field theory, with field spectrum comprising one real scalar field, two real spinors, and a single Abelian 1-form. Section 3 sketches the BRST approach to the problem of constructing consistent interactions mediated by gauge fields. In Section 4, one applies the BRST strategy for deriving the interaction vertices to the considered free model, i.e., one solves the equations that govern the deformation of the solution to the classical master equation under analysis. At this point, one proves that the most general interacting gauge theory, consistently constructed out of the considered starting model and subject to some standard hypotheses in field theory, depends on a degree four polynomial function in the scalar field, two real constants, and six 2×2 real matrices, which are subject to some purely algebraic (consistency) equations. Section 5 displays the previously mentioned twofold 'dichotomy' by showing that the general solution to the consistency equations naturally splits into two complementary classes. Section 6 ends the paper with the main conclusions.

2 Free model: the antifield-antibracket BRST symmetry

The assumed field theory context is a flat one, defined by a four-dimensional Minkowski spacetime of mostly minus signature, $\mathbb{R}^{1|3}$, supplemented with the real (Majorana) representation of the associated Clifford algebra $\mathscr{C}(1,3)$. The field model exhibits four non-interacting real massless fields: a scalar field, two Majorana spinors, and a single Abelian 1-form, with the local dynamics variational generated from the functional

$$S_0^{\mathrm{L}}\left[A,\varphi,\Psi\right] = \int d^4x \Big[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \tfrac{\mathrm{i}}{2}\bar{\Psi}^\top \partial\!\!\!/ \Psi\Big], \tag{1}$$

where $F_{\mu\nu}$ is the usual field-strength, $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$, Ψ is a column spinor-matrix

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix},$$

consisting of real (Majorana) spinors

$$\psi_\Delta^{\mathscr{C}} := (\mathscr{C}\psi_\Delta)^\top = \bar{\psi}_\Delta := \psi_\Delta^\dagger\gamma^0, \quad \Delta = 1,2.$$

By its very definition, the starting model (1) is manifestly invariant under the generating set of gauge transformations

$$\delta_\epsilon A^\mu = \partial^\mu\epsilon, \qquad \delta_\epsilon\varphi = 0, \qquad \delta_\epsilon\psi_\Delta = 0, \tag{2}$$

which is irreducible and Abelian.

The general prescriptions of the antifield-antibracket BRST formalism [7, 8, 9, 10, 11] adapted to the current situation firstly assumes the identification of the BRST complex, that is generated by

$$\{A^{\mu}, \varphi, \psi_{\Delta}, \eta\}, \quad \{A^{*}_{\mu}, \varphi^{*}, \psi^{*}_{\Delta}, \eta^{*}\}, \tag{3}$$

whose degrees and Grassmann parities read

$$\begin{aligned}
&\mathrm{agh}(A^{\mu}) = \mathrm{agh}(\varphi) = 0, && \mathrm{agh}(\psi_{\Delta}) = 0, && \mathrm{agh}(\eta) = 0, && (4)\\
&\mathrm{agh}(A^{*}_{\mu}) = \mathrm{agh}(\varphi^{*}) = 1, && \mathrm{agh}(\psi^{*}_{\Delta}) = 1, && \mathrm{agh}(\eta^{*}) = 2, && (5)\\
&\mathrm{pgh}(A^{\mu}) = \mathrm{pgh}(\varphi) = 0, && \mathrm{pgh}(\psi_{\Delta}) = 0, && \mathrm{pgh}(\eta) = 1, && (6)\\
&\mathrm{pgh}(A^{*}_{\mu}) = \mathrm{pgh}(\varphi^{*}) = 0, && \mathrm{pgh}(\psi^{*}_{\Delta}) = 0, && \mathrm{pgh}(\eta^{*}) = 0, && (7)\\
&\varepsilon(A^{\mu}) = \varepsilon(\varphi) = 0, && \varepsilon(\psi_{\Delta}) = 1, && \varepsilon(\eta) = 1, && (8)\\
&\varepsilon(A^{*}_{\mu}) = \varepsilon(\varphi^{*}) = 0, && \varepsilon(\psi^{*}_{\Delta}) = 0, && \varepsilon(\eta^{*}) = 0. && (9)
\end{aligned}$$

The BRST complex is also equipped with a natural involution according to which the fields are real and the antifields are purely imaginary, i.e.,

$$\begin{aligned}
&(A_{\mu})^{\star} = A_{\mu}, && (\varphi)^{\star} = \varphi, && (\psi_{\Delta})^{\star} = \psi_{\Delta}, && (\eta)^{\star} = \eta,\\
&(A^{*}_{\mu})^{\star} = -A^{*}_{\mu}, && (\varphi^{*})^{\star} = -\varphi^{*}, && (\psi^{*}_{\Delta})^{\star} = -\psi^{*}_{\Delta}, && (\eta^{*})^{\star} = -\eta^{*}.
\end{aligned}$$

One secondly defines the BRST differential associated with the theory (1)–(2). Taking into account the field independence of the gauge generators (2), it results that the BRST differential s simply reduces to

$$s = \delta + \gamma, \tag{10}$$

with δ the Koszul–Tate differential, $\mathrm{agh}(\delta) = -1$, which enforces the algebra of smooth functions on the stationary field equations in its homology, and the longitudinal exterior derivative along gauge orbits, γ, $\mathrm{pgh}(\gamma) = 1$. These two degrees do not interfere [$\mathrm{agh}(\gamma) = 0$, $\mathrm{pgh}(\delta) = 0$]. The overall degree that grades the BRST algebra is known as the ghost number [gh] and is defined like the difference between the pure ghost number and the antifield number, such that $\mathrm{gh}(s) = \mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = 1$. The two differentials act on the BRST generators like

$$\begin{aligned}
&\delta A^{\mu} = 0, && \delta\varphi = 0, && \delta\psi_{\Delta} = 0, && \delta\eta = 0, && (11)\\
&\delta A^{*}_{\mu} = \partial^{\nu} F_{\mu\nu}, && \delta\varphi^{*} = \Box\varphi, && \delta\psi^{*}_{\Delta} = -\mathrm{i}\bar{\psi}_{\Delta}\overleftarrow{\not{\partial}}, && \delta\eta^{*} = -\partial^{\mu} A^{*}_{\mu}, && (12)\\
&\gamma A^{\mu} = \partial^{\mu}\eta, && \gamma\varphi = 0, && \gamma\psi_{\Delta} = 0, && \gamma\eta = 0, && (13)\\
&\gamma A^{*}_{\mu} = 0, && \gamma\varphi^{*} = 0, && \gamma\psi^{*}_{\Delta} = 0, && \gamma\eta^{*} = 0. && (14)
\end{aligned}$$

At last, the BRST complex naturally possesses a Gerstenhaber-like structure, the well-known antibracket, $(\cdot,\cdot)$, defined by decreeing the fields/ghosts conjugated with the corresponding antifields. Within this structure, the BRST differential s admits a canonical $s\cdot = (\cdot, S)$, with S the canonical generator. It is a real [i.e., invariant under the natural involution on the BRST algebra] bosonic functional of ghost number zero, involving both field/ghost and antifield spectra, that encodes the entire gauge structure of the associated theory and obeys the classical master equation

$$(S, S) = 0 \tag{15}$$

which is equivalent to second-order nilpotency of the BRST differential, $s^2 = 0$. In the present context, (1)–(2), the canonical generator of the BRST symmetry takes the simple form

$$S = S_0^{\mathrm{L}} + \int \mathrm{d}^4x \left(A_\mu^* \partial^\mu \eta \right). \tag{16}$$

3 Consistent interactions: the BRST approach

The present part is devoted to a brief review of the BRST approach to the problem of constructing consistent interactions that can be added to a given gauge field theory [12, 13], maintaining the field spectrum and the number of degrees of freedom. As this issue can be formulated as a deformation problem [17], it can be addressed in the BRST context as a deformation problem for the solution to the master equation corresponding to a given "free" theory [12, 13] in the framework of the local BRST cohomology [14, 15, 16]. This means that if an interacting theory can be consistently constructed, then the solution S to the master equation associated with the "free" theory can be deformed into a solution $\bar{S}$

$$S \to \bar{S} = S + \lambda S_1 + \lambda^2 S_2 + \lambda^3 S_3 + \cdots, \quad \varepsilon(\bar{S}) = 0, \quad \mathrm{gh}(\bar{S}) = 0 \tag{17}$$

of the master equation for the deformed theory that pertains to the original "free" BRST algebra, namely,

$$(\bar{S}, \bar{S}) = 0. \tag{18}$$

By projecting the equation (18) on various powers in the deformation parameter λ, one obtains the equivalent tower of equations:

$$\lambda^0 : (S, S) = 0, \tag{19}$$
$$\lambda^1 : sS_1 = 0, \tag{20}$$
$$\lambda^2 : \tfrac{1}{2}(S_1, S_1) + sS_2 = 0, \tag{21}$$
$$\lambda^3 : (S_1, S_2) + sS_3 = 0, \tag{22}$$
$$\lambda^4 : \tfrac{1}{2}(S_2, S_2) + (S_1, S_3) + sS_4 = 0, \tag{23}$$
$$\vdots$$

As S is nothing but the solution to the classical master equation corresponding to the "free" theory, it results that (19) is satisfied by construction. The remaining equations are to be solved recursively, from lower to higher orders, such that each equation corresponding to a given order of perturbation theory, say k ($k \geq 1$), contains a single unknown functional, namely, the deformation of order k, S_k. With the deformed solution to the master equation (17) at hand, from its various antightost number terms, one can read the entire gauge structure of the resulting interacting theory.

4 Cross-couplings between a scalar field, two Majorana fields, and an Abelian 1-form

This section is devoted to the construction of consistent interactions that can be added to a massless "free" theory consisting of a real scalar field, two real spinor fields, and a single Abelian 1-from. This is done through the procedure sketched in Section 3, by constructing, term by term, the functional (17), which starts with the "free" version (16).

From the plethora of solutions to (18), one retains only those that comply with specific hypotheses from field theory as analyticity in the coupling constant, Lorentz covariance, space-time locality, and Poincaré invariance. Moreover, one discards the solutions involving free parameters of strictly negative mass-dimension, such that the power-counting *non-renormalizable* [18] interacting field theories are avoided.

4.1 The deformed solution to the classical master equation

According to the working hypotheses, among other restrictions made at the beginning of this section, only the local solutions to (18) are considered. This particularly implies that the first-order deformation S_1 can be expressed as

$$S_1 = \int \mathrm{d}^4x\, a, \tag{24}$$

where a depends smoothly on the BRST generators (3) and their space-time derivatives up to a finite order, i.e., is a local function. Inserting this realization into (20), one obtains the equivalent equation

$$sa = \partial_\mu j^\mu, \quad \mathrm{gh}(a) = 0, \quad \varepsilon(a) = 0, \tag{25}$$

with j^μ a local current. As only non-trivial couplings are searched for, i.e., those that do not come from some field redefinitions, the solutions of type $a = s\bar{a} + \partial_\mu \bar{j}^\mu$ will not be taken into account [14, 15, 16].

The simplest way for solving equation (25) makes use of the inhomogeneous structure of the field spectrum due to the three kinds of fields: scalar, vector, and spinor. Accordingly, the first-order deformation naturally decomposes into seven components

$$a = a^{(\varphi)} + a^{(\psi)} + a^{(A)} + a^{(\varphi-\psi)} + a^{(\varphi-A)} + a^{(A-\psi)} + a^{(\mathrm{int})}, \tag{26}$$

where $a^{(\varphi)}$, $a^{(\psi)}$, and $a^{(A)}$ govern the self-interactions of the scalar field φ, the spinor fields ψ_Δ, and the vector field A^μ, respectively, $a^{(\varphi-\psi)}$, $a^{(\varphi-A)}$, and $a^{(A-\psi)}$ describe the cross-couplings scalar-spinor, scalar-vector, and vector-spinor, respectively, whereas a^{int} effectively mixes all the three sectors. Inserting decomposition (26) into (25) and invoking the independence of various sectors, it results that equation (25) is equivalent to seven independent equations, one for each piece,

$$sa^{(\mathrm{sector})} = \partial_\mu j^\mu_{(\mathrm{sector})} \tag{27}$$

At this point, in solving equations (27), the working hypotheses made in the beginning come into play. A simple analysis of the mass-dimension corresponding to the assumed field spectrum displays

$$[A] = M = [\varphi]\,, \quad [\psi_\Delta] = M^{3/2}, \tag{28}$$

which, further, exhibits the general real solutions to (27) that comply with the specified hypotheses

$$a^{(\varphi)} = -\mathscr{V}(\varphi), \quad a^{(\psi)} = \bar{\Psi}^\top(\mathrm{V} + \mathrm{i}\gamma_5\tilde{\mathrm{V}})\Psi, \quad a^{(A)} = \tfrac{\alpha}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}, \tag{29}$$

$$a^{(\varphi-\psi)} = \bar{\Psi}^\top(\mathrm{U} + \mathrm{i}\gamma_5\tilde{\mathrm{U}})\Psi\varphi, \quad a^{(\varphi-A)} = n\,(\varphi^*\eta - A^\mu\partial_\mu\varphi)\,, \tag{30}$$

$$a^{(A-\psi)} = \Psi^{\top *}(\mathrm{W} + \mathrm{i}\gamma_5\tilde{\mathrm{W}})\Psi\eta - \tfrac{\mathrm{i}}{2}\bar{\Psi}\gamma^\mu(\mathrm{W} + \mathrm{i}\gamma_5\tilde{\mathrm{W}})\Psi A_\mu, \quad a^{(\mathrm{int})} = 0. \tag{31}$$

Previously, $\mathscr{V}$ is an arbitrary polynomial function in the scalar field, which is of degree at most equal to four, such that its coefficients are of positive mass-dimensions, α and n are arbitrary real numbers, while V, $\tilde{\mathrm{V}}$, U, $\tilde{\mathrm{U}}$, W, and $\tilde{\mathrm{W}}$ are some 2×2 arbitrary real matrices with all symmetric, but W, which is skew-symmetric.

According to the deformation algorithm, once the first-order deformation has been completed, one imposes its consistency, i.e., the requirement for equation (21) to possess solutions. By direct computation one identifies the second-order deformation

$$S_2 = \tfrac{1}{2}\int \mathrm{d}^4x(n^2 A_\mu A^\mu), \tag{32}$$

as well as the consistency equations

$$n\frac{d\mathscr{V}}{d\varphi} = 0, \tag{33}$$

and

$$[\mathrm{U},\mathrm{W}] = \{\tilde{\mathrm{U}},\tilde{\mathrm{W}}\}, \qquad \left[\tilde{\mathrm{U}},\mathrm{W}\right] = -\{\mathrm{U},\tilde{\mathrm{W}}\}, \tag{34}$$

$$[\mathrm{V},\mathrm{W}] = \{\tilde{\mathrm{V}},\tilde{\mathrm{W}}\} - n\mathrm{U}, \qquad \left[\tilde{\mathrm{V}},\mathrm{W}\right] = -\{\mathrm{V},\tilde{\mathrm{W}}\} - n\tilde{\mathrm{U}}. \tag{35}$$

Inspecting the remaining equations, i.e., (22), (23), etc., direct computations based on the results (24) and (32) yield

$$(S_1, S_2) = 0 = (S_2, S_2),$$

which allow to conclude that higher-order deformations can be made trivial

$$S_k = 0, \quad k > 2. \tag{36}$$

At this point we have determined the full deformed solution to the classical master equation

$$\bar{S} = S + \lambda S_1 + \lambda^2 S_2 \tag{37}$$

where the various order deformations S, S_1, and S_2 are given in (16), (24), and (32) respectively. At this point, a whole family of interacting field theories has been displayed. It is expressed in terms of an arbitrary polynomial function of degree at most four in the undifferentiated scalar field, two real constants and six 2×2 real matrices, all subject to the consistency conditions (33)–(35).

4.2 The Lagrangian gauge structure of the interacting field theories family

According to the general rules of the BRST formalism, information about the gauge structure is derived from the antighost-homogeneous terms of the deformed solution (37). More precisely, the antighost number zero piece of the functional (37)

$$\begin{aligned} S^{\mathrm{L}}[A,\varphi,\psi] = \int d^4x \big[&-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(\partial_\mu\varphi - \lambda n A_\mu)(\partial^\mu\varphi - \lambda n A^\mu) + \tfrac{\mathrm{i}}{2}\bar{\Psi}^\top \not{D}\Psi \\ &-\lambda\mathscr{V} + \tfrac{\lambda\alpha}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} + \lambda\bar{\Psi}^\top(\mathrm{V}+\mathrm{i}\gamma_5\tilde{\mathrm{V}})\Psi + \lambda\bar{\Psi}^\top(\mathrm{U}+\mathrm{i}\gamma_5\tilde{\mathrm{U}})\Psi\varphi\big] \end{aligned} \tag{38}$$

is just the Lagrangian action of the family interacting field theories, while the antighost number one terms display the generating set of gauge transformations for (38) as

$$\bar{\delta}_\epsilon A^\mu = \partial^\mu \epsilon, \quad \bar{\delta}_\epsilon \varphi = gn\epsilon, \quad \bar{\delta}_\epsilon \Psi = (\mathrm{W} + \mathrm{i}\gamma_5 \tilde{\mathrm{W}})\Psi\epsilon. \tag{39}$$

As the functional (37) does not contain terms of antighost number greater or equal to two, it results that the generating set of gauge transformations (39) is irreducible and remains Abelian, as the starting one (2). In the Lagrangian action corresponding to the interacting gauge theory (38) one used the covariant derivatives

$$\not{D}\Psi = \gamma^\mu(\partial_\mu - (\mathrm{W} + \mathrm{i}\gamma_5 \tilde{\mathrm{W}})A_\mu)\Psi.$$

It is understood that the free parameters parameterizing the family of gauge theories expressed by (38)–(39) are solutions to the consistency equations (33)–(35).

5 Representative interacting models

In this section, some representative interacting theories are singled out from the family (38)–(39), with the members depending on the potential $\mathscr{V}$, real constants α and n, and the 2×2 real matrices V, $\tilde{\mathrm{V}}$, U, $\tilde{\mathrm{U}}$, W, and $\tilde{\mathrm{W}}$. Remember that these free parameters are subject to consistency conditions (33)–(35).

Equation (33) splits this family into two complementary subfamilies of interacting field theories. The first one involves a *massless* 1-form, but a *possibly massive scalar field*, is governed by

$$n = 0, \quad \mathscr{V}(\varphi) = \tfrac{\alpha_2}{2}\varphi^2 + \tfrac{\alpha_3}{3!}\varphi^3 + \tfrac{\alpha_4}{4!}\varphi^4, \quad \alpha_2, \alpha_3, \alpha_4 \in \mathbb{R}, \tag{40}$$

with the matrix parameters being subject to

$$[\mathrm{U}, \mathrm{W}] = \{\tilde{\mathrm{U}}, \tilde{\mathrm{W}}\}, \qquad \left[\tilde{\mathrm{U}}, \mathrm{W}\right] = -\{\mathrm{U}, \tilde{\mathrm{W}}\}, \tag{41}$$

$$[\mathrm{V}, \mathrm{W}] = \{\tilde{\mathrm{V}}, \tilde{\mathrm{W}}\}, \qquad \left[\tilde{\mathrm{V}}, \mathrm{W}\right] = -\{\mathrm{V}, \tilde{\mathrm{W}}\}. \tag{42}$$

The second one displays a *massive* 1-form, but an *unphysical massless scalar mode*, and is described by

$$n \neq 0, \quad \mathscr{V} = 0, \tag{43}$$

with the matrix parameters verifying the linear equations (34)–(35).

In the last part of this section, a detailed description of the obtained interacting models is done. For the first subfamily, the fermionic dependent part is completely captured in terms of the 2×2 real matrices V, $\tilde{\mathrm{V}}$, U, $\tilde{\mathrm{U}}$, W, and $\tilde{\mathrm{W}}$ subject to the linear equations (41)–(42). The symmetry properties of these matrices lead to the solutions

$$\mathrm{W} = 0, \quad \tilde{\mathrm{W}} = \tilde{w}\sigma_1, \quad \tilde{\mathrm{U}} = \tilde{u}\sigma_3, \quad \tilde{\mathrm{V}} = \tilde{v}\sigma_3, \quad \mathrm{U} = u\sigma_3, \quad \mathrm{V} = v\sigma_3, \tag{44}$$

$$\mathrm{W} = 0, \quad \tilde{\mathrm{W}} = \tilde{w}\sigma_3, \quad \tilde{\mathrm{U}} = \tilde{u}\sigma_1, \quad \tilde{\mathrm{V}} = \tilde{v}\sigma_1, \quad \mathrm{U} = u\sigma_1, \quad \mathrm{V} = v\sigma_1, \tag{45}$$

with the lower-case Latin coefficients being arbitrary real numbers, supplemented with

$$\mathrm{W} = w\mathrm{i}\sigma_2, \quad \tilde{\mathrm{W}} = 0, \quad \tilde{\mathrm{U}} = \tilde{u}\sigma_0, \quad \tilde{\mathrm{V}} = \tilde{v}\sigma_0, \quad \mathrm{U} = u\sigma_0, \quad \mathrm{V} = v\sigma_0, \tag{46}$$

$$\mathrm{W} = w\mathrm{i}\sigma_2, \quad \tilde{\mathrm{W}} = \pm w\sigma_0, \quad \tilde{\mathrm{U}} = \mp u\sigma_3, \quad \tilde{\mathrm{V}} = \mp v\sigma_3, \quad \mathrm{U} = u\sigma_1, \quad \mathrm{V} = v\sigma_1, \tag{47}$$

with the same character of lower-case Latin coefficients as previously.

For the complementary subfamily of interacting gauge theories, namely those that display a massive 1-form, the 2×2 real matrices, which particularly control the masses of fermionic modes, are subject to the linear equations (34)–(35). Invoking again the type of matrices involved, it results the following classes of solutions

$$\mathrm{W} = 0, \quad \tilde{\mathrm{W}} \in \mathbb{R}_s(2), \quad \tilde{\mathrm{U}} = 0, \quad \tilde{\mathrm{V}} = 0, \quad \mathrm{U} = 0, \quad \mathrm{V} = 0, \tag{48}$$

$$\mathrm{W} = 0, \quad \tilde{\mathrm{W}} = 0, \quad \tilde{\mathrm{U}} = 0, \quad \tilde{\mathrm{V}} \in \mathbb{R}_s(2), \quad \mathrm{U} = 0, \quad \mathrm{V} \in \mathbb{R}_s(2), \tag{49}$$

and

$$\mathrm{W} = w\mathrm{i}\sigma_2, \quad \tilde{\mathrm{W}} = 0, \quad \tilde{\mathrm{U}} = 0, \quad \tilde{\mathrm{V}} = 0, \quad \mathrm{U} = 0, \quad \mathrm{V} = 0, \tag{50}$$

$$\mathrm{W} = w\mathrm{i}\sigma_2, \quad \tilde{\mathrm{W}} = \pm w\sigma_0, \quad \tilde{\mathrm{U}} = 0, \quad \tilde{\mathrm{V}} = \pm v\sigma_1, \quad \mathrm{U} = 0, \quad \mathrm{V} = v\sigma_3. \tag{51}$$

Previously, one employed the Pauli matrices σ_1, σ_2, and σ_3, the unit 2×2 matrix σ_0, while by $\mathbb{R}_s(2)$ one understood the set of symmetric 2×2 real matrices.

It is worth noticing that even though the solutions (40) and (44)/ (45)/ (46) or (47), on the one hand, or (43) and (48)/ (49)/ (50) and (51), on the other hand, *do not represent* the general solution to the consistency equations (33)–(35), they display some interacting theories with a non-trivial mass spectrum.

Based on the previous discussion, the first class of interacting gauge theories consists in the Lagrangian action

$$\begin{aligned} S^{\mathrm{I}}[A, \varphi, \Psi] = \int d^4x \, \Big[&-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \tfrac{\mathrm{i}}{2} \bar{\Psi}^\top \not{D} \Psi - \lambda (\tfrac{\alpha_2}{2} \varphi^2 + \tfrac{\alpha_3}{3!} \varphi^3 + \tfrac{\alpha_4}{4!} \varphi^4) \\ &+ \tfrac{\lambda\alpha}{2} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} + \lambda \bar{\Psi}^\top (\mathrm{V} + \mathrm{i}\gamma_5 \tilde{\mathrm{V}}) \Psi + \lambda \bar{\Psi}^\top (\mathrm{U} + \mathrm{i}\gamma_5 \tilde{\mathrm{U}}) \Psi \varphi \Big], \end{aligned} \tag{52}$$

which is invariant under the generating set of gauge transformations

$$\bar{\delta}^I_\epsilon A^\mu = \partial^\mu \epsilon, \quad \bar{\delta}^I_\epsilon \varphi 0, \quad \bar{\delta}^I_\epsilon \Psi = (\mathrm{W} + \mathrm{i}\gamma_5 \tilde{\mathrm{W}}) \Psi \epsilon. \tag{53}$$

Previously, the 2×2 real matrices can be chosen as in (44)/ (45)/ (46) or (47). This class of theories exhibits a massless 1-form and possibly a massive scalar and/or spinor modes. Moreover, all these models exhibit seven physical degrees of freedom, distributed as in the starting model, i.e., two associated with the 1-form A_μ, one corresponding to the scalar field φ, and four fermionic degrees of freedom.

The complementary class of interacting models is characterized by the Lagrangian action

$$\begin{aligned} S^{\mathrm{II}}[A, \varphi, \Psi] = \int d^4x \, \Big[&-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} (\partial_\mu \varphi - \lambda n A_\mu)(\partial^\mu \varphi - \lambda n A^\mu) + \tfrac{\mathrm{i}}{2} \bar{\Psi}^\top \not{D} \Psi \\ &+ \tfrac{\lambda\alpha}{2} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} + \lambda \bar{\Psi}^\top (\mathrm{V} + \mathrm{i}\gamma_5 \tilde{\mathrm{V}}) \Psi + \lambda \bar{\Psi}^\top (\mathrm{U} + \mathrm{i}\gamma_5 \tilde{\mathrm{U}}) \Psi \varphi \Big], \end{aligned} \tag{54}$$

which is found invariant under the generating set of gauge transformations

$$\bar{\delta}^{II}_\epsilon A^\mu = \partial^\mu \epsilon, \quad \bar{\delta}^{II}_\epsilon \varphi = gn\epsilon, \quad \bar{\delta}^{II}_\epsilon \Psi = (\mathrm{W} + \mathrm{i}\gamma_5 \tilde{\mathrm{W}}) \Psi \epsilon. \tag{55}$$

For this class of models, the 2×2 real matrices can be chosen as in (48)/ (49)/ (50) or (51).

It is worth noticing that the Stuekelberg coupling between the scalar field and the Abelian 1-form present in the last models (54), combined with the shift gauge transformation of the scalar field in (55), show that the distribution of physical modes is no longer as in the free model. Here, the seven physical degrees of freedom come from the three bosonic modes due to the now massive 1-form A_μ and the four DOFs corresponding to the fermionic modes ψ_Δ. At the same time, the scalar field is a purely gauge one as it can be seen, at the classical level, from the reparametrization

$$A_\mu \to \bar{A}_\mu := A_\mu - \tfrac{1}{gn}\partial_\mu\varphi, \quad \varphi \to \bar{\varphi} := \varphi, \quad \psi_\Delta \to \bar{\psi}_\Delta := \psi_\Delta$$

of the jet bundle corresponding to the considered field theory.

6 Conclusions

In this paper, we analyzed the consistent couplings that can be added to a massless free field theory comprising one real scalar field, two real spinor fields, and one Abelian 1-form, using the deformation of the solution to the classical master equation [12, 13], combined with specific techniques of local BRST cohomology [14, 15, 16]. This approach, completed with some standard working hypotheses from field theory, led to two classes of interacting theories, each containing quadratic, derivative-free interaction vertices naturally interpreted as mass terms for various field spectrum components. The first of them exhibits a *massless* Abelian 1-form, a *massive* scalar field, and two spinor fields that are *massive* whenever they are not coupled with the 1-form. Also, the physical degrees of freedom of the interacting models are distributed just as in the starting model, i.e., two associated with the 1-form A_μ, one corresponding to the scalar field φ, and four fermionic degrees of freedom. The second class displays a *massive* 1-form, a *massless* scalar field, and two fields that are *massive* unless these are cross-coupled with the massive 1-form. In this context, the scalar field becomes purely gauge, while the 1-form gets an extra physical degree of freedom.

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