Calculation the Potential Energy Surface for *Mo* (A=92-102) Isotopes

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Abstract

A potential energy surface (PES) is a mathematical function that gives the energy of a nucleus as a function of its geometry. This paper calculated the PES for the Molybdenum (A=92-102) even-even isotopes by the special equation for the PES using (IBM-1.For) program, in addition, manual calculations, the relationship between deformation parameters β, asymmetry angle γ, with the PES for each of the mentionable isotopes was plotted also, the results were in good agreement when comparison it with the typical plots of contour lines and triaxial symmetric.

Keywords: Potential energy surface; IBM-1; Mo isotopes; The contour plots; The triaxial symmetric.

1. INTRODUCTION

The potential energy surface (PES) is a mathematical and graphical relationship between the energy of a molecule (or a collection of molecules) and its geometry. PES is a central concept in computational chemistry as it describes nuclei as essentially stationary in a molecule compared to the electrons, and as such is one of the cornerstones of computational chemistry because it makes the concept of molecular shape (geometry) meaningful [1], the same is true of theoretical nuclear physics when dealing with the particles inside nuclei [1,2], so that the PES gives a complete picture of the nuclear structure of the different nuclei isotopes [3].

The PES function $V(N,\beta,\gamma)$ depends on the shape variables β and γ, where β is the magnitude of the nuclear deformation and γ gives the deviations from axial symmetry (asymmetry angle), and they are different for different state of the nucleus. This means that the rigidity of the shape is only a special case because of the natural dynamic of the nucleus [4].

The probability of finding the nucleus at $(\beta = 0)$ or $\gamma = 0^{\circ}(60^{\circ})$ is always zero, and the nucleus is always deformed and nonaxial, but some nuclei are so to $(\beta = 0)$ that may be called them (spherical), while those are close to $\gamma = 0^{\circ}$ may be called prolate and $\gamma = 60^{\circ}$ called oblate [2,4,5].

As in computational chemistry, theoretical nuclear physics has many computational programs used to find PES from them the interacting boson model [6].

The interacting boson model (IBM) represents an important model for studying a nuclear structure for the isotopes; it is also used to calculate the PES for these isotopes [7]. The idea of the program is simply that it deals with parameters for an equation called the Hamiltonian equation, it is characterized by its ability to describe nuclear properties of a wide variety of nuclei; from these properties PES, that can be calculated using the same program with a special equation and parameters [8], it is founded on a rather general algebraic group of theoretical techniques. It has also found its last application to problems in atomic, molecular, and low-energy physics, where it deals with the heavy and medium nuclei, and the application of this model to deformed nuclei is a subject of great interest nowadays [9].

There are many versions of IBM, called IBM-1, 2, 3… etc. One of them (IBM-1) is used in the present work [10]. The calculations have been performed with the PHINT and FBEM codes [11]. In this model, the deformation of the nuclear surface is represented by six parameters from which a Hamiltonian of a six-dimensional oscillator results [12].

Many researchers in theoretical nuclear physics use IBM-1 to study different nuclear properties for the various nuclei isotopes [13], it is a successful program to determine the nuclear properties theoretically, and the results are somewhat in agreement with the experimental data [14]. Molybdenum isotopes *Mo* (A=92-102) belong to the medium nuclei, therefore IBM-1 is suitable to study their nuclear properties, Rana (2005) has studied the energy levels and calculation of delta mixing ratios for *Mo* (A=92-102) even – even isotopes using the IBM-1 model [15].

In the present work, the PES of *Mo* (A=92-102) even-even isotopes will be studied using IBM-1, as well as study the relationship between deformed parameters β, asymmetry angle γ, and the PES for each isotope from the mentionable isotopes graphically and comparison it with the typical plots of contour lines and triaxial symmetric.

2. THEORETICAL FRAMEWORK

2.1. THE HAMILTONIAN OPERATOR OF IBM-1

In the IBM-1, the Hamiltonian operator can be written in many forms, such as in terms of (one-body and two-body interaction), or in terms of creation and annihilation operators, and other many forms, but the formula most commonly to writing the Hamiltonian equation is [7,9,10]:

$$\hat{H} = \varepsilon\hat{n}_d + a_0(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \quad (1)$$

$\hat{H}, \hat{n}_d, \hat{P}, \hat{L}, \hat{Q}, \hat{T}_3, \hat{T}_4$ are the Hamiltonian, the bosons number, the pairing bosons, the angular momentum, the quadrupole, the octupole, and the hexadecapole operators, respectively.
ε is the boson energy, and a_0, a_1, a_2, a_3, a_4 are the phenomenological parameters.

2.2. THE DYNAMICAL SYMMETRIES IN IBM-1

To solve the eigenvalue problem for the Hamiltonian equation and represented by the equation (1), there are three limits called the dynamical symmetries [9,10,16] are vibrational U(5), rotational SU(3), and the Gamma unstable O(6) [10], in addition, there are transitional regions in which some isotopes are located represented with U(5)-SU(3), SU(3)-O(6), O(6)-U(5) [7,9,10], and U(5)-SU(3)-O(6) in this class, where the nuclei have middle properties between previous three limits [3,10]. The isotope affiliation of any of these limits is classified

according to the energy ratios R_4, R_6, and R_8 as shown in Table (1), otherwise, the isotope is located in one of the transitional regions [2,7,10].

Table 1. The energy ratios of corresponding limits [7,9,10].

Limit	U(5)	SU(3)	O(6)
$R_4 = E(4_1^+)/E(2_1^+)$	2	3.33	2.5
$R_6 = E(6_1^+)/E(2_1^+)$	3	7	4.5
$R_8 = E(8_1^+)/E(2_1^+)$	4	12	7

Each isotope of nuclei isotopes has special dynamical symmetry [3,9], which is dependent on the energy ratios for the isotope [7,9], at the same time; each dynamical symmetry has a special Hamiltonian operator, which is derived from the original equation, and dependent on the used parameters for the defined dynamical symmetry [2-4].

The Hamiltonian operators take the following forms as shown in Table (2).

Table 2. The Hamiltonian equations corresponding to the dynamical symmetries [3,9,10,16].

Dynamical symmetry	Hamiltonian equation
U(5)	$\widehat{H} = \varepsilon\hat{n}_d + a_1(\hat{L}.\hat{L}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4)$
SU(3)	$\widehat{H} = a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q})$
O(6)	$\widehat{H} = a_0(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_3(\hat{T}_3.\hat{T}_3)$
U(5)-SU(3)	$\widehat{H} = \varepsilon\hat{n}_d + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q})$
SU(3)-O(6)	$\widehat{H} = a_0(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3)$
O(6)-U(5)	$\widehat{H} = \varepsilon\hat{n}_d + a_0(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_3(\hat{T}_3.\hat{T}_3)$
U(5)-SU(3)-O(6)	$\widehat{H} = \varepsilon\hat{n}_d + a_0(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4)$

2.3. POTENTIAL ENERGY SURFACE BASIS

The potential energy surface $V(N,\beta,\gamma)$ gives a final shape to the nucleus that corresponds to the function of Hamiltonian [7,10].The general formula for the PES as a function of geometrical variables β and γ is given by [7,9]:

$$V(N,\beta,\gamma) = \frac{N(\varepsilon_s+\varepsilon_d\beta^2)}{1+\beta^2} + \frac{N(N-1)}{(1+\beta^2)^2}(\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4) \quad (2)$$

Where: N= is the total boson number.
$\hat{\beta}^2$ = is the quadrupole deformation parameter operator from $0 \rightarrow 2.4$
$\hat{\gamma}$ = is the distortion parameter operator or (asymmetry angle) from $0^\circ \rightarrow 60^\circ$.
The variables $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ are related to the parameters C_L, V_L, and U_L which is given in Hamiltonian equation when written in terms of creation and annihilation operators.

The relationships between the variables (α's) and these parameters have been expressed by Van Isacker and Chen (1981) as [17]:

$$\alpha_1 = \frac{C_0}{10} + \frac{C_2}{7} + \frac{9}{35} * C_4 \quad (3)$$

$$\alpha_2 = -SQRT\left(\frac{8}{35}\right) * V_2 \quad (4)$$

$$\alpha_3 = (V_0 + U_2)/SQRT(5) \quad (5)$$

$$\alpha_4 = U_0 \quad (6)$$

One must take into account that the asymmetry angle which occurs only in the term cos3γ.

Thus, the energy surfaces has minima only at $\gamma = 0^{\circ}$ and $\gamma = 60^{\circ}$. The energy expressions in their limits, can display the essential dependence on β and γ, which are been given as [8,9]:

$$E(N,\beta,\gamma) = \varepsilon_d N \frac{\beta^2}{1+\beta^2} \quad \text{in U(5) limit} \quad (7)$$

$$E(N,\beta,\gamma) = KN(N-1)\frac{1+\frac{3}{4}\beta^4-\sqrt{2}\beta^2\cos 3\gamma}{(1+\beta^2)} \quad \text{in SU(3) limit} \quad (8)$$

$$E(N,\beta,\gamma) = K'N(N-1)\left[\frac{1-\beta^2}{1+\beta^2}\right] \quad \text{in O(6) limit} \quad (9)$$

Where $K \propto a_2$ and $K' \propto a_0$ in equation (1).

3. RESULTS AND DISCUSSION

3.1. ENERGY RATIOS AND DYNAMICAL SYMMETRIES

To introduce an accurate description of a nuclear structure in general and for the geometric model of PES in particular of the *Mo* (A=92-102) isotopes, we have first of all on finding the dynamical symmetries of these isotopes, by comparing the experimental energy ratios values for the isotopes under study with the identical values for all dynamical symmetries as shown in Table (3).

Table 3. The experimental energy ratio values (R_4, R_6, and R_8) for the isotopes under study [18,19].

Isotopes	R_4	R_6	R_8
$^{92}_{42}Mo_{50}$	1.51216	1.73104	1.82842
$^{94}_{42}Mo_{52}$	1.80680	2.78186	3.39265
$^{96}_{42}Mo_{54}$	2.09226	3.13634	3.82806
$^{98}_{42}Mo_{56}$	1.91758	2.98120	3.81128

$^{100}_{42}Mo_{58}$	2.12276	3.69208	5.08221
$^{102}_{42}Mo_{60}$	2.51014	4.40878	6.61149

The experimental energy ratio values calculated for five decimal places have been compared to the energy scheme of each isotope in the present work with the typical scheme to obtain the best fit of the dynamical symmetry.

An equivalent equation of the Hamiltonian operator function is used for each dynamical symmetry to calculate the energy levels, their ratios, and therefore the PES.

Table 4. The dynamical symmetries and corresponding Hamiltonian equations for the isotopes used in the present work.

Isotopes	Dynamical Symmetry	Hamiltonian
$^{92}_{42}Mo_{50}$	SU(5) – O(6)	$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{P}^{\dagger}.\hat{P} + a_1\hat{L}^2 + a_3\hat{T}_3{}^2 + a_4\hat{T}_4{}^2$
$^{94}_{42}Mo_{52}$	SU(5) – O(6)	$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{P}^{\dagger}.\hat{P} + a_1\hat{L}^2 + a_3\hat{T}_3{}^2 + a_4\hat{T}_4{}^2$
$^{96}_{42}Mo_{54}$	SU(5) – O(6)	$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{P}^{\dagger}.\hat{P} + a_1\hat{L}^2 + a_3\hat{T}_3{}^2 + a_4\hat{T}_4{}^2$
$^{98}_{42}Mo_{56}$	SU(5) – O(6)	$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{P}^{\dagger}.\hat{P} + a_1\hat{L}^2 + a_3\hat{T}_3{}^2 + a_4\hat{T}_4{}^2$
$^{100}_{42}Mo_{58}$	SU(5)	$\hat{H} = \varepsilon\hat{n}_d + a_1\hat{L}^2 + a_3\hat{T}_3{}^2 + a_4\hat{T}_4{}^2$
$^{102}_{42}Mo_{60}$	O(6)	$\hat{H} = a_0\hat{P}^{\dagger}.\hat{P} + a_1\hat{L}^2 + a_3\hat{T}_3{}^2$

Table 5. The parameters of the Hamiltonian operator for even-even *Mo* (A=92-102) isotopes by using (IBM-1.For) program [15].

Isotopes	N_π	N_ν	N	ε	a_0	a_1	a_2	a_3	a_4
$^{92}_{42}Mo_{50}$	4	0	4	2.0000	0.0000	-0.0545	0.0000	0.0200	-0.2000
$^{94}_{42}Mo_{52}$	4	1	5	1.0400	0.0000	-0.0101	0.0000	-0.0010	-0.0600
$^{96}_{42}Mo_{54}$	4	2	6	0.9000	0.0000	-0.0019	0.0000	0.0320	-0.0899
$^{98}_{42}Mo_{56}$	4	3	7	0.2800	0.0000	0.0042	0.0000	0.2800	0.0250
$^{100}_{42}Mo_{58}$	4	4	8	0.2800	0.0000	0.0150	0.0000	0.0798	0.0112
$^{102}_{42}Mo_{60}$	4	5	9	0.0000	0.0570	0.0104	0.0000	0.1705	0.0810

3.2. THE POTENTIAL ENERGY SURFACE (PES)

The present study included calculating the PES for *Mo* (A=92-102) isotopes, that's based on a special equation of calculating the PES and used in (IBM-1.For) program. Accompaniment parameters to resulting energy levels output from IBM-1 $(\varepsilon_s, \varepsilon_d, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$ were used to find the PES as in Table (6).

Table 6. The potential energy surface (PES) parameters values for even – even *Mo* (A=92-102) isotopes.

Isotopes	N_π	N_ν	N	ε_s	ε_d	α_1	α_2	α_3	α_4
$^{92}_{42}Mo_{50}$	4	0	4	0.0000	1. 3410	-0.1030	0.0000	0.0000	0.0000
$^{94}_{42}Mo_{52}$	4	1	5	0.0000	0. 8700	-0.0310	0.0000	0.0000	0.0000
$^{96}_{42}Mo_{54}$	4	2	6	0.0000	0. 7720	-0.0460	0.0000	0.0000	0.0000
$^{98}_{42}Mo_{56}$	4	3	7	0.0000	0. 7420	-0.0130	0.0000	0.0000	0.0000
$^{100}_{42}Mo_{58}$	4	4	8	0.0000	0. 5020	0.0060	0.0000	0.0000	0.0000
$^{102}_{42}Mo_{60}$	4	5	9	0.0000	0. 4470	0.0560	0.0000	-0.0290	0.0000

In the present work, the same equation was used, but with manual calculations, and more accurate results were obtained that reached almost nine decimal places, which were rounded approximations for nearly five decimal places as in Tables (7 – 12).

The first four isotopes under study belong to the dynamical symmetry SU(5)-O(6) transitional region, where be the nearest to SU(5) from it to O(6), the fifth isotope belongs to the dynamical symmetry SU(5), which describes the nucleus vibrational, while the sixth isotope belongs to the dynamical symmetry O(6) unstable.

Been studying each isotope separately and applying the values of the variables used in the equation (N, β, γ), and calculated the energy values change of each where the values of *N* ranging from (4 – 9), and for each value of *N* the value of β ranging from (0 – 2.4), also the value of γ for each value of the value of *N* and β ranging from $(0^\circ, 60^\circ)$, and therefore the value of the PES change based on these three variables. In addition, the value of the PES depends on the parameters (α's), since these parameters are commonly used in (PES.For) program to get the value of the energy and the results of the program were rounded to three decimal places.

The current work analysis of the contour plots of the potential energy function $V(N, \beta, \gamma)$ was calculated by using the parameters (α's) that were deduced from the (IBM-1.For) program.

Show that, all the isotopes under study take the same energy values for all asymmetry angles $(\gamma = 0^\circ - 60^\circ)$ at each value from deformed parameters β, because the parameter (α) associated with the $\cos 3\gamma$ in the PES equation takes the value (0.0000), as shown in the below Tables (7 – 12).

Figures (1a – 6a) elucidate the PES as a function of deformed parameters (β, γ), the contour lines are in good agreement with the typical plots.

The axially symmetric $(\gamma = 0^\circ - 60^\circ)$ plots of the potential function are calculated in the present work, for *Mo* (A=92-102) isotopes, which is explained as follows:

1- The isotope $^{92}_{42}Mo_{50}$: from Table (7) and the axially symmetric for the mentionable isotope, as shown in figure (1b), the behavior of the PES of (-16.51613 MeV) on the prolate shape at $\beta = 2.4$, $\gamma = 0^\circ$ and for oblate shape at $\beta = 2.4$, $\gamma = 60^\circ$ which is (-16.51613 MeV). This agrees with the behavior of the SU(5) – O(6) dynamical symmetry compared with the typical triaxial symmetric.

2- The isotope $^{94}_{42}Mo_{52}$: from Table (8) and the axially symmetric for the mentionable isotope, as shown in figure (2b), the behavior of the PES of (-4.26230 MeV) on the prolate shape at $\beta = 2.4$, $\gamma = 0^\circ$ and for oblate shape at $\beta = 2.4$, $\gamma = 60^\circ$ which is (-4.26230 MeV). This agrees with the behavior of the SU(5) – O(6) dynamical symmetry compared with the typical triaxial symmetric.

3- The isotope $^{96}_{42}Mo_{54}$: from Table (9) and the axially symmetric for the mentionable isotope, as shown in figure (3b), the behavior of the PES of (-7.02539 MeV) on the prolate shape at

$\beta = 2.4$, $\gamma = 0^{\circ}$ and for oblate shape at $\beta = 2.4$, $\gamma = 60^{\circ}$ which is (-7.02539 MeV). This agrees with the behavior of the SU(5) – O(6) dynamical symmetry compared with the typical triaxial symmetric.

4- The isotope $^{98}_{42}Mo_{56}$: from Table (10) and the axially symmetric for the mentionable isotope, as shown in figure (4b), the behavior of the PES of (-2.30523 MeV) on the prolate shape at $\beta = 2.4$, $\gamma = 0^{\circ}$ and for oblate shape at $\beta = 2.4$, $\gamma = 60^{\circ}$ which is (-2.30523 MeV). This agrees with the behavior of the SU(5) – O(6) dynamical symmetry compared with the typical triaxial symmetric.

5- The isotope $^{100}_{42}Mo_{58}$: from Table (11) and the axially symmetric for the mentionable isotope, as shown in figure (5b), the behavior of the PES of (0.92513 MeV) on the prolate shape at $\beta = 2.4$, $\gamma = 0^{\circ}$ and for oblate shape at $\beta = 2.4$, $\gamma = 60^{\circ}$ which is (0.92513 MeV). This agrees with the behavior of the SU(5) limit compared with the typical triaxial symmetric.

6- The isotope $^{102}_{42}Mo_{60}$: from Table (12) and the axially symmetric for the mentionable isotope, as shown in figure (6b), the behavior of the PES of (8.46037MeV) on the prolate shape at $\beta = 2.4$, $\gamma = 0^{\circ}$ and for oblate shape at $\beta = 2.4$, $\gamma = 60^{\circ}$ which is (8.46037MeV). This agrees with the behavior of the O(6) limit compared with the typical triaxial symmetric.

Table 7. The potential energy surface values (PES) in the present work for $^{92}_{42}Mo_{50}$ isotope.

β \ γ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	-0.00186	-0.00186	-0.00186	-0.00186	-0.00186	-0.00186	-0.00186	-0.00186	-0.00186	-0.00186	-0.00186	-0.00186	-0.00186
0.4	-0.02546	-0.02546	-0.02546	-0.02546	-0.02546	-0.02546	-0.02546	-0.02546	-0.02546	-0.02546	-0.02546	-0.02546	-0.02546
0.6	-0.10556	-0.10556	-0.10556	-0.10556	-0.10556	-0.10556	-0.10556	-0.10556	-0.10556	-0.10556	-0.10556	-0.10556	-0.10556
0.8	-0.21031	-0.21031	-0.21031	-0.21031	-0.21031	-0.21031	-0.21031	-0.21031	-0.21031	-0.21031	-0.21031	-0.21031	-0.21031
1	-0.58525	-0.58525	-0.58525	-0.58525	-0.58525	-0.58525	-0.58525	-0.58525	-0.58525	-0.58525	-0.58525	-0.58525	-0.58525
1.2	-1.10661	-1.10661	-1.10661	-1.10661	-1.10661	-1.10661	-1.10661	-1.10661	-1.10661	-1.10661	-1.10661	-1.10661	-1.10661
1.4	-1.94734	-1.94734	-1.94734	-1.94734	-1.94734	-1.94734	-1.94734	-1.94734	-1.94734	-1.94734	-1.94734	-1.94734	-1.94734
1.6	-3.24287	-3.24287	-3.24287	-3.24287	-3.24287	-3.24287	-3.24287	-3.24287	-3.24287	-3.24287	-3.24287	-3.24287	-3.24287
1.8	-5.15369	-5.15369	-5.15369	-5.15369	-5.15369	-5.15369	-5.15369	-5.15369	-5.15369	-5.15369	-5.15369	-5.15369	-5.15369
2	-7.86294	-7.86294	-7.86294	-7.86294	-7.86294	-7.86294	-7.86294	-7.86294	-7.86294	-7.86294	-7.86294	-7.86294	-7.86294
2.2	-11.57523	-11.57523	-11.57523	-11.57523	-11.57523	-11.57523	-11.57523	-11.57523	-11.57523	-11.57523	-11.57523	-11.57523	-11.57523
2.4	-16.51613	-16.51613	-16.51613	-16.51613	-16.51613	-16.51613	-16.51613	-16.51613	-16.51613	-16.51613	-16.51613	-16.51613	-16.51613

Table 8. The potential energy surface values (PES) in the present work for $^{94}_{42}Mo_{52}$ isotope.

β \ γ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	-0.00092	-0.00092	-0.00092	-0.00092	-0.00092	-0.00092	-0.00092	-0.00092	-0.00092	-0.00092	-0.00092	-0.00092	-0.00092
0.4	-0.01227	-0.01227	-0.01227	-0.01227	-0.01227	-0.01227	-0.01227	-0.01227	-0.01227	-0.01227	-0.01227	-0.01227	-0.01227
0.6	-0.04807	-0.04807	-0.04807	-0.04807	-0.04807	-0.04807	-0.04807	-0.04807	-0.04807	-0.04807	-0.04807	-0.04807	-0.04807
0.8	-0.11597	-0.11597	-0.11597	-0.11597	-0.11597	-0.11597	-0.11597	-0.11597	-0.11597	-0.11597	-0.11597	-0.11597	-0.11597
1	-0.22242	-0.22242	-0.22242	-0.22242	-0.22242	-0.22242	-0.22242	-0.22242	-0.22242	-0.22242	-0.22242	-0.22242	-0.22242
1.2	-0.38097	-0.38097	-0.38097	-0.38097	-0.38097	-0.38097	-0.38097	-0.38097	-0.38097	-0.38097	-0.38097	-0.38097	-0.38097
1.4	-0.61487	-0.61487	-0.61487	-0.61487	-0.61487	-0.61487	-0.61487	-0.61487	-0.61487	-0.61487	-0.61487	-0.61487	-0.61487
1.6	-0.95611	-0.95611	-0.95611	-0.95611	-0.95611	-0.95611	-0.95611	-0.95611	-0.95611	-0.95611	-0.95611	-0.95611	-0.95611
1.8	-1.44377	-1.44377	-1.44377	-1.44377	-1.44377	-1.44377	-1.44377	-1.44377	-1.44377	-1.44377	-1.44377	-1.44377	-1.44377
2	-2.12288	-2.12288	-2.12288	-2.12288	-2.12288	-2.12288	-2.12288	-2.12288	-2.12288	-2.12288	-2.12288	-2.12288	-2.12288
2.2	-3.04388	-3.04388	-3.04388	-3.04388	-3.04388	-3.04388	-3.04388	-3.04388	-3.04388	-3.04388	-3.04388	-3.04388	-3.04388
2.4	-4.26230	-4.26230	-4.26230	-4.26230	-4.26230	-4.26230	-4.26230	-4.26230	-4.26230	-4.26230	-4.26230	-4.26230	-4.26230

Table 9. The potential energy surface values (PES) in the present work for $^{96}_{42}Mo_{54}$ isotope.

β \ γ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	-0.00205	-0.00205	-0.00205	-0.00205	-0.00205	-0.00205	-0.00205	-0.00205	-0.00205	-0.00205	-0.00205	-0.00205	-0.00205
0.4	-0.03341	-0.03341	-0.03341	-0.03341	-0.03341	-0.03341	-0.03341	-0.03341	-0.03341	-0.03341	-0.03341	-0.03341	-0.03341
0.6	-0.10400	-0.10400	-0.10400	-0.10400	-0.10400	-0.10400	-0.10400	-0.10400	-0.10400	-0.10400	-0.10400	-0.10400	-0.10400
0.8	-0.24422	-0.24422	-0.24422	-0.24422	-0.24422	-0.24422	-0.24422	-0.24422	-0.24422	-0.24422	-0.24422	-0.24422	-0.24422
1	-0.45154	-0.45154	-0.45154	-0.45154	-0.45154	-0.45154	-0.45154	-0.45154	-0.45154	-0.45154	-0.45154	-0.45154	-0.45154
1.2	-0.74139	-0.74139	-0.74139	-0.74139	-0.74139	-0.74139	-0.74139	-0.74139	-0.74139	-0.74139	-0.74139	-0.74139	-0.74139
1.4	-1.14708	-1.14708	-1.14708	-1.14708	-1.14708	-1.14708	-1.14708	-1.14708	-1.14708	-1.14708	-1.14708	-1.14708	-1.14708
1.6	-1.71775	-1.71775	-1.71775	-1.71775	-1.71775	-1.71775	-1.71775	-1.71775	-1.71775	-1.71775	-1.71775	-1.71775	-1.71775
1.8	-2.51503	-2.51503	-2.51503	-2.51503	-2.51503	-2.51503	-2.51503	-2.51503	-2.51503	-2.51503	-2.51503	-2.51503	-2.51503
2	-3.61052	-3.61052	-3.61052	-3.61052	-3.61052	-3.61052	-3.61052	-3.61052	-3.61052	-3.61052	-3.61052	-3.61052	-3.61052
2.2	-5.08452	-5.08452	-5.08452	-5.08452	-5.08452	-5.08452	-5.08452	-5.08452	-5.08452	-5.08452	-5.08452	-5.08452	-5.08452
2.4	-7.02539	-7.02539	-7.02539	-7.02539	-7.02539	-7.02539	-7.02539	-7.02539	-7.02539	-7.02539	-7.02539	-7.02539	-7.02539

Table 10. The potential energy surface values (PES) in the present work for $^{98}_{42}Mo_{56}$ isotope.

β \ γ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	-0.00081	-0.00081	-0.00081	-0.00081	-0.00081	-0.00081	-0.00081	-0.00081	-0.00081	-0.00081	-0.00081	-0.00081	-0.00081
0.4	-0.01063	-0.01063	-0.01063	-0.01063	-0.01063	-0.01063	-0.01063	-0.01063	-0.01063	-0.01063	-0.01063	-0.01063	-0.01063
0.6	-0.04057	-0.04057	-0.04057	-0.04057	-0.04057	-0.04057	-0.04057	-0.04057	-0.04057	-0.04057	-0.04057	-0.04057	-0.04057
0.8	-0.09394	-0.09394	-0.09394	-0.09394	-0.09394	-0.09394	-0.09394	-0.09394	-0.09394	-0.09394	-0.09394	-0.09394	-0.09394
1	-0.17026	-0.17026	-0.17026	-0.17026	-0.17026	-0.17026	-0.17026	-0.17026	-0.17026	-0.17026	-0.17026	-0.17026	-0.17026
1.2	-0.27280	-0.27280	-0.27280	-0.27280	-0.27280	-0.27280	-0.27280	-0.27280	-0.27280	-0.27280	-0.27280	-0.27280	-0.27280
1.4	-0.41116	-0.41116	-0.41116	-0.41116	-0.41116	-0.41116	-0.41116	-0.41116	-0.41116	-0.41116	-0.41116	-0.41116	-0.41116
1.6	-0.60055	-0.60055	-0.60055	-0.60055	-0.60055	-0.60055	-0.60055	-0.60055	-0.60055	-0.60055	-0.60055	-0.60055	-0.60055
1.8	-0.86047	-0.86047	-0.86047	-0.86047	-0.86047	-0.86047	-0.86047	-0.86047	-0.86047	-0.86047	-0.86047	-0.86047	-0.86047
2	-1.21372	-1.21372	-1.21372	-1.21372	-1.21372	-1.21372	-1.21372	-1.21372	-1.21372	-1.21372	-1.21372	-1.21372	-1.21372
2.2	-1.68592	-1.68592	-1.68592	-1.68592	-1.68592	-1.68592	-1.68592	-1.68592	-1.68592	-1.68592	-1.68592	-1.68592	-1.68592
2.4	-2.30523	-2.30523	-2.30523	-2.30523	-2.30523	-2.30523	-2.30523	-2.30523	-2.30523	-2.30523	-2.30523	-2.30523	-2.30523

Table 11. The potential energy surface values (PES) in the present work for $^{100}_{42}Mo_{58}$ isotope.

β \ γ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	0.00050	0.00050	0.00050	0.00050	0.00050	0.00050	0.00050	0.00050	0.00050	0.00050	0.00050	0.00050	0.00050
0.4	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648	0.00648
0.6	0.02437	0.02437	0.02437	0.02437	0.02437	0.02437	0.02437	0.02437	0.02437	0.02437	0.02437	0.02437	0.02437
0.8	0.05502	0.05502	0.05502	0.05502	0.05502	0.05502	0.05502	0.05502	0.05502	0.05502	0.05502	0.05502	0.05502
1	0.09605	0.09605	0.09605	0.09605	0.09605	0.09605	0.09605	0.09605	0.09605	0.09605	0.09605	0.09605	0.09605
1.2	0.14651	0.14651	0.14651	0.14651	0.14651	0.14651	0.14651	0.14651	0.14651	0.14651	0.14651	0.14651	0.14651
1.4	0.20862	0.20862	0.20862	0.20862	0.20862	0.20862	0.20862	0.20862	0.20862	0.20862	0.20862	0.20862	0.20862
1.6	0.28730	0.28730	0.28730	0.28730	0.28730	0.28730	0.28730	0.28730	0.28730	0.28730	0.28730	0.28730	0.28730
1.8	0.38949	0.38949	0.38949	0.38949	0.38949	0.38949	0.38949	0.38949	0.38949	0.38949	0.38949	0.38949	0.38949
2	0.52347	0.52347	0.52347	0.52347	0.52347	0.52347	0.52347	0.52347	0.52347	0.52347	0.52347	0.52347	0.52347
2.2	0.69859	0.69859	0.69859	0.69859	0.69859	0.69859	0.69859	0.69859	0.69859	0.69859	0.69859	0.69859	0.69859
2.4	0.92513	0.92513	0.92513	0.92513	0.92513	0.92513	0.92513	0.92513	0.92513	0.92513	0.92513	0.92513	0.92513

Table 12. The potential energy surface values (PES) in the present work for $^{102}_{42}Mo_{60}$ isotope.

β \ γ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	-0.07142	-0.07142	-0.07142	-0.07142	-0.07142	-0.07142	-0.07142	-0.07142	-0.07142	-0.07142	-0.07142	-0.07142	-0.07142
0.4	-0.17335	-0.17335	-0.17335	-0.17335	-0.17335	-0.17335	-0.17335	-0.17335	-0.17335	-0.17335	-0.17335	-0.17335	-0.17335
0.6	-0.12727	-0.12727	-0.12727	-0.12727	-0.12727	-0.12727	-0.12727	-0.12727	-0.12727	-0.12727	-0.12727	-0.12727	-0.12727
0.8	0.12406	0.12406	0.12406	0.12406	0.12406	0.12406	0.12406	0.12406	0.12406	0.12406	0.12406	0.12406	0.12406
1	0.54031	0.54031	0.54031	0.54031	0.54031	0.54031	0.54031	0.54031	0.54031	0.54031	0.54031	0.54031	0.54031
1.2	1.07584	1.07584	1.07584	1.07584	1.07584	1.07584	1.07584	1.07584	1.07584	1.07584	1.07584	1.07584	1.07584
1.4	1.80853	1.80853	1.80853	1.80853	1.80853	1.80853	1.80853	1.80853	1.80853	1.80853	1.80853	1.80853	1.80853
1.6	2.51015	2.51015	2.51015	2.51015	2.51015	2.51015	2.51015	2.51015	2.51015	2.51015	2.51015	2.51015	2.51015
1.8	3.49644	3.49644	3.49644	3.49644	3.49644	3.49644	3.49644	3.49644	3.49644	3.49644	3.49644	3.49644	3.49644
2	4.75675	4.75675	4.75675	4.75675	4.75675	4.75675	4.75675	4.75675	4.75675	4.75675	4.75675	4.75675	4.75675
2.2	6.37893	6.37893	6.37893	6.37893	6.37893	6.37893	6.37893	6.37893	6.37893	6.37893	6.37893	6.37893	6.37893
2.4	8.46037	8.46037	8.46037	8.46037	8.46037	8.46037	8.46037	8.46037	8.46037	8.46037	8.46037	8.46037	8.46037

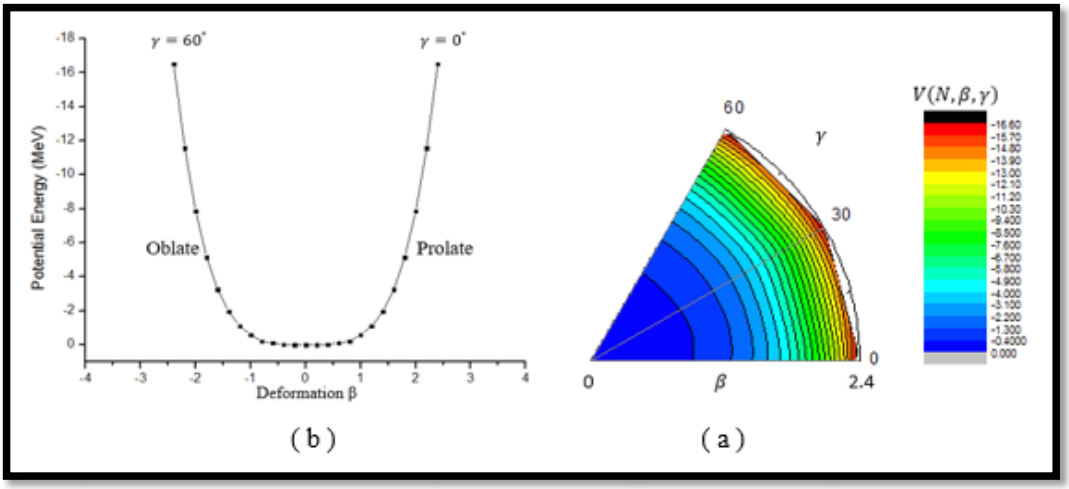

Figure 1. a) The Contour plots; b) The axial symmetric for the $^{92}_{42}Mo_{50}$ isotope of the SU(5) – O(6) limit.

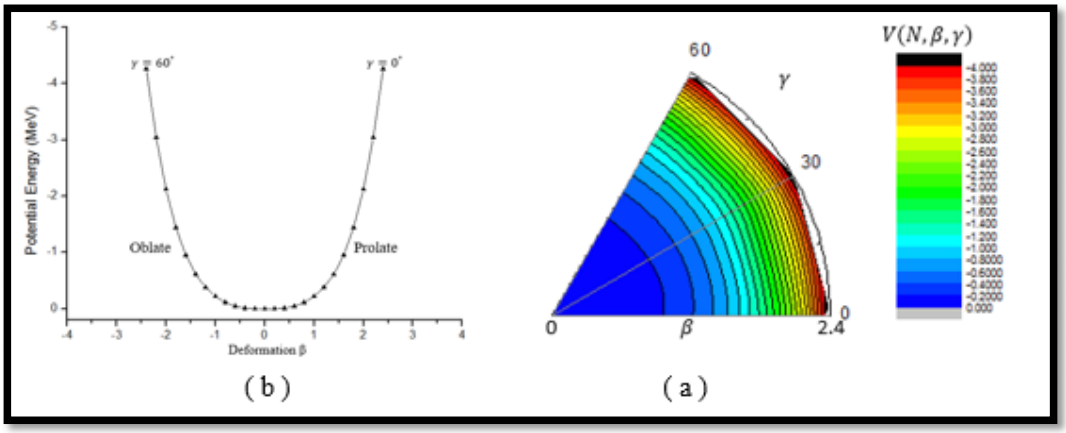

Figure 2. a) The Contour plots; b) The axial symmetric for the $^{94}_{42}Mo_{52}$ isotope of the SU(5) – O(6) limit.

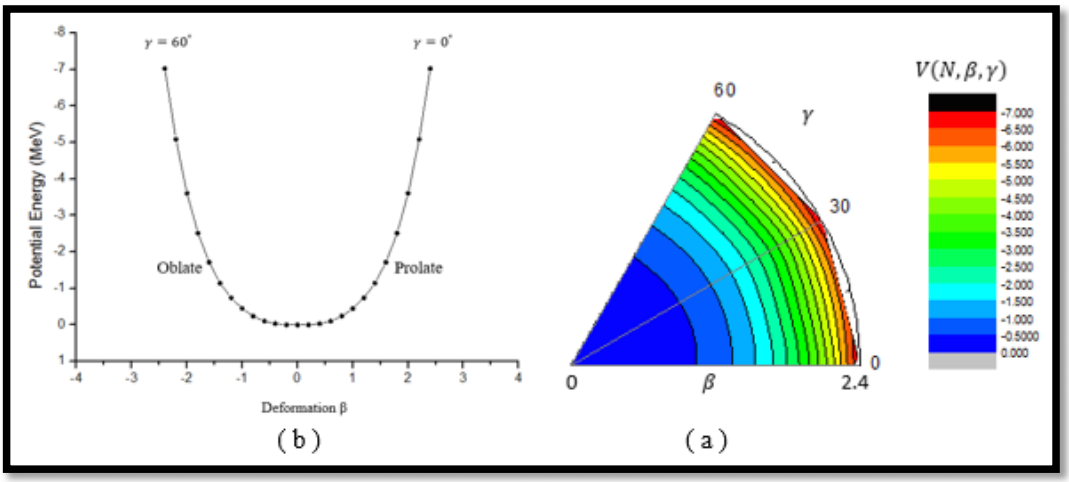

Figure 3. a) The Contour plots; b) The axial symmetric for the $^{96}_{42}Mo_{54}$ isotope of the SU(5) – O(6) limit.

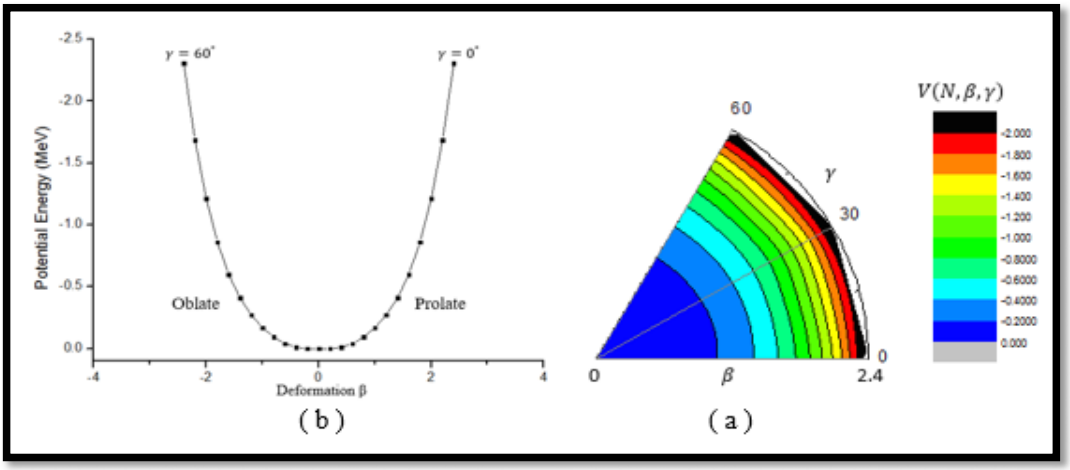

Figure 4. a) The Contour plots; b) The axial symmetric for the $^{98}_{42}Mo_{56}$ isotope of the SU(5) – O(6) limit.

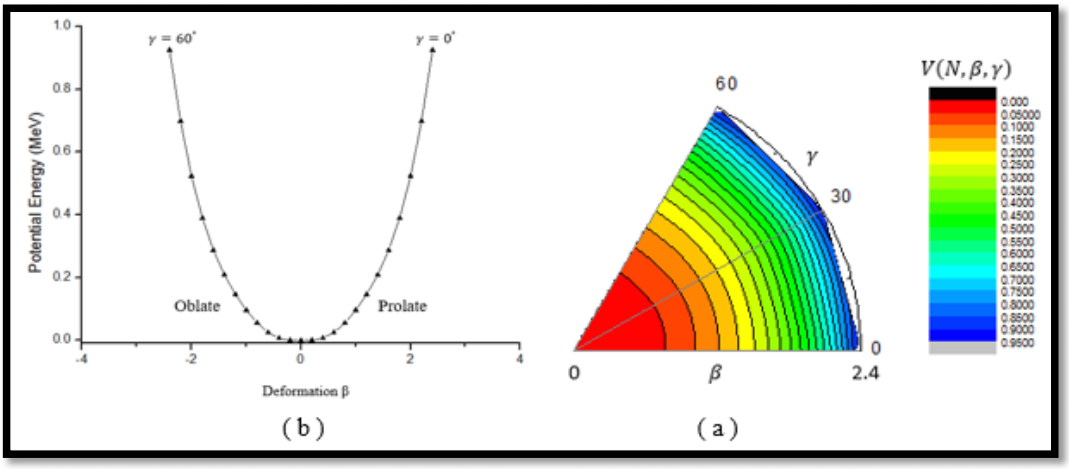

Figure 5. a) The Contour plots; b) The axial symmetric for the $^{100}_{42}Mo_{58}$ isotope of the SU(5) limit.

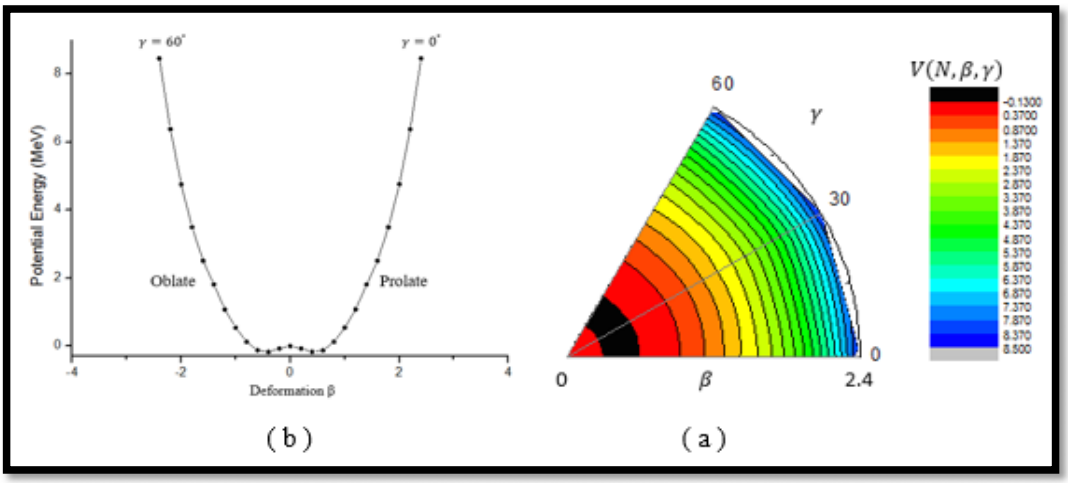

Figure 6. a) The Contour plots; b) The axial symmetric for the $^{102}_{42}Mo_{60}$ isotope of the O(6) limit.

4. CONCLUSIONS

From previous calculations, we can conclude the following:

1– Determine the dynamical symmetries to the isotopes under study, where turns out that, *Mo* (A= 92-98) even – even isotopes are belonging to the dynamical symmetry SU(5) – O(6), *Mo* (A= 100) isotope is belonging to the dynamical symmetry SU(5), and *Mo* (A= 102) isotope is belonging to the dynamical symmetry O(6).

2– Calculation of the potential energy surface for the isotopes in the present work mathematically.

3– The potential energy surface $V(N,\beta,\gamma)$ gives the final shape to the nucleus.

4– The potential energy surface $V(N,\beta,\gamma)$ depends on the parameters (N,β,γ) and the variables $(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$ respectively.

5 – When the variable (α_2) is equal to zero, the limit that includes $(\alpha_2\beta^3\cos 3\gamma)$ is equal to zero, consequently the energy values are the same at each value from β values.

REFERENCES

[1] Lewars, E. G., *Computational Chemistry Introduction to the Theory and Applications of Molecular and Quantum Mechanics*, 2nd Ed., Springer Science+Business Media B.V., 2011. DOI 10.1007/978-90-481-3862-3

[2] Abdulkadhim, A., Investigation Energy Levels and Energy Transitions for $^{250}_{98}Cf_{152}$ Isotope Using Interacting Boson Model, *Physics AUC*, (33), (120-132), 2023.

[3] Abdulkadhim, A., Investigation of nuclear structure for $^{142}_{54}Xe_{88}$ isotope using interacting boson model, *Kuwait Journal of Science*, **3**(50), (204-208), 2023.
https://doi.org/10.1016/j.kjs.2022.12.005

[4] Abdulkadhim, A., A Study for the Energy Levels and Potential Energy Surface of $^{246}_{96}Cm_{150}$ Isotope, *Journal of Science and Arts*, **3**(36), (265-274), 2016.

[5] Casten, R. F., Lipas, P. O., Warner, D. D, Otsuka, T., Heyde, K., and Draayer, J. P., *Algebraic Approaches to Nuclear Structure*, Volume 6, Harwood Academic Publishers GmbH, U.S.A., P. 149, 1993.

[6] Greiner, W., and Maruhn, J. A., *Nuclear Models*, Springer-Verlag Berlin Heidelberg, Germany, 1996.

[7] Casten, R. F., and Warner, D. D., The interacting boson approximation, *Reviews of Modern Physics*, **2**(60), (389-469), 1988.

[8] Iachello, F., Analytic Description of Critical Point Nuclei in a Spherical-Axially Deformed Shape Phase Transition, *Physical Review Letters*, **5**(87), (052502-1-052502-4), 2001. DOI: 10.1103/PhysRevLett.87.052502

[9] Bonatsos, D., *Interacting Boson Models of Nuclear Structure*, Pub. Oxford University Press, New York Ed. Hodgion P.E., 1988.

[10] Iachello, F., and Arima, A., *The interacting boson model*, Cambridge University Press, 2011. ISBN: 0511895518, 9780511895517

[11] Scholten, O., Computer program PHINT, KVT, University of Groningen, Netherlands– Holland, 1980; User's Manual of the PHINT package, Michigan state University, Michigan 48824, 1984.

[12] Pfeifer, W., *Introduction to interacting boson model of atomic nucleus*, (PI) P. 5, 1998.

[13] Abdulkadhim A., and Hussain K. A., Energy Levels, Reduced Matrix Elements & the Probability of Electric Transitions for $^{248}_{96}Cm_{152}$ Isotope, *Journal of Science and Arts*, **2**(39), (347-356), 2017.

[14] Abdulkadhim, A., The Square of Rotational Energy & the Moment of Inertia for Pt (A=192–198) Isotopes, *Research & Reviews: Journal of Physics*, **2**(6), (1–7), 2017.

[15] Rana, O., *Study of the energy levels and calculation of the delta mixing ratios for the some even – even isotopes*, M. Sc. Thesis, Al-Mustansiriyah University, Iraq, 2005.

[16] McGrory, J. B. (auth.), Abrahams, K., Allaart, K., and Dieperink, A. E. L. (eds.), *Nuclear Structure*, NATO Advanced Study Institutes Series, Series B, Physics, Vol. 67, Springer, US, 1981. ISBN 9781468439526, 9781468439502, DOI 10.1007/978-1-4684-3950-2

[17] Isacker, P. V., and Chen, J. Q., Classical limit of the interacting boson Hamiltonian, *Physical Review C*, **2**(24), (684-689), 1981.

[18] Endt, P. M., Strengths of Gamma-ray Transitions in A = 91-150 Nuclei, *Atomic Data and Nuclear Data Tables*, **1**(26), (47-91), 1981.

[19] National Nuclear Data Center Datasets for $^{92\text{-}102}$Mo, https://www.nndc.bnl.gov/ensdf/.