Transit Cosmic Expansion of Two-Fluid Cosmological Models in Modified Gravity

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Abstract

We have looked into Kaluza-Klein two-fluid cosmological models in the setting of $f(R)$ gravitational theory. In these models, the cosmic microwave background is represented by the radiation distribution, while the matter composition of the cosmos is represented by the first fluid. We take the γ law that is $p_m = (\gamma - 1)\rho_m$ the equation of state for a perfect fluid, $1 \le \gamma \le 2$ as given. For both exponential and power law volumetric growth, exact solutions to the field equations are found. We determined the models' cosmic jerk and statefinder parameters and found that the cosmos smoothly transitions from its decelerating to accelerating phase. Finally, some of the models' geometrical and physical characteristics are discussed.

Keywords: Gravitational theory; Two-fluid; Kaluza-Klein space-time.
PACS: 04.20.Jb, 04.50.Cd, 98.80.JK, 04.50.Kd.

1. Introduction

The most recent observable evidence for the accelerated cosmic expansion of the universe came from high redshift supernovae of type Ia (SN Ia) (Riess et al. 1998; Perlmutter et al. 1999; Bennett et al. 2003). The late-time accelerated expansion of the universe was also discovered by recent cosmic observations, including the Cosmic Microwave Background (CMB) (Spergel et al. 2003; Oli 2012), the large-scale structure (Tegmark et al. 2004), and the CMB radiation (CMBR) (Caldwell & Doran 2004; Huang et al. 2006). Additionally, it is hypothesised that the cosmic acceleration is caused by the enormous negative pressure known as dark energy. Two different ways have been suggested to explain this. Investigating other dark energy contenders (Bento 2003, Cohen 1999, Sheykhi 2009, Padmanabhan 2003, Copeland 1998) and modifying Einstein's theory of gravitation are two options.

Several modified theories of gravity have been put up as alternatives to Einstein's general theory of relativity in light of the universe's late-time acceleration and the existence of dark energy and dark matter. The cosmologically significant $f(R)$ gravity hypothesis stands out among them and has been thoroughly studied by various authors (Capozziello et al. 2005; Nojiri et al. 2006; Nojiri & Odintsov 2007). It has been demonstrated that $f(R)$ gravity theory, which is compatible in the dark epoch, is a plausible alternative to general relativity. It has been proposed that a general function of Ricci scalar R might be used to replace the general relativity's Einstein's Hilbert action in order to accelerate the universe. First of all, the class of modified theories of gravity was put forth by Buchdahl in 1970. Copeland et al. provide an extensive review of changed $f(R)$ gravity (2006). Researchers like Caroll et al. (2004), Nojiri and Odintsov (2003, 2004), Akbar and Cai (2006), and Chiba et al. (2007) have researched several aspects of $f(R)$ gravity. Gravitational Bianchi type I models have been studied by Gurovich and Starobinsky (1979). Multamki and Vilja discuss vacuum solutions for the spherically symmetric metric in $f(R)$ gravity (2006, 2007).

The moving star gains a constant asymptotic speed at great distances, according to Sobouti's analysis of the modified Schwarzschild de Sitter metric in the $f(R)$ gravity, whereas in the weak field limit, one receives a minor logarithmic correction to the Newtonian potential. Sharif and Shamir (2010) investigate the cosmological models for the ideal fluid in $f(R)$ gravity of the Bianchi type I and V. Shamir (2010) researches cosmological models of gravity of the Bianchi type. In their 2012 study, Shojai and Shojai examined the static, spherically symmetric interior solution to $f(R)$ gravity. By adopting symmetric space time for dark energy and the mesonic scalar field, Aktas et al. (2012) were able to create anisotropic universe models in terms of $f(R)$ gravity. In 2018, Capozziello et al. looked at the significance of energy conditions in $f(R)$ cosmology. Numerous physicists and cosmologists have discovered different contexts for $f(R)$ gravity, including Katore et al. (2016), Vijaya Santhi et al. (2017), Heba Sami et al. (2017), Roushan et al. (2019), Chaichain et al.(2017), Shaikh and Katore(2016), Vijaya Santhi et al. (2019), Shah and Samantha (2019), Dagwal (2020), Maurya and Myrzakulov (2024).

The early universe was thought to have been higher dimensional. Due to experimental restrictions, it is not now possible to detect more dimensions. However, Kaluza (1921) and Klein (1926) demonstrated that both four-dimensional gravity and Maxwell's electromagnetic unification are included in five-dimensional general relativity. According to the Kaluza-Klein theory, empty five-dimensional space's geometry is the sole source of four-dimensional matter (Patrico 2013). The fifth dimension is thought to be compressed and wrapped up into a little circle. Numerous theories, including the super symmetric Kaluza-Klein theory, the multidimensional unified theories, the string theory, etc., were founded on the concept of an additional dimension. According to Marciano (1984), there should be compelling evidence for the presence of extra dimensions if experiments show that fundamental constants vary throughout time. The equation of motion in higher dimensional gravity is stated in terms of $N(>4)$ dimension and typically exhibits additional effects when applied to four-dimensional space-time. Additional terms related to the fifth dimension are present in the generic five-dimensional geodesic and can be seen as modifications to the conventional four-dimensional equation of motion (Wesson 2011). Some of the challenging problems in Big Bang cosmology and other areas of physics have been successfully addressed by five-dimensional models within the framework of Kaluza-Klein theory. The Kaluza-Klein cosmological model was examined in several contexts by Gross and Perry (1983), Sharif and Khanum (2011), Reddy and Vijaya Laxmi (2015), Adhav et al. (2012), Tikekar and Patel (2000), Santhi Kumar and Reddy (2015), Khadekar and Wanjari (2015), Pawar et. al.(2018) , Reddy and Aditya (2018) , Naidu et. al. (2021), Lambat and Pund (2023).

Two-fluid cosmological models have become more significant in recent years. In general relativity, Coley and Dunn (1990) created a Bianchi type VI_0 model with a two-fluid source. In their 2002 study, Pant and Oli used space-time of Bianchi type II to examine two-fluid cosmological models. Oli (2008a,2008b) studies Bianchi type I two-fluid cosmological models with and without variables G and Λ. In the FRW universe, Amirahashchi et al. (2011) created a two-fluid dark energy model with a time-dependent deceleration parameter.

In 2011, Harko and Lobo looked into the two-fluid dark matter models. The interacting two-fluid viscous dark energy models in the non-flat cosmos have been studied by Amirahashchi et al. (2012). Mishra et al. (2017) have examined the accelerating dark energy cosmological model in a two-fluid with a hybrid scale factor. Mishra et al. (2018) investigate anisotropic cosmological models using two fluids. Anisotropic Bianchi type VI_0 two fluid cosmological models coupled with massless scalar field and time-varying G and Λ have been studied by Satish and Venkateswarlu (2019), Solanke et al.(2021) have obtained accelerating cosmic model with mixture of fluids, first fluid show the perfect fluid and other show the dark energy. Shekh et. al (2023) studied Interacting two fluid models in modified theories of gravitation.

In this study, we explore the Kaluza-Klein cosmological model with two-fluid in $f(R)$ gravity, which is inspired by the aforementioned research. The work is structured as follows: Section 2 describes the field equation of $f(R)$ gravity. In section 3, we used the Kaluza-Klein metric to construct the field equations in the presence of minimally interacting two fluids. The volumetric exponential expansion concept is covered in Section 4. We looked into the volumetric power law expansion model in Section 5. Conclusion is found in Section 6.

2. Formalism for the $f(R)$ gravity

The mechanism for the $f(R)$ gravitational field is provided

$$S = \int \sqrt{-g} \ \left(\frac{1}{16\pi G} f(R) + L_m \right) d^4 x \ . \tag{1}$$

L_m is the matter Lagrangian and the Ricci scalar's general function $f(R)$. Simply substituting $f(R)$ for R in the typical Einstein-Hilbert action yields this action. By changing the action in relation to the metric g_{ij} as shown, the corresponding field equations are discovered

$$F(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - \nabla_i \nabla_j F(R) - g_{ij} \nabla^i \nabla_j F(R) = K^2 g_{ij} T_j^i \ , \tag{2}$$

where the covariant derivative is ∇_i and $F(R) = \dfrac{\partial f(R)}{\partial R}$ .In their 2016 study, Katore and Hatkar looked into two-fluid FRW cosmological models. Reddy et al. (2014) investigated vacuum solutions of $f(R)$ gravity-based Bianchi type I and V models with a unique deceleration parameter. In an anisotropic model, Singh et al. (2013) investigated the functional form with power law growth. In their 2016 paper, Katore et al. offered a unified description of the gravitational Bianchi type-I universe. Anisotropic plane symmetric two-fluid cosmological models with time-varying G and Λ have been examined by Verma et al. (2015).

3. Solutions to field equations

We consider a Kaluza-Klein metric with five dimensions that has the following form:

$$ds^2 = dt^2 - A^2(t)[dx^2 + dy^2 + dz^2] - B^2(t)d\varphi^2 \ , \tag{3}$$

where φ the fifth coordinate is taken to the space, $A(t)$, $B(t)$ as the metric potentials. Kaluza-Klein cosmological model in $f(R,T)$ gravity with domain wall was researched by Biswal et al. (2015). Lyra Geometry was used by Sahu et al. (2017) to study the Kaluza-Klein slanted cosmological model. The energy momentum tensor for two-fluids is established by Letelier (1980) and Bayin (1982) $T_j^i = (T^m)_j^i + (T^r)_j^i$,where $(T^m)_j^i$ the energy momentum tensor of the radiation field and $(T^r)_j^i$ the matter field are both present, which are (Coley and Dunn

1990) $(T^m)_{ij} = (p_m + \rho_m) u_i^m u_j^m - p_m g_{ij}$, $(T^r)_{ij} = \frac{4}{3}\rho_r u_i^r u_j^r - \frac{1}{3}\rho_r g_{ij}$, where, ρ_m, p_m and ρ_r are the energy densities of the radiation, the pressure of the matter, and the energy density of the matter, respectively. $g^{ij} u_i^m u_j^m = 1$, $g^{ij} u_i^r u_j^r = 1$ and $u_i^m = (0,0,0,0,1)$, $u_j^r = (0,0,0,0,1)$ .The form of the total energy momentum tensor is

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = -(p_m + \frac{1}{3}\rho_r) = -p \ , T_5^5 = \rho_m + \rho_r = \rho \ . \tag{4}$$

Adhav et al. look into two-fluid cosmological models in space-time of Bianchi type-III in 2011. Induced-matter theory with two-fluid four-dimensional FRW cosmological models was examined by Guang and Qian in 2006. Mete et al. (2013) have investigated the two-fluid source Kasner cosmological models in general relativity. As per Akarsu and Kilinic (2010), we first assume that the two sources interact minimally in order for their individual energy momentum tensors to be conserved. As a result, the single conservation equation $T^{ij}_{;j} = 0$ for a system of two fluids leads to the following conservation equations:

$$(\rho_m)_5 + \left(3\frac{A_5}{A} + \frac{B_5}{B}\right)(\rho_m + p_m) = 0 \ , \ (\rho_r)_5 + \frac{4}{3}\left(\frac{3A_5}{A} + \frac{B_5}{B}\right)\rho_r = 0 \ .$$

With (2), (3), and (7), the field equations are stated as

$$F\left[2\frac{A_5^2}{A^2} + \frac{A_{55}}{A} + \frac{A_5 B_5}{AB}\right] - \frac{1}{2} f - 2\frac{A_5 F_5}{A} - \frac{B_5 F_5}{B} - F_{55} = -K^2\left(p_m + \frac{1}{3}\rho_r\right), \tag{5}$$

$$F\left[\frac{B_{55}}{B} + 3\frac{A_5 B_5}{AB}\right] - \frac{1}{2} f - 3\frac{A_5 F_5}{A} - F_{55} = -K^2\left(p_m + \frac{1}{3}\rho_r\right) \ , \tag{6}$$

$$F\left[-3\frac{A_{55}}{A} - \frac{B_{55}}{B}\right] + \frac{1}{2} f + 3\frac{A_5 F_5}{A} + \frac{B_5 F_5}{B} = K^2(\rho_m + \rho_r). \tag{7}$$

Here, the subscript "5" stands for a derivative with regard to *t*.

Over three equations and seven unknowns are included (*A*, *B*, *F*, *f*, p_m, ρ_m , ρ_r). Therefore, one might take into account extra factors as per a physical circumstance or an arbitrary mathematical assumption in order to arrive at the solution. For assuming the condition, one should keep in mind that a physical scenario may result in a differential equation that is not integrable, whereas a mathematical assumption may produce conclusions that are not consistent with reality. Johari and Desikan (1994) employed the Robertson-Walker model's power law relationship between the scale factor and scalar field in the context of Brans-Dicke theory. The relationship between the scale factor and the derivative of $f(R)$ gravity, as employed by Sharif and Shamir (2009, 2010), demonstrates that $F \,\alpha\, a^n$, where $a = V^{\frac{1}{4}}$ is a scalar factor and *n* an arbitrary constant. We include above equation to solve the field equations in the form $F = ba^n$, where the proportionality constant *b* is used. We use as the specific value of the constants $b = 1$, $n = 3$.

With (5), and (6) give us

$$\frac{d}{dt}\left(\frac{A_5}{A} - \frac{B_5}{B}\right) + \left(\frac{A_5}{A} - \frac{B_5}{B}\right)\left(\frac{A_5}{A} + \frac{B_5}{B}\right) + 2\frac{A_5}{A}\left(\frac{A_5}{A} - \frac{B_5}{B}\right) + \left(\frac{A_5}{A} - \frac{B_5}{B}\right)\frac{F_5}{F} = 0 \ . \tag{8}$$

On incorporating (8), there is $\frac{A}{B} = \exp\left(\int \frac{c}{A^3BF}dt\right) + c_1$, (9)

where c_1 is the integrating constant, which is assumed to be zero for convenience. We have $\frac{A}{B} = \exp\left(\int \frac{c}{V^2}dt\right)$, where $F = V$ and $V = A^3B$. We choose a γ-law as an equation of state and follow the literature of Oli (2008b), Pant and Oli (2002), and Verma et al. (2015) $p_m = (\gamma - 1)\rho_m$, where $1 \leq \gamma \leq 2$ .The model's spatial volume , mean Hubble's parameter are determined by $V = A^3B$, $H = \frac{1}{4}\left[3\frac{A_5}{A} + \frac{B_5}{B}\right]$. The deceleration parameter is used to determine whether the cosmos is accelerating or decelerating. The universe is said to be decelerating when q is positive, while the cosmos is speeding up when q is negative. The definition of the deceleration parameter is

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 . \tag{10}$$

The model's mean anisotropic parameter, expansion scalar and shear scalar are $\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2$, where x, y, z and, φ respectively, are the directional Hubble's parameters, $\theta = 4H$, $\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - H^2\right) = \frac{4}{2}\Delta^2 H$.

We have taken into consideration two volumetric expansion rules, namely provided by the volumetric power law and the volumetric exponential expansion law

$$V = c_2\, e^{4lt}, \tag{11}$$

$$V = c_3 t^{4m} , \tag{12}$$

where c_2, c_3 , l , m arbitrary positive constants and are present. The volumetric expansion is speeding up in the models with exponential growth and power law (for $m > 1$). For $m = 1$, it displays constant volumetric expansion, whereas the model for displays decelerating volumetric expansion. Singh and Beesham(2011), Sahoo et al. (2016), and Akarsu and Kilnic (2010) have all explored these volumetric expansions.

4. Exponential Expansion Model

With (12) , the metric potentials are written as

$$A = c_2^{\frac{1}{4}} \exp\left(lt - \frac{c}{32c_2^2 l}e^{-8lt}\right), \tag{13}$$

$$B = c_2^{\frac{1}{4}} \exp\left(lt + \frac{3c}{32c_2^2 l}e^{-8lt}\right). \tag{14}$$

With (13) and (14), the Kaluza-Klein metric changes in

$$ds^2 = dt^2 - c_2^{\frac{1}{2}} \exp\{2lt - \frac{c}{16c_2^2 l}e^{-8lt}\}[dx^2 + dy^2 + dz^2] - c_2^{\frac{1}{2}} \exp\{2lt + \frac{3c}{16c_2^2 l}e^{-8lt}\}d\varphi^2 . \tag{15}$$

As can be seen, the scale factor permits constant values at first, but as they go through time without experiencing any singularities, they eventually diverge to infinity. The model's Ricci scalar is determined to be $R = 20l^2 + \frac{3c^2}{4c_2^4}e^{-16lt}$ .It has been seen that at $t = 0$, R is constant, i.e., $R = 20l^2 + \frac{3c^2}{4c_2^4}$ and R exponentially diminishes, and tends to $20l^2$ at late time (i.e. $t \to \infty$).The Ricci scalar's time-based $f(R)$ function is calculated as $f(R) = \frac{c^2}{c_2^3} e^{-12lt}$. It can be seen from the diagram above that the function of the Ricci scalar $f(R)$ is constant at the start of the model, diminishes as time goes on, and finally becomes zero at enormous time.

4.1 The Physical and Kinematical Properties

The Spatial volume, mean Hubble's parameter, deceleration parameter, expansion scalar, shear scalar, mean anisotropic parameter are calculated as $V = c_2 e^{4lt}, H = l, q = -1, \theta = 4l$ $\sigma^2 = \frac{3c^2 e^{-16lt}}{8c_2^4}, \Delta = \frac{3\,c^2\ e^{-16lt}}{16\ c_2^4\ l^2}$ respectively. The spatial volume of the model is seen to be initially bounded at $t = 0$ and then exponentially increase as t increases and become infinite at $t \to \infty$. We have found in this model $q = -1$ and $\frac{dH}{dt} = 0$. As a result, it provides both the highest Hubble parameter value and the quickest pace of cosmic expansion. Therefore, this model might capture the extremely late period of the cosmos as well as the inflationary epoch in the early universe. This result is comparable to that of Sahoo et al (2014). It is significant to remember that, in general relativity $q > 1$, whereas in $f(R)$ gravity $q = -1$, (Samanta and Debata 2013). With passing time, the mean anisotropic parameter falls off exponentially until it reaches zero. As a result, the examined model is isotropic in the long run. The shear scalar vanishes at $t \to \infty$ whereas it is finite at $t = 0$. Additionally, the expansion scalar does not change over the course of the universe's expansion.

The formula for calculating matter's energy density is

$$\rho_m = -\frac{60c_2 l^2 e^{4lt}}{k^2(4-3\gamma)} - \frac{7c^2 e^{-12lt}}{4k^2(4-3\gamma)c_2^3}, \qquad (16)$$

Radiation's energy density is calculated as

$$\rho_r = \left[\frac{60}{4-3\gamma} + 12\right]\frac{c_2 l^2}{k^2} e^{4lt} + \left[\frac{7}{4-3\gamma} - 1\right]\frac{c^2}{4k^2 c_2^3} e^{-12lt} . \qquad (17)$$

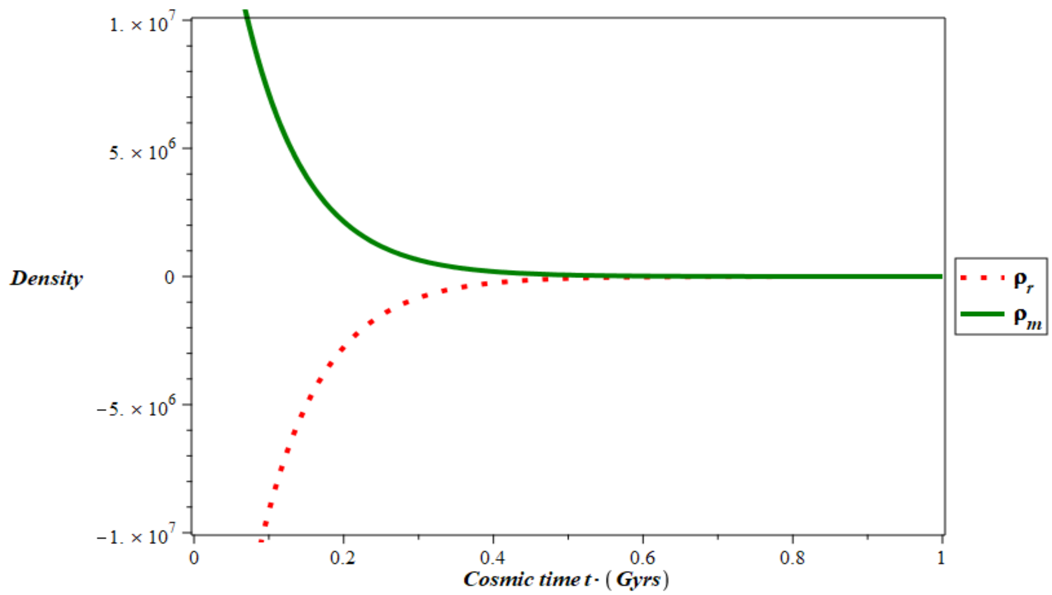

Fig. 1 Plot of Energy density verses cosmic time for $\gamma = 2, k = l = c = 1,\ c_2 = \dfrac{1}{300}$.

It can be shown from Fig. 1 that the universe is matter-dominated. The radiation energy density is negative for $\gamma = 2$, as shown by expressions (16) and (17), whereas the matter energy density is positive. As a result, matter energy density and radiation energy density are dominant in the cosmos, respectively. These results are similar to those of Samanta and Debata (2013).

The parameter for matter density is calculated as

$$\Omega_m = \frac{\rho_m}{4H^2F} = -\frac{1}{k^2(4-3\gamma)}\left[15 + \frac{7c^2}{16c_2^4 l^2} e^{-16lt}\right], \tag{18}$$

The parameter for radiation density is calculated as

$$\Omega_r = \frac{\rho_r}{4H^2F} = \frac{1}{k^2}\left[\left(\frac{15}{4-3\gamma} + 3\right) + \left(\frac{7}{4-3\gamma} - 1\right)\frac{c^2}{16c_2^4 l^2} e^{-16lt}\right], \tag{19}$$

The model's energy density parameter is determined to be

$$\Omega = \Omega_m + \Omega_r = \frac{1}{k^2}\left[3 - \frac{c^2}{16c_2^4 l^2} e^{-16lt}\right]. \tag{20}$$

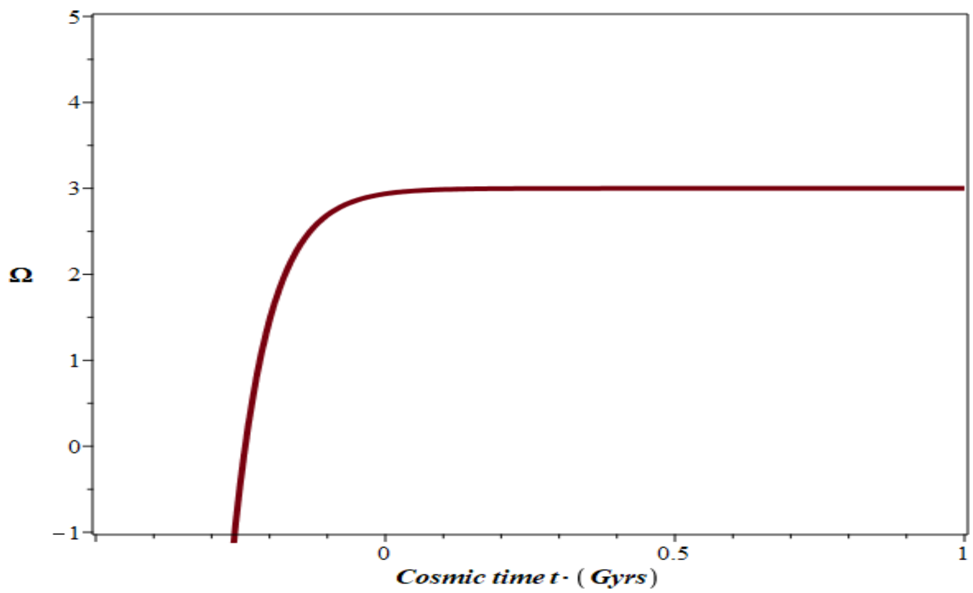

Fig. 2 Plot of density parameter verses cosmic time for $k = l = c = c_2 = 1$

Fig. 2 depicts the evolution of the density parameter across cosmic time. Throughout the evolution, Ω has a finite and bounded value. We noticed that close to $t = 0$, $\Omega > 1$, and as $t \to \infty$, $\Omega \to 3$.

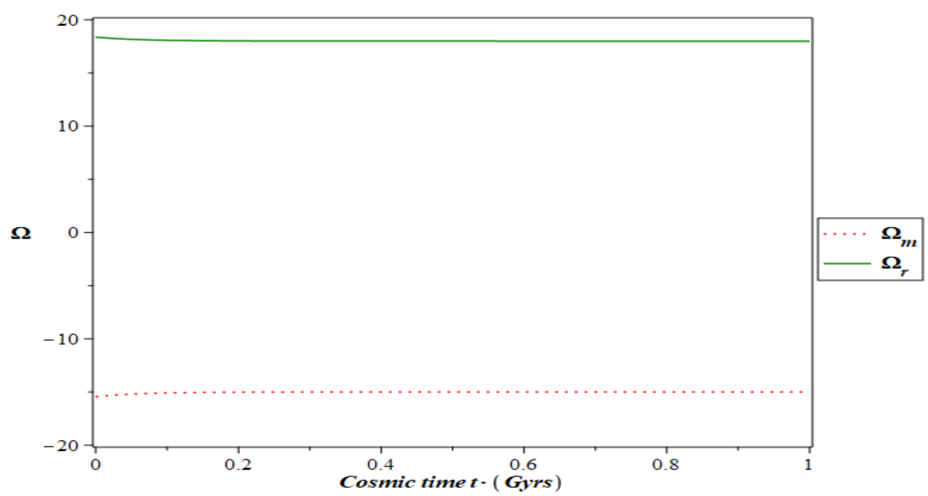

Fig. 3 Plot of density parameters verses cosmic time for $\gamma = 1, k = l = c = c_2 = 1$.

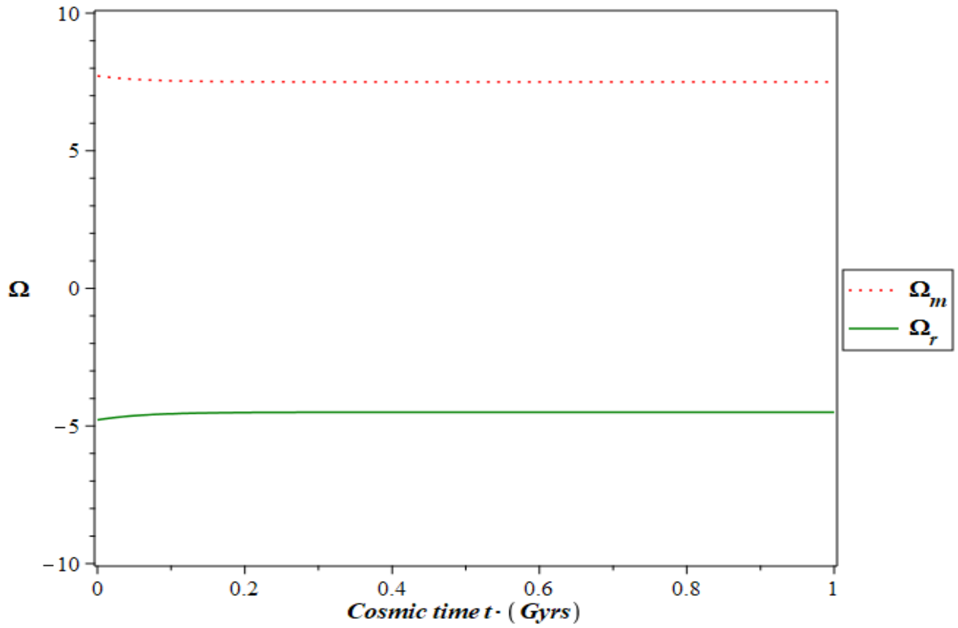

Fig. 4 Plot of density parameters verses cosmic time for $\gamma = 2, k = l = c = c_2 = 1$.

Via Fig. 3,we noted that throughout the evolution, Ω_m and Ω_r are both finite and bounded. For $\gamma = 1$, $\Omega_m < 0$, as $-20 < \Omega_m < -10$, and $\Omega_r > 0$ as $10 < \Omega_r < 20$ similarly via Fig. 4, it was discovered that for, $\gamma = 2$, $\Omega_m > 0$ as $5 < \Omega_m < 10$, and $\Omega_r < 0$ as $-6 < \Omega_r < -4$.

4.2. Jerk Cosmic Parameter

According to Capozziello (2006), the dark energy predominated the matter as the cosmos expanded larger over time and began to accelerate between five and six billion years ago. A dimensionless quantity known as the cosmic jerk parameter is what causes the universe to smoothly switch from a decelerating to an accelerating phase. It is provided by

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} \tag{22}$$

With (22), we come to $j(t) = 1$. For some models with positive values for the jerk parameter and negative values for the deceleration parameter, the universe transitions from a decelerating to an accelerating phase (Blandford et al. 2004, Chiba and Nakamura 1998). The behaviour like ΛCDM is clearly visible in the investigated model.

4.3. Statefinder Parameter

Statefinder diagnostic pair, introduced by Sahani et al. (2003) and Alam et al. (2003), is a geometrical pair with the coordinates {r, s} . Given by the statefinder pair is

$$r = \frac{\ddot{a}}{aH^3} \ , \quad s = \frac{r-1}{3(q-\frac{1}{2})} \quad . \tag{23}$$

A space-time metric is directly used to build the state finder. Consequently, it has a wider use than physical factors. The statefinder pair for the ΛCDM model is defined as {r.s}={1,0}, while the fixed value for SCDM is {r,s}={1,1} (Feng 2008). This model corresponds to ΛCDM because we found that {r, s}={1,0}in it.

5. Power Law Model

With (11), the metric potentials are written as

$$A = c_3^{\frac{1}{4}} t^m \exp\{\frac{c}{4c_3^2}\frac{t^{1-8m}}{1-8m}\}, \tag{24}$$

$$B = c_3^{\frac{1}{4}} t^m \exp\left[-\frac{3c}{4c_3^2}\frac{t^{1-8m}}{1-8m}\right]. \tag{25}$$

With (24) and (25), the Kaluza-Klein metric changes

$$ds^2 = dt^2 - c_3^{\frac{1}{2}} t^m \exp\{\frac{c}{2c_3^2}\frac{t^{1-8m}}{1-8m}\}[dx^2 + dy^2 + dz^2] - c_3^{\frac{1}{2}} t^m \exp\{-\frac{3c}{2c_3^2}\frac{t^{1-8m}}{1-8m}\} d\varphi^2. \tag{26}$$

It is obvious that the scale factor disappears very close. Then, as time goes on, they begin to change, eventually diverging over a long period of time. The model's Ricci scalar is determined to be $R = \frac{20m^2}{t^2} - \frac{8m}{t^2} + \frac{3c^2}{4c_3^4} t^{-16m}$.It is observed that, R is infinite at the beginning of the model and decreases as time increases and ultimately become zero at late time .The Ricci scalar's time-based function is calculated as

$f(R) = -40c_3 m^2 \frac{t^{4m-2}}{4m-2} + 16c_3 m \frac{t^{4m-2}}{4m-2} + \frac{c^2}{c_3^3} t^{-12m}$ .Here, we saw that it was impossible to write in the form of R a single sentence. Here, $f(R)$ values tend to be high initially and last a long time. $f(R)$ is achievable only for finite. It should be remembered that as m changes, so does their behaviour.

5.1 The Physical and Kinematical Properties

The Spatial volume, mean Hubble's parameter, deceleration parameter, expansion scalar, shear scalar, mean anisotropic parameter are calculated as $V = c_3 t^{4m}$, $H = \frac{m}{t}$, $q = \frac{1}{m} - 1$, $\theta = \frac{4m}{t}$, $\sigma^2 = \frac{3c^2 t^{-16m}}{8c_3^4}$, $\Delta = \frac{3\, c^2\, t^{-16m+2}}{16\, c_3^4\, m^2}$.The cosmos is currently accelerating, and the value of the deceleration parameter is somewhere in the range $-1 < q < 0$, according to recent observations of SN Ia. As a result, one can select a deceleration parameter value in our derived model that is compatible with the observed. It has been noted that the spatial volume V is zero at this time. It keeps growing forever for all positive values. The spatial volume continues to expand indefinitely over a long period of time. The Hubble's directional parameters are varied at $t \to 0$ and go close to zero at $t \to \infty$. The average Hubble's parameter decreases over time. As a result, the cosmos was expanding quickly when it first began, but that speed has now decreased as time has passed. The model under investigation accelerates for $m > 1$, decelerates for $0 < m < 1$, and displays constant-velocity volumetric expansion for $m = 1$. These findings agree with those of Akarsu and Kilnic (2010), Sahoo et al. (2016), Singh and Beesham (2011), and others. The shear and expansion scalars at $t = 0$ indicate that the cosmos begins to evolve from the original singularity with an infinite rate of shear, which ultimately becomes zero as $t \to \infty$. The anisotropic parameter of expansion is infinite at the beginning, indicating that the model was strongly anisotropic during the universe's history. The parameter $\Delta \to 0$ indicates that the cosmos has been isotropic for a very long time.

The formula for calculating matter's energy density is

$$\rho_m = \frac{20c_3 m^2 t^{4m-2}}{k(4-3\gamma)}\left[\frac{2}{4m-2}-3\right]+\frac{c_3 m t^{4m-2}}{k(4-3\gamma)}\left[13-\frac{16}{4m-2}\right]-\frac{7c^2 t^{-12m}}{4k^2(4-3\gamma)c_3^3}, \quad (27)$$

Radiation's energy density is calculated as

$$\rho_r = \frac{12c_3 m^2 t^{4m-2}}{k^2}\left[\frac{5}{4-3\gamma}+1\right]+\frac{c_3 m t^{4m-2}}{k^2}\left[-\frac{13}{4-3\gamma}+4\right]-\frac{20c_3 m^2 t^{4m-2}}{k^2(4m-2)}\left[\frac{2}{4-3\gamma}+1\right]$$
$$+\frac{8c_3 m t^{4m-2}}{k^2(4m-2)}\left[\frac{2}{4-3\gamma}+1\right]-\frac{c^2 t^{-12m}}{4k^2 c_3^3}+\frac{7c^2 t^{-12m}}{4k^2(4-3\gamma)c_3^3}. \quad (28)$$

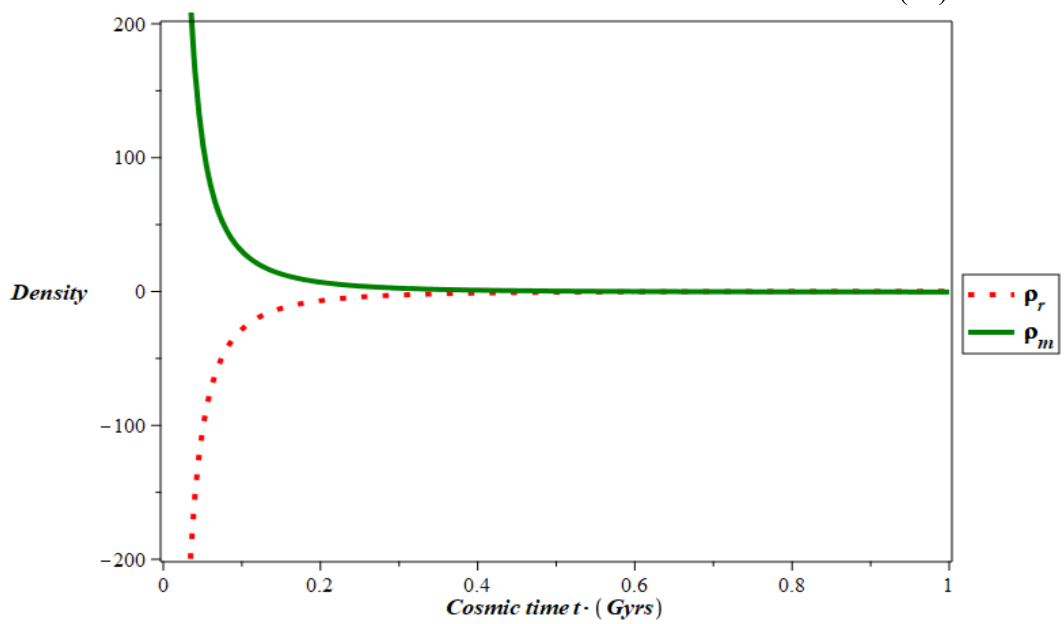

Fig. 5 Plot of energy density verses cosmic time for $\gamma=1$, $m=0.1$, $c=k=c_3=1$.

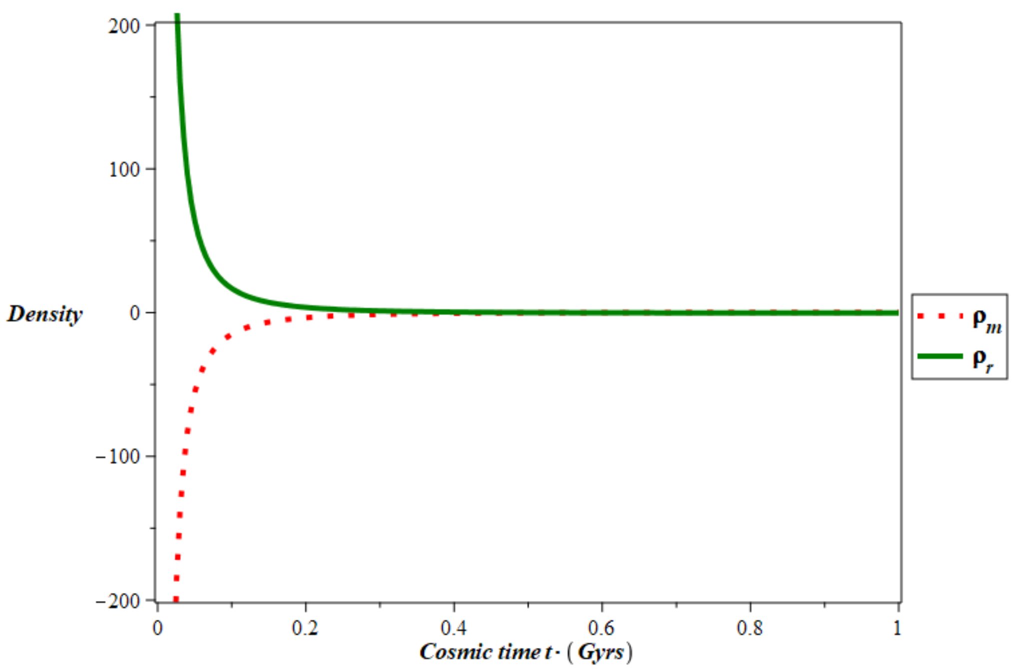

Fig. 6 Plot of energy density verses cosmic time for $\gamma=2$, $m=0.1$, $c=k=c_3=1$.

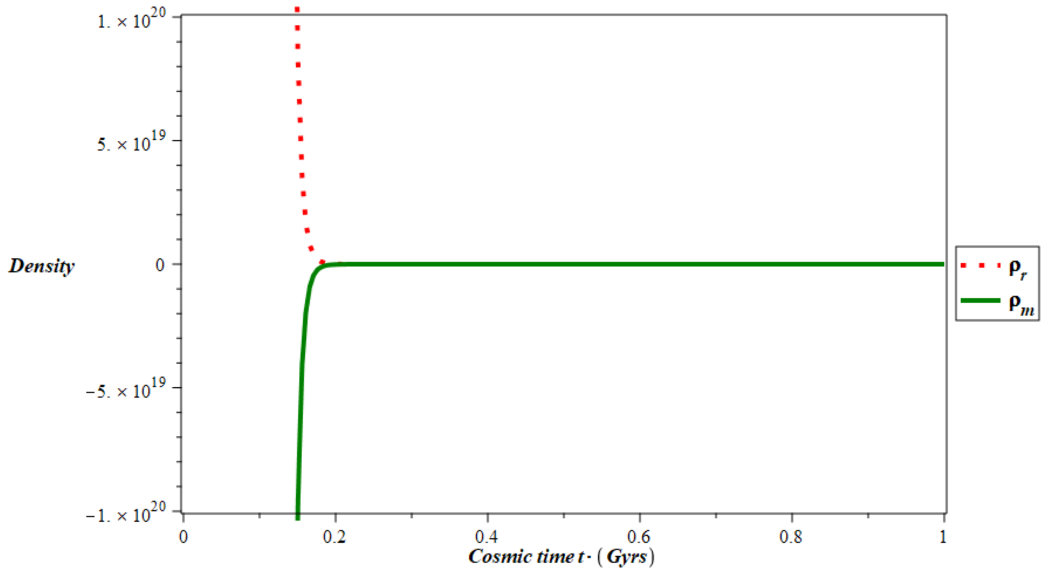

Fig. 7 Plot of energy density verses cosmic time for $\gamma = 1,\ m = 2,\ c = k = c_3 = 1$.

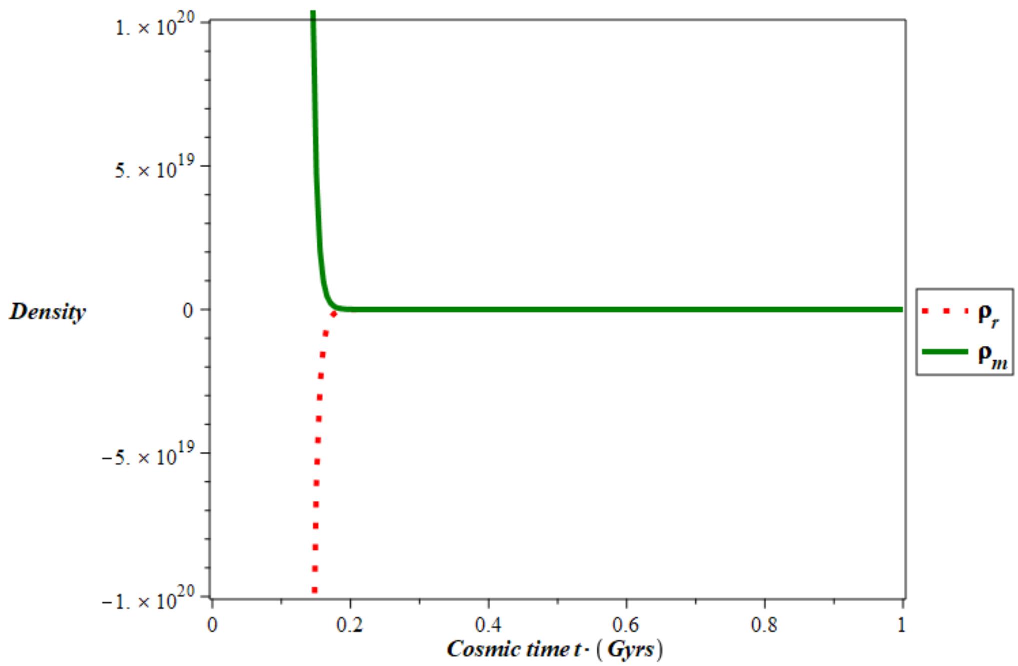

Fig. 8 Plot of energy density verses cosmic time for $\gamma = 2,\ m = 2,\ c = k = c_3 = 1$.

Via fig. 5-8, we found that the matter dominance of the dust ($\gamma = 1$) and stiff ($\gamma = 2$) models are different for $m = 0.1$, however the radiation dominance of the dust ($\gamma = 1$) and stiff ($\gamma = 2$) universes are also different for $m = 2$. In this instance, we have seen that the outcomes of our investigation differ with the general relativity results reported by Samanta and Debata (2013). This appears to be a general theory of relativity effect.

The parameter for matter density is calculated as

$$\Omega_m = \frac{\rho_m}{4H^2F} = \frac{1}{k^2(4-3\gamma)}\left\{5\left[\frac{2}{4m-2}-3\right]+\frac{1}{4m}\left[13-\frac{16}{4m-2}\right]-\frac{7c^2t^{-16m+2}}{16m^2c_3^4}\right\}, (29)$$

The parameter for radiation density is calculated as

$$\Omega_r = \frac{\rho_r}{4H^2F},$$

$$\Omega_r = \frac{1}{k^2}\left\{\begin{array}{l} 3\left[\frac{5}{4-3\gamma}+1\right]+\frac{1}{4m}\left[-\frac{13}{4-3\gamma}+4\right]-\frac{(5m-2)}{m(4m-2)}\left[\frac{2}{4-3\gamma}+1\right] \\ \quad -\frac{c^2}{16m^2c_3^4}\left[1-\frac{7}{4-3\gamma}\right]t^{-16m+2} \end{array}\right\} \quad (30)$$

The model's energy density parameter is determined to be

$$\Omega = \Omega_m + \Omega_r = \frac{1}{k^2}\left\{3-\frac{1}{4m-2}-\frac{c^2t^{-16m+2}}{16m^2c_3^4}\right\}. \quad (31)$$

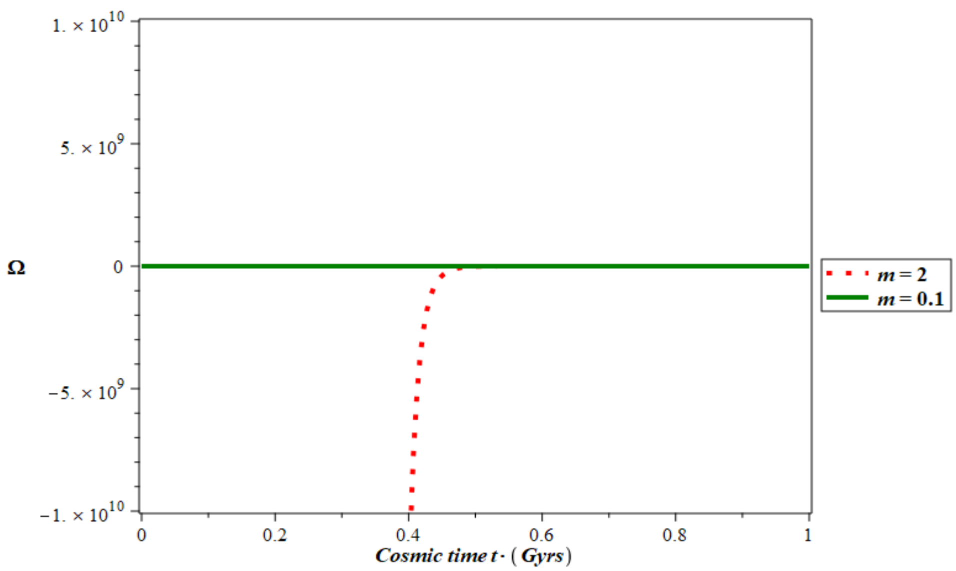

Fig. 9 Plot of Density parameter versus time for $c = k = c_3 = 1$.

We noticed that for $m = 0.1$, the universe $\Omega > 1$ is closed throughout its evolution, but for $m = 2$, we have $\Omega < 1$ for $0 \le t \le 0.4$.

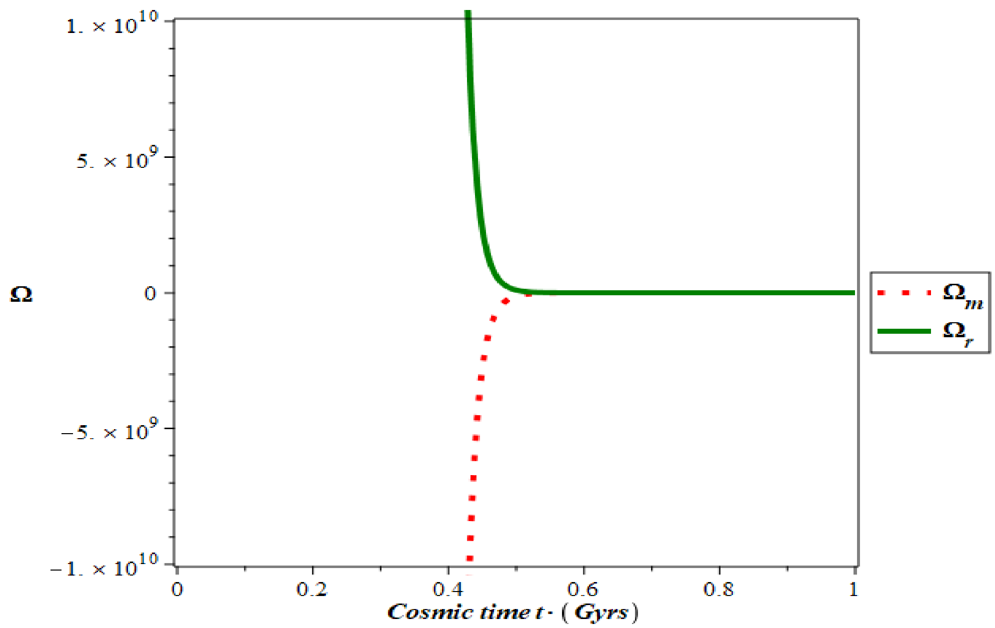

Fig. 10 Plot of Density parameter versus time for $\gamma = 1,\ m = 2,\ c = k = c_3 = 1$.

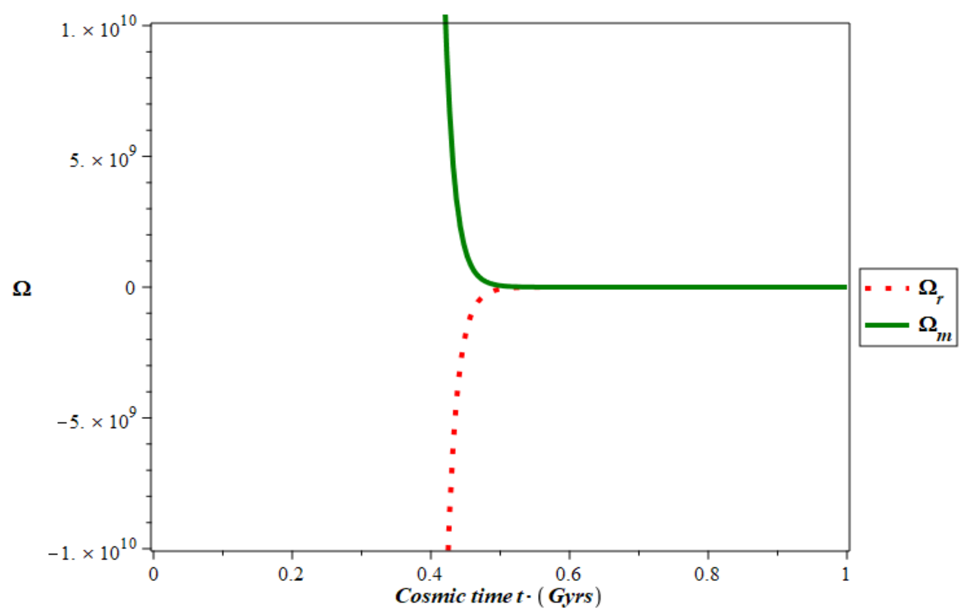

Fig. 11 Plot of Density parameter versus time for $\gamma = 2,\ m = 2,\ c = k = c_3 = 1$.

Fig. 10 and 11depict how Ω_m and Ω_r have changed throughout time. It is evident from Fig. 10 that for the dust universe $\gamma = 1$, $\Omega_m < 0$, $\Omega_r > 0$, early time and as $t \to \infty$ both $\Omega_m, \Omega_r \to 0$. Figure (11), which depicts the stiff universe $\gamma = 2$, shows that $\Omega_m > 0$, $\Omega_r < 0$, all through the universe's expansion.

5.2. Cosmic Jerk Parameter

The formula for the Cosmic Jerk parameter is $j(t) = \dfrac{m^2 - 3m + 2}{m^2}$. (32)

$m > 0$; given a suitable selection of m, Eq. (32) yields a constant number that is positive. As a result, the cosmos smoothly switches from its decelerating to accelerating phase.

5.3. State finder Parameter

For this model, the state finder parameters, *r* and *s*, are determined as

$$r = \frac{m^2 - 3m + 2}{m^2}, \qquad s = \frac{2}{3}m. \tag{33}$$

It's interesting to observe that the parameters solely depend on the value of m and are independent of time.

6. Conclusion

In the theory of $f(R)$ gravity, two-fluid cosmological models have been examined in this study. There are two volumetric expansions, to find the precise answers to the field equation, exponential and power law expansions are taken into consideration.

It is discovered that the model approaches isotropy in exponential expansion, which provides further insight on the universe's accelerating expansion (Tiwari 2013, Reddy et al. 2012, Shri Ram and Priyanka 2013). It is significant to remember that, $q > 1$ in general relativity, whereas $q = -1$ in $f(R)$ gravity, (Samanta and Debata 2013). The cosmos is dominated by radiation and matter, respectively. This result is similar to that of Samanta and Debata (2013). The investigated model exhibits behaviour ΛCDM consistent with an early closed cosmos that eventually became open.

When the power law expands, it is noticed that the cosmos begins to expand at an endless rate but then stops for a very long time. The expanding universe is speeding up. The cosmos smoothly switches from its decelerating phase to its speeding phase. The examined model starts out strongly anisotropic and then moves toward isotropy. We found that the matter dominance of the dust ($\gamma = 1$) and stiff ($\gamma = 2$) models are different for $m = 0.1$, however the radiation dominance of the dust ($\gamma = 1$) and stiff ($\gamma = 2$) universes are also different for $m = 2$. In this instance, we have seen that the outcomes of our investigation differ with the general relativity results reported by Samanta and Debata (2013). This appears to be a general theory of relativity effect. We noticed that for $m = 0.1$, the universe $\Omega > 1$ is closed throughout its evolution, but for $m = 2$, we have $\Omega < 1$ for $0 \le t \le 0.4$. It is evident that for the dust universe $\gamma = 1$, $\Omega_m < 0$, $\Omega_r > 0$, early time and as $t \to \infty$ both $\Omega_m, \Omega_r \to 0$. The stiff universe $\gamma = 2$, shows that $\Omega_m > 0$, $\Omega_r < 0$, all through the universe's expansion.

Declarations

Ethical Approval: This declaration is not applicable.

Competing interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions: Conceptualization, S.D.K..; Data curation, C.D.W and S.V.G.; Formal analysis, A.Y.S.; Investigation, C.D.W.; Methodology, C.D.W. and S.V.G.; Project administration, S.D.K.; Software, S.V.G. and A.Y.S.; Supervision, S.D.K.; Validation, A.Y.S.; Visualization, S.V.G. Writing—original draft, A.Y.S.; Writing— review & editing, C.D.W. and A.Y.S. All authors have read and agreed to the unpublished version of the manuscript.

Funding: This research received no external funding.

Availability of data and Material: There are no new data associated with this article.

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