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Thermodynamics of Quantum Measurement

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Abstract

Von Neumann's quantum measurement protocol is outlined in terms of thermodynamics. In particular, based on the separation between quantum and classical realms implied in the Copenhagen interpretation, entanglement generated from the interaction between the system and the apparatus yields a negative conditional entropy in the quantum part. On the other hand, the Heisenberg cut implies the mandatory tracing out of the system part, and the resulting apparatus corresponds to a classical system. This measurement process exhibits a conservation of energy between the quantum and classical systems, analogous to the first law of thermodynamics. Moreover, an exact amount of entropy increase in apparatus may shed light on a connection between the first and the second law of thermodynamics.

1 Introduction

Thermodynamics which outlines a relation between heat, work, temperature etc. has been placed at a special status in the history of physics. While quantum theory and relativity are generally considered to be fundamental laws describing the way our universe operates, the rules of thermodynamics withstood longer than the two. Indeed, some people have expressed an opinion on the reliability of thermodynamics, such that, it may never be overthrown. One of the reasons why thermodynamics has enjoyed not only great success but reliability at a full range is partly due to its compatibility, especially the first and the second laws, with people's common sense. Indeed, the energy conservation and the irreversibility of time flow are consistent with the experience of ordinary people.

On the other hand, while quantum theory has enjoyed unprecedented success in precision and prediction ever since its birth about a century ago, its exact nature has been debated. At the heart of the conceptual confusion lies the so-called measurement problem. Unlike a deterministic and causal unitary evolution of state vectors, measurement process insists on the collapse of wavefunction yielding an observable outcome probabilistically. There has been numerous attempts in resolving the apparent predicaments generating a various interpretation of quantum theory such as the many worlds [1], pilot waves [2], and certainly the Copenhagen interpretation.

With the rapid development of quantum computation last few decades [3], there has been an active research in connecting quantum information and thermodynamics [4]. In particular, with the pioneering work from Landauer [5], thermodynamical nature of information has been derived. Moreover, while the traditional statistical understanding of thermodynamics generally focused on a large number of microstates, there has also been a number of attempts in applying thermodynamics for a small quantum system with just a few qubits [6, 7].

It is a goal of this paper to outline quantum measurement protocol in terms of energy conservation analogous to the first law of thermodynamics. In particular, it will be argued that the collapse of wavefunction corresponds to the transfer of energy from quantum to classical realms.

2 Heisenberg Cut

There are various versions of quantum interpretations and this often causes confusion in regards to the precise assumptions in formulating the theory. Here we wish to outline orthodox axioms of quantum theory often used in quantum computation but with a strong emphasis of the Copenhagen spirit:

1. State Vector: it is defined in a complex vector space and a complete description of physical system or an object that is being observed.

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2. Observable: it is also defined in a complex vector space and corresponds to a measurable quantity of a physical system. It may also be identified as the observer's reference frame [8].

3. Dynamics: the state vector or observable evolves under unitary operation. That is, the dynamical variable is either a state vector (Schrödinger picture) or an observable (Heisenberg picture).

4. Measurement: it corresponds to the physical system represented by state vector, defined in quantum realm, being observed in the reference frame of observable, also defined in quantum world, that yields an outcome, in classical realm, probabilistically. One of the intrinsic elements in the Copenhagen interpretation is that it treats quantum and classical realms separately, also known as the Heisenberg cut.

It is noted that one of the central issues in quantum foundations involves the probability that is introduced in the measuring process. Indeed, the so-called the collapse of wavefunction presents a problem not only it is different from the transformation rule of unitary evolution but also its nature associated with observation. Let us review a measurement model involving this probability outlined by von Neumann [9]. Given a quantum state to be measured, an apparatus corresponds to a system with pointer that has a definite position in classical space. The scheme assumes that there is an interaction between the system and the apparatus such that quantum entanglement is created between the two. In particular, we wish to review this interaction of the measurement in the Heisenberg picture using the notation introduced in [10], with the Pauli matrices that are defined as follows:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{1}$$

Now, the system, Q, and the apparatus, A, are initially prepared uncorrelated as follows:

$$Q = (\sigma_x \otimes I, \sigma_y \otimes I, \sigma_z \otimes I) \tag{2}$$
$$A = (I \otimes \sigma_x, I \otimes \sigma_y, I \otimes \sigma_z) \tag{3}$$

Next, under unitary operations, the system and the apparatus become maximally entangled as follows,

$$Q = (\sigma_z \otimes \sigma_x, -\sigma_y \otimes \sigma_x, -\sigma_x \otimes I) \tag{4}$$
$$A = (I \otimes \sigma_x, \sigma_x \otimes \sigma_y, \sigma_x \otimes \sigma_z) \tag{5}$$

A conditional entropy corresponds to the unknown information about one variable given the value of the other. Therefore, it is easy to see that ordinarily the value of the conditional entropy is non-negative because the amount of remainder cannot be negative. The quantum version of conditional entropy was introduced in [11], and it has been shown that, in the case of entangled quantum states, such as the one in (4,5), it is given as follows:

$$\begin{aligned} S(Q|A) &= S(QA) - S(A) \\ &= -S(A) \end{aligned} \tag{6}$$

That is, it can be seen that the entanglement between the system and the apparatus yields the entropy of QA, a pure state, to be zero, which then leads the conditional entropy to have a negative value. There has been a number of notable works that provided meaning to this unfamiliar negativity in quantum entropy, such as negative quantum information [12], or an extractable work [13]

The identification of observable with the reference frame, an attempt to take the Copenhagen interpretation at a full scale, may lead to inconsistency within quantum theory. Indeed, it was argued [8] that when the observable is not only a reference frame but also the very object that is being observed, the two-picture formulation of quantum theory, namely the Heisenberg and the Schrödinger pictures, does not work. This inconsistency was then led to conjecture that it is the Heisenberg picture with time going backwards that yields a correct description of nature. It is noted that the negativity shown with the quantum conditional entropy as in (6) is in fact consistent with the conclusion drawn in [8] where the observer's reference frame, i.e., observable, unitarily evolving backwards in time.

Now, we wish to consider what happens in the classical realm: As explained in the Schrödinger's cat thought experiment, a superposition is not observed in the classical world. If we take the Heisenberg cut in the axiom fully, then there are two separate realms to be considered and, in particular, the process of entanglement between the system and the apparatus described above occurs only within the quantum

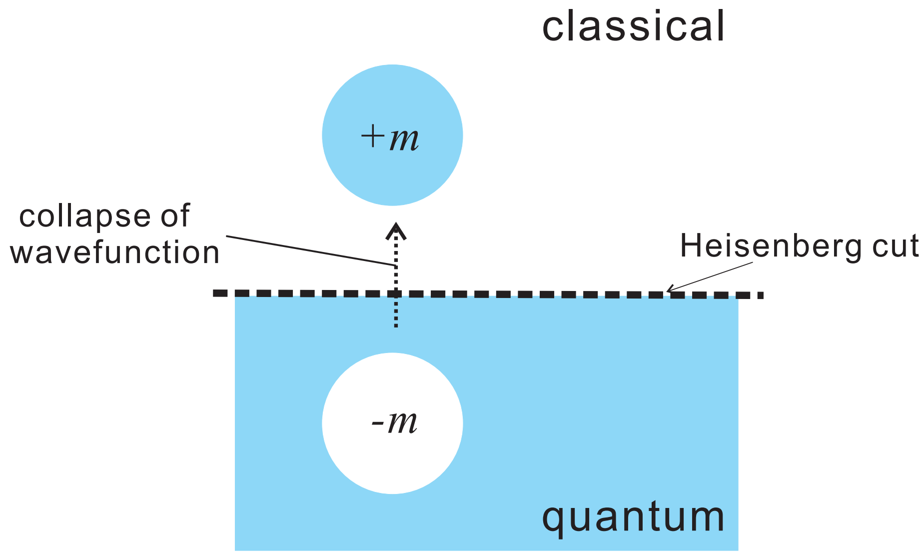

Figure 1: Energy conservation in quantum measurement process. There exists an energy transfer from quantum to classical realm, separated by Heisenberg cut, which corresponds to the collapse of wavefunction.

world. In the classical realm, the tracing out the system is mandatory and an observer, Alice, is left with the following state,

$$\frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)_A \tag{7}$$

This is equivalent to a *classical* system and it yields one of the two outcomes that is distinguishable in classical realm.

In fact, this explains why a physical system, such as a cat or an apparatus, in the classical realm does not exhibit quantum superposition. In particular, one of the confusions with the von Neumann's protocol was that how can an apparatus with a definite pointer in the classical world be entangled with a quantum system, i.e., a classical system to be in more than one status simultaneously. However, as discussed above, if one takes the Heisenberg cut seriously as implied in the Copenhagen interpretation, the quantum correlation exists only in the realm behind the cut, similar to the inside of black holes behind the event horizon, and in the classical realm, the apparatus is equivalent to a classical system.

3 Energy Transfer in Quantum Measurement

The intricate connection between information and thermodynamics has been discussed by Landauer, who showed that erasing a bit would require $kT\ln 2$ amount of work [5]. This result is surprising in a sense that rather vague concept called information is connected to the tangible work [13]. This indeed exhibits the well-known phrase in quantum information science that *information is physical* [14].

Moreover, Landauer's result and related work imply physical reality may in fact not be the same for different observers. For example, given a physical system of a qubit, Alice with the knowledge of preparation would perceive it as a pure state, therefore entropy vanishes, while Bob, without the information, would see the system as completely mixed, therefore non-zero entropy exists. That is, information is subjective [13] and if work or energy is connected to information as hinted by Landauer's principle, then the physical reality itself may be subjective, too. In fact, there has been a well-known result that the existence of particles is observer dependent. In the Unruh effect [15], Alice in the Rindler coordinate experiences particles while Bob in Minkowski reference frame do not observe them.

Similar to the Alice and Bob experiencing different reality, i.e., creation of particles, in the Unruh effect, one may consider a similar situation with the quantum measurement process. That is, while Alice experiences the mixed state in the classical realm as in (7) (similar to particle creation in Unruh effect), Bob is given the following state,

$$|0\rangle|0\rangle \tag{8}$$

that has no quantum correlation, and does not experience the energy that comes from the quantum realm and therefore no heat or energy is generated in the classical realm. Therefore, it may be said that Alice, given (4,5), and Bob, given (8), experience different physical realities similar to the Unruh effect.

The relation between the quantum and classical realms, particularly in the process of quantum measurement outlined above, may be considered with the following energy conservation rule between the two,

$$\Delta E_Q = T\Delta S_C \tag{9}$$

That is, the energy from the quantum realm of negative conditional entropy is equal to the increase of entropy of apparatus in the classical region multiplied by the temperature of the environment. One of the central issues regarding the measurement process has been exactly when the collapse of wavefunction occurs. The equation in (9) shows not only the energy is conserved between quantum and classical realms, but the transfer of energy is when the collapse of the quantum state occurs (Fig. 1).

One of the puzzling aspects of the second law is that while the validity of it is widely accepted, it is not always clear exactly how much irreversible increment of entropy is generated. According to (9), it is the change of entropy in the apparatus that corresponds to the increase of entropy, which then is responsible for the irreversible flow of time. That is, (9) exhibits not only the energy conservation but the non-decreasing rule of the second law of thermodynamics as well.

4 Remarks

Let us make some comments on the peculiarity of observable being the observer's reference frame as conjectured in the axiom of quantum theory above. It was noted that quantum states and observables are defined in complex vector space rather than a classical realm. While there is much debate in regards to whether the wavefunction defined in this hypothetical space to be taken to describe reality in a strict sense, it is certainly possible to consider the observable to be the observer's mental reference frame. In particular, the continuous unitary evolution of observer's mental reference frame that is going backwards in time may be considered as the Dirac-type negative sea that is filling up the vacuum. In this line of thought, the Heisenberg cut may be considered as a clear separation between the discrete physical world and the continuous mental realm generated from quantum evolution process. That is, the measurement process in quantum theory corresponds to the interaction between the mental and physical world where there exists energy conservation between the two.

In 1970's, black hole entropy and radiation were derived by Bekenstein [16] and Hawking [17] which showed the intimate connection between black holes and thermodynamics. Interestingly, the thermodynamical relation in (9) discussed in this paper also exhibited a relation similar to the black hole case: inside the horizon is analogous to the unreachable quantum realm behind the Heisenberg cut while the black hole exterior region corresponds to the classical world in the measurement process.

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