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Quantum Conditional Entropy in Entangled States

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Abstract

This paper seeks to outline a connection between physical reality and the observer. To this end, it delves into the fundamental tenets of quantum theory, utilizing computational terminology alongside the von Neumann measurement protocol. Particularly, the paper asserts that the selection of an observable within the quantum theory's axioms could be interpreted as a reference frame of the observer, rather than a physical one. This frame of reference is subsequently actualized through the quantum system as defined by the von Neumann measurement protocol. Drawing on the concept of negative conditional entropy that emerges from the correlation between the quantum system and the classical apparatus, the state of the observer undergoes a retrograde evolution. This evolution leads it to fill the classical domain beyond the horizon, or the Heisenberg cut, akin to Dirac's notion of a negative sea. In essence, this protocol furnishes a rudimentary framework for comprehending the subjective essence of physical reality. This understanding is grounded in the currently acknowledged axioms of quantum theory.

1 Introduction

Throughout history, science has aimed to establish unchanging objective facts, irrespective of differing personal outlooks. This approach has facilitated the progression of knowledge, enabling people to build on the work of predecessors and encouraging collaborative efforts across generations, culminating in a profoundly influential repository of wisdom. However, the advent of quantum theory in the early 20th century, particularly the Copenhagen interpretation championed by Bohr and Heisenberg, posed formidable challenges to the robust foundation of science's quest for objective truth. Quantum theory introduced a complex interplay between the observed object and the observing subject [1].

Interestingly, certain prominent philosophers of science from the 20th century began to posit inherent boundaries on science's role as a conveyor of objective truths. For instance, Karl Popper argued that science comprises a collection of hypotheses, whose validity must be continually tested through falsifiability [2]. Similarly, Thomas Kuhn, a science historian, expounded on the subjective constraints within science. He suggested that 'normal science' advances within the confines of a shared intersubjective paradigm, underscoring the inherently subjective nature of scientific progress within this framework [3].

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Indeed, a strong indication of subjectivity also emerges in the phenomenon of entanglement. Following numerous experiments [4, 5] that strongly validated quantum theory's predictions subsequent to Bell's insightful proposal [6], these findings were occasionally considered an invalidation of the incompleteness argument [7] (see [8] for a review). Nonetheless, nonlocality and entanglement introduce a seemingly paradoxical scenario [9]:

1. Superluminality exists between objects.
2. Superluminality does not exist between observers.

In essence, the restriction against signaling between observers leads to an uneasy situation, particularly when observers are often regarded as entities within the system. This quandary might find resolution through the assumption that reality is subjective. From this perspective, reality hinges on contextual factors or assigned meanings. Notably, the notion of subjectivity in quantum theory has been deliberated by numerous scholars, such as [10, 11, 12, 13, 14, 15, 16], and has also been explored via arguments like Wigner's friend [17, 18].

If physical reality does indeed possess subjectivity, as suggested by quantum theory's axioms and the concept of entanglement - implying a reliance on the observer - then how does this mechanism function? How can we even define the consciousness of an observer, not to mention its relationship with physical reality? This paper endeavors to address these inquiries, grounded in the existing axioms of quantum theory.

2 Axioms of Quantum Theory

In this section, let us examine the axioms of quantum theory, particularly using the language of quantum information. Why might computer notation be useful in grasping the true nature of quantum theory? Firstly, it significantly simplifies the description of a physical system using only 0's and 1's. In other words, without resorting to terms such as electrons, photons, etc., this notation represents complex entities through simple quantum bits, or qubits.

Furthermore, the notation of quantum information directly addresses the subjectivity inherent in quantum theory. For instance, when a student initially embarks on learning about quantum mechanics, the theory is often presented in a historical sequence, commencing with Planck's work on black body radiation. This approach is pragmatically aligned with the 'shut up and calculate' philosophy. This methodology provides a practical means to engage with the immensely powerful quantum theory without delving into the intricate foundational matters surrounding it.

However, the advent of quantum technologies like quantum computers, cryptography, etc. (refer to [19] for a review) began to embrace the subjective nature of quantum theory and transformed the probability crisis into a remarkable opportunity. Notably, the vigorous research in quantum information subsequent to Bell's inequalities and the development of potent quantum algorithms contributed to directly confronting and embracing the subjective aspect of quantum theory. This is noteworthy considering that the conventional approach in quantum mechanics has frequently aimed to downplay or circumvent this subjective dimension of the theory at its core.

The principles of quantum theory, when conveyed through the language of quantum computation notation, can be succinctly encapsulated as follows:

1. States: The mathematical portrayal of an object intended for observation takes the form of a state vector defined within complex space. It is postulated that this state encompasses a comprehensive and exhaustive representation of an observable physical system.[1] In the notation of a qubit, this state can be expressed as follows:

$$|\psi\rangle = e^{\frac{-i\nu}{2}} \cos\frac{\mu}{2}|0\rangle + e^{\frac{i\nu}{2}} \sin\frac{\mu}{2}|1\rangle \tag{1}$$

where $0 \leq \mu \leq \pi$ and $0 \leq \nu \leq 2\pi$. In fact, a qubit may be envisioned as a unit vector $\hat{\alpha}$ pointing in (μ, ν) in a Bloch sphere where $|\psi\rangle\langle\psi| = \frac{1}{2}(1 + \hat{\alpha} \cdot \vec{\sigma})$ and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ with

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2}$$

2. Observable: Within the framework of quantum theory, the observable pertains to the quantifiable attribute of the object, specifically the state vector mentioned in axiom 1. Notably, when employing qubit notation, it becomes more evident that this observable can be interpreted as the selection of a reference frame along the direction defined by $\hat{\beta} = (\beta_x, \beta_y, \beta_z) = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$ within the Bloch sphere associated with an observing entity:

$$\hat{\beta} \cdot \vec{\sigma} = \begin{pmatrix} \cos\vartheta & \sin\vartheta\cos\varphi - i\sin\vartheta\sin\varphi \\ \sin\vartheta\cos\varphi + i\sin\vartheta\sin\varphi & -\cos\vartheta \end{pmatrix} \tag{3}$$

In [21], it has been argued that observables, or the reference frame, can be considered a mathematical description of the observer's state. This notion is supported by the fact that observables are defined in a complex vector space.[2]

3. Measurement: Observation constitutes a fundamental element within the axioms of quantum theory. The measurement process establishes a connection between the object (i.e., the states in axiom 1) and the reference frame of the observing party (i.e., the observables in axiom 2). With the choice of the reference frame, the measurement outcome is provided probabilistically. The randomness inherent in the observation process raises important questions in the foundations of quantum mechanics. This randomness contrasts with the continuous evolution of quantum states, which will be discussed in the next axiom.

4. Dynamics: Time evolution is elucidated through the utilization of unitary operators denoted as U, satisfying the conditions $UU^\dagger = U^\dagger U = 1$. Within the Schrödinger picture, it is the state vector (or the object under observation) that undergoes evolution.

$$U|\psi\rangle \rightarrow |\psi'\rangle \tag{4}$$

whereas in the case of the Heisenberg picture, it is the observable, $\mathcal{O}$, that is evolving under unitary transformation as follows

$$\hat{\mathcal{O}} \rightarrow \hat{\mathcal{O}}' = U^\dagger \hat{\mathcal{O}} U \tag{5}$$

[1] It's important to acknowledge that the term 'observable system' is employed, rather than referring directly to the physical system itself.

[2] It's worth noting that in [11], quantum states are assumed to represent the knowledge of an observer, rather than a description of the physical system. In our approach, observables, which refer to the reference frame when observing the object, are considered to be the mental reference frame of the observer.

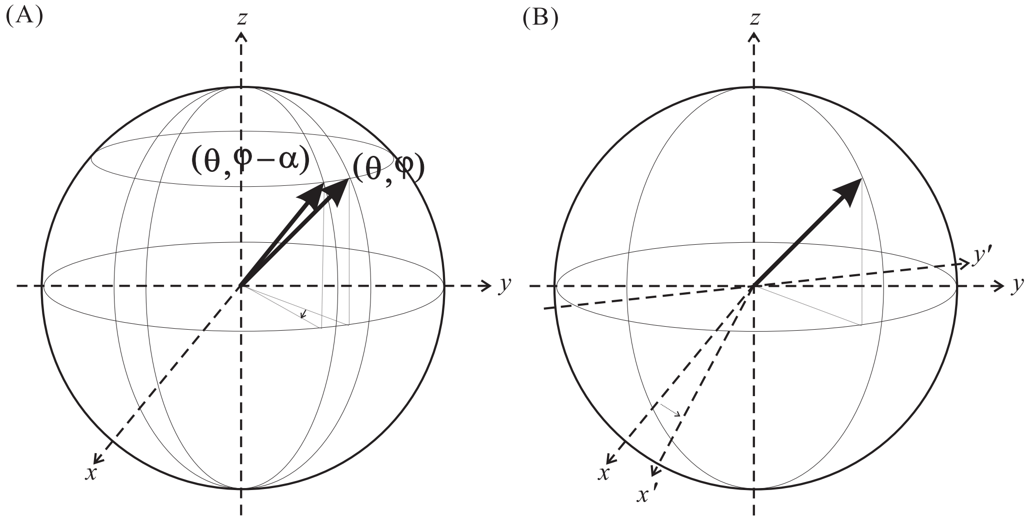

Figure 1: Quantum theory's dynamics give rise to the Schrödinger (A) and Heisenberg (B) pictures. In the former (A), the evolution pertains to the object, whereas in the latter (B), it involves the observer's reference frame evolving unitarily.

In this framework, the dynamics of quantum evolutions may be understood as follows: the Schrödinger picture corresponds to the case where the object is evolving, while the Heisenberg picture posits that it is the observer's mental state that is evolving. In Figure 1, (A) depicts the Schrödinger picture where the object is evolving, while (B) illustrates the evolution of the observer's mental reference frame. Certainly, both pictures would provide the same measurement outcome; therefore, they may be considered equivalent.

In [21], the argument was made that when self-observation occurs, wherein the observer's reference frame is aligned with the object being observed, the Heisenberg picture, featuring the evolution of the observer's reference frame backward in time, offers a more accurate description of quantum dynamics than the Schrödinger picture.

3 Vacuum

With a distinct demarcation between the microscopic realm and the macroscopic matter - referred to as the Heisenberg cut - let's delve into the potential realization of the observer's reference frame evolving backward in time, as proposed in [21], through the von Neumann measurement protocol [22] (also see [23] for a review). Within this framework, we consider a quantum system designated for measurement alongside an apparatus, which serves as a classical system. In this context, it's important to highlight that the quantum realm aligns with the observer's reference frame, while the classical domain encompasses the physical world that the observer directly experiences.

The von Neumann measurement model comprises an initial quantum system, denoted as Q, to be subjected to measurement, alongside a classical apparatus represented as C. This configuration is initially described as follows:

$$|\psi_0\rangle_Q \otimes |0\rangle_C \tag{6}$$

The next step involves applying the chosen measurement basis to the quantum system using the Hamiltonian:

$$\hat{H} = -\hat{\mathcal{O}} \cdot \hat{P} \tag{7}$$

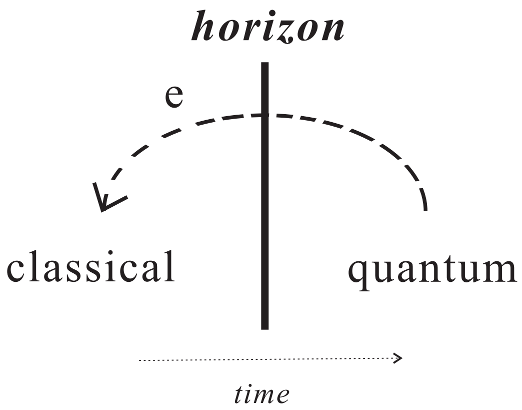

Figure 2: Using the Cerf and Adami notation [24], the antiparticle in an entangled state can be conceptualized as a qubit moving in reverse through time. When this notion is extended to the context of entanglement within the von Neumann measurement protocol, it's the selection of observables embedded within the quantum system, denoted as Q, that voyages backward through time, traversing the horizon or the Heisenberg cut.

Here, $\hat{\mathcal{O}}$ stands for the observable, and $\hat{P}$ is the momentum operator of the apparatus. When adopting the reference frame parameters ϑ and φ in qubit notation, the observable is expressed as:

$$\hat{\mathcal{O}} \equiv (+1)|\psi(\vartheta,\varphi)\rangle\langle\psi(\vartheta,\varphi)| + (-1)|\psi(\vartheta,\varphi)^{\perp}\rangle\langle\psi(\vartheta,\varphi)^{\perp}| \tag{8}$$

The eigenvectors for the selected measurement basis are given by:

$$|\psi(\vartheta,\varphi)\rangle = \frac{1}{M}\begin{pmatrix} (\csc\vartheta + \cot\vartheta)(\cos\varphi - i\sin\varphi) \\ 1 \end{pmatrix} \tag{9}$$

$$|\psi(\vartheta,\varphi)^{\perp}\rangle = \frac{1}{N}\begin{pmatrix} (\csc\vartheta - \cot\vartheta)(\cos\varphi - i\sin\varphi) \\ -1 \end{pmatrix} \tag{10}$$

where M and N are defined as:

$$M = \sqrt{1 + \left|\cot\frac{\vartheta}{2}(-\cos\varphi + i\sin\varphi)\right|^2} \tag{11}$$

$$N = \sqrt{1 + \left|\tan\frac{\vartheta}{2}(-\cos\varphi + i\sin\varphi)\right|^2} \tag{12}$$

By applying a unitary transformation to Equation (6), an interaction between the quantum system and the classical apparatus is generated, resulting in the following expression:

$$\begin{aligned} U|\psi_0\rangle_Q \otimes |0\rangle_C \;\; &\rightarrow \;\; \langle\psi(\vartheta,\varphi)|\psi_0\rangle|\psi(\vartheta,\varphi)\rangle_Q|0\rangle_C \\ &+ \;\; \langle\psi(\vartheta,\varphi)^{\perp}|\psi_0\rangle|\psi(\vartheta,\varphi)^{\perp}\rangle_Q|1\rangle_C \end{aligned} \tag{13}$$

In this expression, the choice of the observable is encoded in the quantum system Q, and it leads to a classically distinguishable outcome in the apparatus C. Essentially, the observer's mental reference frame, represented by ϑ and φ, becomes encoded within the quantum system Q.

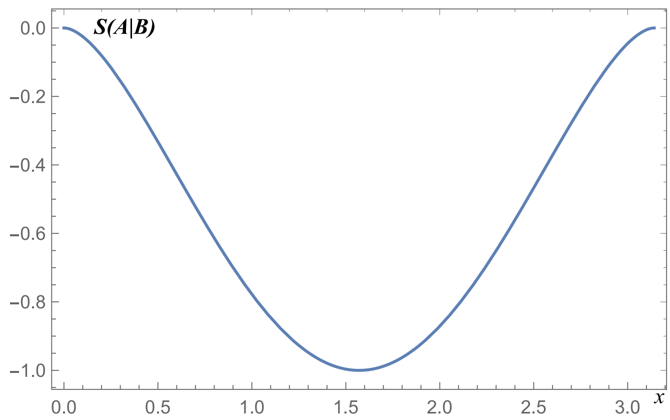

Figure 3: The negativity of the conditional entropy $S(A|B)$ is shown as a function of x where $0 \leq x \leq \pi$.

In the realm of information theory, diverse methods exist for quantifying the relationship between two entities. Specifically, conditional entropy gauges the requisite information for delineating system A 'in relation to the knowledge about system B.' The magnitude of this value hinges on the degree of correlation between entities A and B. In a publication by Cerf et al. [24], the conditional entropy for entangled quantum systems is elucidated through the equation:

$$S(A|B) = S(AB) - S(B) \tag{14}$$

An intriguing revelation is that $S(A|B)$ within equation (14) can assume a negative value, an occurrence absent in classically correlated systems. For instance, for correlated states such as $\cos\frac{x}{2}|00\rangle_{AB} + \sin\frac{x}{2}|11\rangle_{AB}$, Figure 3 shows the negativity of the conditional entropy $S(A|B)$ as a function of x. This anomalous negative entropy has been construed as representing an anti-qubit that harbors negative or virtual information. Drawing a parallel, we can apply this analogy to our discourse on the quantum measurement process in equation (13), wherein Q can be regarded as an anti-system. Conforming to the rationale of [24], this anti-system within equation (13) might be conceptualized as a system journeying backward in time beyond the Heisenberg cut, akin to the horizon.

In our conceptual framework, the quantum system--that is, the reference frame delineated by (ϑ, φ) of the observer--evolves in a retrograde fashion, permeating the physical vacuum in a manner reminiscent of the Dirac-type negative sea. Consequently, the state described by (ϑ, φ) operates as a context for observing the physical outcomes of the eigenvalues ± 1. A comparable line of reasoning has been explored in the context of black holes, as discussed in [25].

4 Conclusion

This paper outlines a model of physical reality that encompasses a universe imbued with an observer, expressed using quantum information notation. A notable innovation of this approach lies in its provision of a straightforward method for describing the enigmatic concept of consciousness. Prior endeavors in scientifically grasping consciousness encountered challenges due to attempts to apprehend this inherently subjective experience in an objective manner, akin to other observable physical systems. Nevertheless, the present protocol takes a different stance by embracing the Copenhagen interpretation of quantum theory, devoid of additional assumptions or ambiguities.

The current predicament of discerning whether physical reality is objective or not holds parallels with the perplexing situation faced during the transition from geocentric to heliocentric models of our solar system in the 16th century. With a finite pool of observational data, scientists of that era had to choose between these competing theories. Interestingly, contemporary employment of neural networks by scientists, as demonstrated in [26], favored the heliocentric model when provided with data on the movements of the Sun and Mars.

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