Investigation Energy Levels and Energy Transitions for $^{250}_{98}Cf_{152}$ Isotope Using Interacting Boson Model

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Abstract

Different methods and programs are used to study the nuclear properties of the isotopes. In this study, the interacting boson model (IBM-1) was used on $^{250}_{98}\mathrm{Cf}_{152}$ isotope, where the model was used to account for energy ratios, energy levels, and energy transitions of the isotope nucleus. Californium (A=250) isotope belongs to transition region SU(3)-U(5), their properties lie between two dynamical symmetries but are closer to the rotational region than to the vibrational region. It was found that the energy levels are in good compatibility compared to experimental data, some energy values and some energy transitions have also been confirmed, in addition to predicting many of their values.

Keywords: Interacting boson model; Energy levels; Energy transitions; Dynamical symmetry; IBM-1.

1. INTRODUCTION

The interacting boson model (IBM) in all versions is very appropriate for describing the low-lying collective states in even-even nuclei isotopes [1] through a system of interacting s-bosons with angular momentum's 0 and d-bosons with angular momentum's 2 [2,3]. In IBM-1 no difference between proton bosons about neutron bosons [2]. The total number of bosons ($N = N_\pi + N_\nu$) is finite and conserved in a specific nucleus and is simply given by half the total valence nucleus number, where $N_\pi(N_\nu)$ both its proton (neutron) bosons [3,4], that get it from the difference between the number of protons (neutrons) for the isotope and the nearest of the magic number and correspond to pairs of particles or holes [1,5]. The s(L=0) and d(L=2) bosons of the IBM-1 define a six-dimensional space therefore that one can describe it in terms of the unitary group in six dimensions U(6), because have six sub-states [2,3]. This leads to the derivation of many of the unique characteristics of the isotopes by different theoretical methods and programs including IBM-1 program. Then, it is expressed analytically [1,2,6].

The unitary groups U(6) have 36 generators, which are written down explicitly, as the following [2,7]:

$$G_0^0(ss), G_0^0(dd), G_k^1(dd), G_k^2(dd), G_k^3(dd), G_k^4(dd), G_k^2(ds), G_k^2(sd)$$

The value of k relies on the above value G in the negative and positive terms.
The analysis of this unitary group will lead to the three chains of U(6), all of which end in O(2) which is [8,9]:

$\mathrm{U}(6) \supset \mathrm{SU}(5) \supset \mathrm{O}(5) \supset \mathrm{O}(3) \supset \mathrm{O}(2)$
$\mathrm{U}(6) \supset \mathrm{SU}(3) \supset \mathrm{O}(3) \supset \mathrm{O}(2)$
$\mathrm{U}(6) \supset \mathrm{O}(6) \supset \mathrm{O}(5) \supset \mathrm{O}(3) \supset \mathrm{O}(2)$

Californium isotopes have different dynamical symmetries according to the energy ratios and the number of bosons for each isotope, this study aims to investigate ${}^{250}_{98}Cf_{152}$ isotope by finding the dynamical symmetry, calculating the energy levels, and energy transitions using the Interacting Boson Model (IBM-1) and comparing them with experimental data.

2. PRINCIPLES AND METHODS OF CALCULATION

2.1. DYNAMICAL SYMMETRY

In the deformed nuclei, there are three kinds of excitations, that is vibration, rotation, and Gamma unstable [10], they represent the dynamical symmetries U(5), SU(3), and O(6), respectively [9]. In addition, there are transition regions in which some isotopes are located represented by U(5)-SU(3), SU(3)-O(6), O(6)-U(5) [3,9], and U(5)-SU(3)-O(6) in this class, where the nuclei have middle properties between previous three limits [11]. The isotope affiliation of any of these regions is classified according to the energy ratios in Table 1.

Table 1. Energy ratios of the corresponding dynamical symmetries [2,9,11]

Energy rations	**Dynamical symmetries**		
	U(5)	**SU(3)**	**O(6)**
$R_4 = E(4_1^+)/E(2_1^+)$	2	3.33	2.5
$R_6 = E(6_1^+)/E(2_1^+)$	3	7	4.5
$R_8 = E(8_1^+)/E(2_1^+)$	4	12	7

Some isotopes nuclei can be described exactly by one limited that previously mentionable (U(5), SU(3), and O(6)) that are pure nuclei, dependent on the energy ratios represented by Table 1, where several isotopes nuclei far out with properties about these limits, to lie between two limits properties or three limits, to transition regions [9], the three limits that show previous very useful, when that give some analytical solutions, that can be experimental examination, where found some nucleus spectra don't agreement with any these three limits mentionable, therefore, the Hamiltonian write by two limits operators and sometimes write by denotation all operators [12].
If the ratios of the previous energy levels are not met, the isotope belongs to one of the transition regions [6,9,13].

2.2. HAMILTONIAN OPERATOR OF IBM-1

Hamiltonian operator represents the energy equation according to the dynamical symmetry of the isotope under study. There are many forms to describe Hamiltonian operator, and each one from it contains several parameters that lie between 9,8, and 6 depending on the form of the used Hamiltonian equation and dynamical symmetry for the isotope [2].
In the IBM-1, the Hamiltonian operator can be expressed by [9,11]:

$$\hat{H} = \varepsilon\, \hat{n}_d + a_0(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \quad (1)$$

The parameters:
$\varepsilon = \varepsilon_d - \varepsilon_s \Rightarrow$ the energy of the boson.
$a_0, a_1, a_2, a_3, a_4 \Rightarrow$ the phenomenological parameters.
The operators:
$\hat{n}_d = (\hat{d}^{\dagger}.\hat{\tilde{d}}) \Rightarrow$ the bosons number.
$\hat{P} = \frac{1}{2}(\hat{\tilde{d}}.\hat{\tilde{d}}) - \frac{1}{2}(\hat{\tilde{S}}.\hat{\tilde{S}}) \Rightarrow$ the pairing bosons.
$\hat{L} = \sqrt{10}[\hat{d}^{\dagger} \times \hat{\tilde{d}}\,]^{(1)} \Rightarrow$ the angular momentum.
$\hat{Q} = \left[\left(\hat{d}^{\dagger} \times \hat{\tilde{S}}\right) + \left(\hat{S}^{\dagger} \times \hat{\tilde{d}}\right)\right]^{(2)} - \frac{1}{2}\sqrt{7}\left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(2)} \Rightarrow$ the quadrupole.
$\hat{T}_3 = \left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(3)} \Rightarrow$ the octupole.
$\hat{T}_4 = \left[\hat{d}^{\dagger} \times \hat{\tilde{d}}\right]^{(4)} \Rightarrow$ the hexadecapole.

3. RESULTS AND DISCUSSION

3.1. DYNAMICAL SYMMETRY

The isotope $^{250}_{98}Cf_{152}$ belongs to the transition region SU(3)-U(5), therefore the total Hamiltonian is written as [9]:

$$H = T_{\text{rot}} + T_{\text{vib}} + V \quad (2)$$

Then the Hamiltonian gets the form [2,9]:

$$\hat{H}^{SU(3)-U(5)} = \varepsilon\, \hat{n}_d + a_1\hat{L}^2 + a_2\hat{Q}^2 \quad (3)$$

Here ε, a_1, and a_2 are free parameters that depend on the number of proton bosons and neutron bosons N_π and N_ν, respectively.
The energy ratios R_4, R_6, and R_8, respectively, for isotope $^{250}_{98}Cf_{152}$ are given in Table 2, representing experimental values for the isotope and comparing them with the values resulting from IBM-1.

Table 2. The experimental values for energy ratios and comparison with the theoretical values resulting from IBM-1 of the $^{250}_{98}Cf_{152}$ isotope

Isotope	$^{250}_{98}Cf_{152}$	
Energy ratios	EXP. [14,15]	IBM-1
R_4	3.3210	3.3432
R_6	6.9338	6.9235
R_8	11.7038	11.9639

Since $E_{4_1^+}$ approaches the value $E_{4_1^+} \simeq \frac{10}{3} E_{2_1^+}$, the isotope $^{250}_{98}Cf_{152}$ in the region ε is small [16], therefore, the isotope is nearer to the region of symmetry SU(3) than to the symmetry U(5).

3.2. ENERGY LEVELS

The parameters of equation (3) were calculated from the experimental schemes of the nucleus of this isotope [14,15]. These parameters were tabulated in Table 3.

Table 3. The parameter values of the Hamiltonian operator for the $^{250}_{98}Cf_{152}$ isotope using (IBM-1. For) program.

Isotope	$^{250}_{98}Cf_{152}$	**Dynamical Symmetry**		**SU(3)-U(5)**
No. of bosons	**Parameters**		**Parameters**	
$N_\pi = 8$	EPS (Mev)	0.0013	$\hat{T}_3.\hat{T}_3$ (Mev)	0.0000
$N_\nu = 13$	$\hat{P}.\hat{P}$ (Mev)	0.0000	$\hat{T}_4.\hat{T}_4$ (Mev)	0.0000
$N = 21$	$\hat{L}.\hat{L}$ (Mev)	0.0021	CHI (Mev)	-0.0480
----------	$\hat{Q}.\hat{Q}$ (Mev)	-0.0145	$SO6$ (Mev)	0.0000

The Hamiltonian equation is equivalent to the dynamical symmetry of the $^{250}_{98}Cf_{152}$ isotope, which is used to calculate the energy ratios, energy levels, energy transitions, and energy band spectrum (g, β, and γ).

Table 4. Comparison between the experimental [14,15] and theoretical energy bands for the $^{250}_{98}Cf_{152}$ isotope using IBM-1.

Spin / Band	0^+ 2^+	2^+ 3^+	4^+ 4^+	6^+ 5^+	8^+ 6^+	10^+ 7^+	12^+ 8^+	14^+ 9^+
g- exp.	0.000000	0.042721	0.141875	0.296220	0.500000	0.762800	1.070200	----
g-pw	0.000000	0.042390	0.141720	0.293490	0.507150	0.767160	1.072800	1.423780
γ_1-exp.	1.031852	1.071370	(1.123000)	----	----	----	----	----
γ_1-pw	1.034360	1.073730	1.129360	1.344140	1.578730	1.688030	1.722960	1.755410
β_1-exp.	1.154240	1.189390						
β_1-pw	1.150000	1.190830						
γ_2-exp.	1.244500							
γ_2-pw	1.271000							
β_2-exp.	1.266600	1.296600	-------	-------	-------			
β_2-pw	1.275910	1.294620	1.322800	1.369890	1.431180			

γ_3-exp.	1.658000	(1.695150)						
γ_3-pw	1.694200	1.714000						

Table 4 illustrates the energy levels of $^{250}_{98}Cf_{152}$ isotope, and their comparison with available experimental values, the results show that there is good compatibility for the energy levels according to IBM-1 program used in the current study compared with the experimental data, where using the mentioned program some energy levels were confirmed at the spin and parity (4^+) for γ_1 band and (3^+) for γ_3 band, which was not exactly determined in experimental data to be (1.129360, 1.714000), respectively. Experimentally, there are many energy levels, but the spin or parity has been unknown, so it was confirmed by choosing the most suitable for the existing energy bands. Also, it predicted some energy levels of a spin (14^+) for the ground state band to be (1.423780), for spins $(5^+, 6^+, 7^+, 8^+, 9^+)$ for γ_1 band to be (1.344140, 1.578730, 1.688030, 1.722960, 1.755410), respectively, and for spins $(4^+, 6^+, 8^+)$ for β_2 band to be (1.322800, 1.369890, 1.431180) respectively.

Figure 1 represents the comparison between the experimental and theoretical energy levels for $^{250}_{98}Cf_{152}$ isotope, noticed that there is good compatibility of the level sequences for each band with the typical sequences of $(g-band)(0^+, 2^+, 4^+ \ldots.)$, $(\gamma-bands)(2^+, 3^+, 4^+ \ldots.)$ and $(\beta-bands)(0^+, 2^+, 4^+ \ldots.)$.

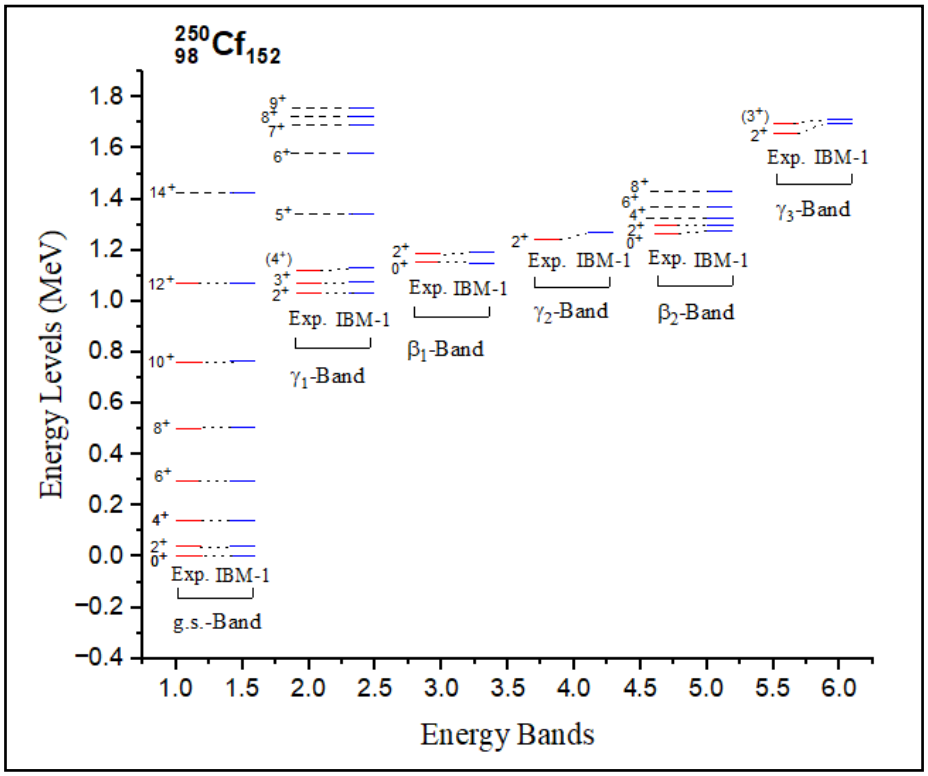

Figure 1. Comparison between experimental values [14,15] and calculated IBM-1 energy bands states for $^{250}_{98}Cf_{152}$ isotope

3.3. ENERGY TRANSITIONS

Table 5 elucidates the energy transitions were getting from IBM-1 program of $^{250}_{98}Cf_{152}$ isotope. There is good compatibility between available experimental data and theoretical results using an aforementioned program for the isotope under study. In addition to predicting energy transitions for multiple levels that are not mentioned in the experimental data.

Table 5. Comparison between theoretical energy transitions with the experimental data of $^{250}_{98}Cf_{152}$ isotope using IBM-1 program.

Spin sequences $I_f^+ - I_i^+$	Transition Energy (MeV)		Spin sequences $I_f^+ - I_i^+$	Transition Energy (MeV)	
	Exp.[14,15]	IBM-1		Exp.[14,15]	IBM-1
$0_1^+(g)$			$14_1^+(g)$		
----	----	----	$14_1^+ - 6_2^+$	----	0.05389
$2_1^+(g)$			$14_1^+ - 5_1^+$	----	0.07964
$2_1^+ - 0_1^+$	0.042721	0 .04239	$14_1^+ - 4_3^+$	----	0.10098
$4_1^+(g)$			$14_1^+ - 2_5^+$	----	0.12916
$4_1^+ - 2_1^+$	(0.09916)	0.09933	$14_1^+ - 0_3^+$	----	0.14787
$4_1^+ - 0_1^+$	----	0.14172	$14_1^+ - 2_4^+$	----	0.15278
$6_1^+(g)$			$14_1^+ - 2_3^+$	----	0.23295
$6_1^+ - 4_1^+$	----	0.15177	$14_1^+ - 0_2^+$	----	0.27378
$6_1^+ - 2_1^+$	----	0.25110	$14_1^+ - 4_2^+$	----	0.29442
$6_1^+ - 0_1^+$	----	0.29349	$14_1^+ - 3_1^+$	----	0.35005
$8_1^+(g)$			$14_1^+ - 12_1^+$	----	0.35098
$8_1^+ - 6_1^+$	----	0.21366	$14_1^+ - 2_2^+$	----	0.38942
$8_1^+ - 4_1^+$	----	0.36543	$14_1^+ - 10_1^+$	----	0.65662
$8_1^+ - 2_1^+$	----	0.46476	$14_1^+ - 8_1^+$	----	0.91663
$8_1^+ - 0_1^+$	----	0.50715	$14_1^+ - 6_1^+$	----	1.13029
$10_1^+(g)$			$14_1^+ - 4_1^+$	----	1.28206
$10_1^+ - 8_1^+$	----	0.26001	$14_1^+ - 2_1^+$	----	1.38139
$10_1^+ - 6_1^+$	----	0.47367	$14_1^+ - 0_1^+$	----	1.42378
$10_1^+ - 4_1^+$	----	0.62544	$8_2^+(\beta_2)$		
$10_1^+ - 2_1^+$	----	0.72477	$8_2^+ - 14_1^+$	----	0.00780
$10_1^+ - 0_1^+$	----	0.76716	$8_2^+ - 6_2^+$	----	0.06129
$2_2^+(\gamma_1)$			$8_2^+ - 5_1^+$	----	0.08704
$2_2^+ - 10_1^+$	----	0.26720	$8_2^+ - 4_3^+$	----	0.10838
$2_2^+ - 8_1^+$	----	0.52721	$8_2^+ - 2_5^+$	----	0.13656
$2_2^+ - 6_1^+$	----	0.74087	$8_2^+ - 0_3^+$	----	0.15527
$2_2^+ - 4_1^+$	0.8899	0.89264	$8_2^+ - 2_4^+$	----	0.16018
$2_2^+ - 2_1^+$	0.9891	0.99197	$8_2^+ - 2_3^+$	----	0.24035
$2_2^+ - 0_1^+$	1.0319	1.03436	$8_2^+ - 0_2^+$	----	0.28118
$12_1^+(g)$			$8_2^+ - 4_2^+$	----	0.30182
$12_1^+ - 2_2^+$	----	0.03844	$8_2^+ - 3_1^+$	----	0.35745
$12_1^+ - 10_1^+$	----	0.30564	$8_2^+ - 12_1^+$	----	0.35838
$12_1^+ - 8_1^+$	----	0.56565	$8_2^+ - 2_2^+$	----	0.39682
$12_1^+ - 6_1^+$	----	0.77931	$8_2^+ - 10_1^+$	----	0.66402
$12_1^+ - 4_1^+$	----	0.93108	$8_2^+ - 8_1^+$	----	0.92403
$12_1^+ - 2_1^+$	----	1.03041	$8_2^+ - 6_1^+$	----	1.13769
$12_1^+ - 0_1^+$	----	1.07280	$8_2^+ - 4_1^+$	----	1.28946
$3_1^+(\gamma_1)$			$8_2^+ - 2_1^+$	----	1.38879
$3_1^+ - 12_1^+$	----	0.00093	$8_2^+ - 0_1^+$	----	1.43118
$3_1^+ - 2_2^+$	----	0.03937	$6_3^+(\gamma_1)$		
$3_1^+ - 10_1^+$	----	0.30657	$6_3^+ - 8_2^+$	----	0.14755
$3_1^+ - 8_1^+$	----	0.56658	$6_3^+ - 14_1^+$	----	0.15495
$3_1^+ - 6_1^+$	----	0.78024	$6_3^+ - 6_2^+$	----	0.20884
$3_1^+ - 4_1^+$	0.9294	0.93201	$6_3^+ - 5_1^+$	----	0.23459

$3_1^+ - 2_1^+$	1.0285	1.03134	$6_3^+ - 4_3^+$	----	0.25593
$3_1^+ - 0_1^+$	----	1.07373	$6_3^+ - 2_5^+$	----	0.28411
$4_2^+(\gamma_1)$			$6_3^+ - 0_3^+$	----	0.30282
$4_2^+ - 3_1^+$	----	0.05563	$6_3^+ - 2_4^+$	----	0.30773
$4_2^+ - 12_1^+$	----	0.05656	$6_3^+ - 2_3^+$	----	0.38790
$4_2^+ - 2_2^+$	----	0.09500	$6_3^+ - 0_2^+$	----	0.42873
$4_2^+ - 10_1^+$	----	0.36220	$6_3^+ - 4_2^+$	----	0.44937
$4_2^+ - 8_1^+$	----	0.62221	$6_3^+ - 3_1^+$	----	0.50500
$4_2^+ - 6_1^+$	----	0.83587	$6_3^+ - 12_1^+$	----	0.50593
$4_2^+ - 4_1^+$	----	0.98764	$6_3^+ - 2_2^+$	----	0.54437
$4_2^+ - 2_1^+$	----	1.08697	$6_3^+ - 10_1^+$	----	0.81157
$4_2^+ - 0_1^+$	----	1.12936	$6_3^+ - 8_1^+$	----	1.07158
$0_2^+(\beta_1)$			$6_3^+ - 6_1^+$	----	1.28524
$0_2^+ - 4_2^+$	----	0.02064	$6_3^+ - 4_1^+$	----	1.43701
$0_2^+ - 3_1^+$	----	0.07627	$6_3^+ - 2_1^+$	----	1.53634
$0_2^+ - 12_1^+$	----	0.07720	$6_3^+ - 0_1^+$	----	1.57873
$0_2^+ - 2_2^+$	----	0.11564	$7_1^+(\gamma_1)$		
$0_2^+ - 10_1^+$	----	0.38284	$7_1^+ - 6_3^+$	----	0.10930
$0_2^+ - 8_1^+$	----	0.64285	$7_1^+ - 8_2^+$	----	0.25685
$0_2^+ - 6_1^+$	----	0.85651	$7_1^+ - 14_1^+$	----	0.26425
$0_2^+ - 4_1^+$	----	1.00828	$7_1^+ - 6_2^+$	----	0.31814
$0_2^+ - 2_1^+$	1.1115	1.10761	$7_1^+ - 5_1^+$	----	0.34389
$0_2^+ - 0_1^+$	1.1543	1.15000	$7_1^+ - 4_3^+$	----	0.36523
$2_3^+(\beta_1)$			$7_1^+ - 2_5^+$	----	0.39341
$2_3^+ - 0_2^+$	----	0.04083	$7_1^+ - 0_3^+$	----	0.41212
$2_3^+ - 4_2^+$	----	0.06147	$7_1^+ - 2_4^+$	----	0.41703
$2_3^+ - 3_1^+$	----	0.11710	$7_1^+ - 2_3^+$	----	0.49720
$2_3^+ - 12_1^+$	----	0.11803	$7_1^+ - 0_2^+$	----	0.53803
$2_3^+ - 2_2^+$	----	0.15647	$7_1^+ - 4_2^+$	----	0.55867
$2_3^+ - 10_1^+$	----	0.42367	$7_1^+ - 3_1^+$	----	0.61430
$2_3^+ - 8_1^+$	----	0.68368	$7_1^+ - 12_1^+$	----	0.61523
$2_3^+ - 6_1^+$	----	0.89734	$7_1^+ - 2_2^+$	----	0.65367
$2_3^+ - 4_1^+$	1.0478	1.04911	$7_1^+ - 10_1^+$	----	0.92087
$2_3^+ - 2_1^+$	1.1467	1.14844	$7_1^+ - 8_1^+$	----	1.18088
$2_3^+ - 0_1^+$	----	1.19083	$7_1^+ - 6_1^+$	----	1.39454
$2_4^+(\gamma_2)$			$7_1^+ - 4_1^+$	----	1.54631
$2_4^+ - 2_3^+$	----	0.08017	$7_1^+ - 2_1^+$	----	1.64564
$2_4^+ - 0_2^+$	----	0.12100	$7_1^+ - 0_1^+$	----	1.68803
$2_4^+ - 4_2^+$	----	0.14164	$2_6^+(\gamma_3)$		
$2_4^+ - 3_1^+$	----	0.19727	$2_6^+ - 7_1^+$	----	0.00617
$2_4^+ - 12_1^+$	----	0.19820	$2_6^+ - 6_3^+$	----	0.11547
$2_4^+ - 2_2^+$	----	0.23664	$2_6^+ - 8_2^+$	----	0.26302
$2_4^+ - 10_1^+$	----	0.50384	$2_6^+ - 14_1^+$	----	0.27042
$2_4^+ - 8_1^+$	----	0.76385	$2_6^+ - 6_2^+$	----	0.32431
$2_4^+ - 6_1^+$	----	0.97751	$2_6^+ - 5_1^+$	----	0.35006
$2_4^+ - 4_1^+$	1.1030	1.12928	$2_6^+ - 4_3^+$	----	0.37140
$2_4^+ - 2_1^+$	1.2017	1.22861	$2_6^+ - 2_5^+$	----	0.39958
$2_4^+ - 0_1^+$	1.2444	1.27100	$2_6^+ - 0_3^+$	----	0.41829
$0_3^+(\beta_2)$			$2_6^+ - 2_4^+$	----	0.42320
$0_3^+ - 2_4^+$	----	0.00491	$2_6^+ - 2_3^+$	----	0.50337
$0_3^+ - 2_3^+$	----	0.08508	$2_6^+ - 0_2^+$	----	0.54420
$0_3^+ - 0_2^+$	----	0.12591	$2_6^+ - 4_2^+$	----	0.56484
$0_3^+ - 4_2^+$	----	0.14655	$2_6^+ - 3_1^+$	0.5866	0.62047
$0_3^+ - 3_1^+$	----	0.20218	$2_6^+ - 12_1^+$	----	0.62140
$0_3^+ - 12_1^+$	----	0.23110	$2_6^+ - 2_2^+$	0.6261	0.65984
$0_3^+ - 2_2^+$	----	0.24155	$2_6^+ - 10_1^+$	----	0.92704
$0_3^+ - 10_1^+$	----	0.50875	$2_6^+ - 8_1^+$	----	1.18705

$0_3^+ - 8_1^+$	----	0.76876	$2_6^+ - 6_1^+$	----	1.40071
$0_3^+ - 6_1^+$	----	0.98242	$2_6^+ - 4_1^+$	(1.51622)	1.55248
$0_3^+ - 4_1^+$	----	1.13419	$2_6^+ - 2_1^+$	1.6153	1.65181
$0_3^+ - 2_1^+$	1.2238	1.23352	$2_6^+ - 0_1^+$	1.6581	1.69420
$0_3^+ - 0_1^+$	1.2666	1.27591	$3_2^+(\gamma_3)$		
$2_5^+(\beta_2)$			$3_2^+ - 2_6^+$	----	0.01980
$2_5^+ - 0_3^+$	----	0.01871	$3_2^+ - 7_1^+$	----	0.02597
$2_5^+ - 2_4^+$	----	0.02362	$3_2^+ - 6_3^+$	----	0.13527
$2_5^+ - 2_3^+$	----	0.10379	$3_2^+ - 8_2^+$	----	0.28282
$2_5^+ - 0_2^+$	----	0.14462	$3_2^+ - 14_1^+$	----	0.29022
$2_5^+ - 4_2^+$	----	0.16526	$3_2^+ - 6_2^+$	----	0.34411
$2_5^+ - 3_1^+$	----	0.22089	$3_2^+ - 5_1^+$	----	0.36986
$2_5^+ - 12_1^+$	----	0.22182	$3_2^+ - 4_3^+$	----	0.39120
$2_5^+ - 2_2^+$	----	0.26026	$3_2^+ - 2_5^+$	----	0.41938
$2_5^+ - 10_1^+$	----	0.52746	$3_2^+ - 0_3^+$	----	0.43809
$2_5^+ - 8_1^+$	----	0.78747	$3_2^+ - 2_4^+$	----	0.44300
$2_5^+ - 6_1^+$	----	1.00113	$3_2^+ - 2_3^+$	----	0.52317
$2_5^+ - 4_1^+$	1.1549	1.15290	$3_2^+ - 0_2^+$	----	0.56400
$2_5^+ - 2_1^+$	1.2540	1.25223	$3_2^+ - 4_2^+$	----	0.58464
$2_5^+ - 0_1^+$	(1.29654)	1.29462	$3_2^+ - 3_1^+$	----	0.64027
$4_3^+(\beta_2)$			$3_2^+ - 12_1^+$	----	0.64120
$4_3^+ - 2_5^+$	----	0.02818	$3_2^+ - 2_2^+$	----	0.67964
$4_3^+ - 0_3^+$	----	0.04689	$3_2^+ - 10_1^+$	----	0.94684
$4_3^+ - 2_4^+$	----	0.05180	$3_2^+ - 8_1^+$	----	1.20685
$4_3^+ - 2_3^+$	----	0.13197	$3_2^+ - 6_1^+$	----	1.42051
$4_3^+ - 0_2^+$	----	0.17280	$3_2^+ - 4_1^+$	----	1.57228
$4_3^+ - 4_2^+$	----	0.19344	$3_2^+ - 2_1^+$	----	1.67161
$4_3^+ - 3_1^+$	----	0.24907	$3_2^+ - 0_1^+$	----	1.71400
$4_3^+ - 12_1^+$	----	0.25000	$8_3^+(\gamma_1)$		
$4_3^+ - 2_2^+$	----	0.28844	$8_3^+ - 3_2^+$	----	0.00896
$4_3^+ - 10_1^+$	----	0.55564	$8_3^+ - 2_6^+$	----	0.02876
$4_3^+ - 8_1^+$	----	0.81565	$8_3^+ - 7_1^+$	----	0.03493
$4_3^+ - 6_1^+$	----	1.02931	$8_3^+ - 6_3^+$	----	0.14423
$4_3^+ - 4_1^+$	----	1.18108	$8_3^+ - 8_2^+$	----	0.29178
$4_3^+ - 2_1^+$	----	1.28041	$8_3^+ - 14_1^+$	----	0.29918
$4_3^+ - 0_1^+$	----	1.32280	$8_3^+ - 6_2^+$	----	0.35307
$5_1^+(\gamma_1)$			$8_3^+ - 5_1^+$	----	0.37882
$5_1^+ - 4_3^+$	----	0.02134	$8_3^+ - 4_3^+$	----	0.40016
$5_1^+ - 2_5^+$	----	0.04952	$8_3^+ - 2_5^+$	----	0.42834
$5_1^+ - 0_3^+$	----	0.06823	$8_3^+ - 0_3^+$	----	0.44705
$5_1^+ - 2_4^+$	----	0.07314	$8_3^+ - 2_4^+$	----	0.45196
$5_1^+ - 2_3^+$	----	0.15331	$8_3^+ - 2_3^+$	----	0.53213
$5_1^+ - 0_2^+$	----	0.19414	$8_3^+ - 0_2^+$	----	0.57296
$5_1^+ - 4_2^+$	----	0.21478	$8_3^+ - 4_2^+$	----	0.59360
$5_1^+ - 3_1^+$	----	0.27041	$8_3^+ - 3_1^+$	----	0.64923
$5_1^+ - 12_1^+$	----	0.27134	$8_3^+ - 12_1^+$	----	0.65016
$5_1^+ - 2_2^+$	----	0.30978	$8_3^+ - 2_2^+$	----	0.68860
$5_1^+ - 10_1^+$	----	0.57698	$8_3^+ - 10_1^+$	----	0.95580
$5_1^+ - 8_1^+$	----	0.83699	$8_3^+ - 8_1^+$	----	1.21581
$5_1^+ - 6_1^+$	----	1.05065	$8_3^+ - 6_1^+$	----	1.42947
$5_1^+ - 4_1^+$	----	1.20242	$8_3^+ - 4_1^+$	----	1.58124
$5_1^+ - 2_1^+$	----	1.30175	$8_3^+ - 2_1^+$	----	1.68057
$5_1^+ - 0_1^+$	----	1.34414	$8_3^+ - 0_1^+$	----	1.72296
$6_2^+(\beta_2)$			$9_1^+(\gamma_1)$		
$6_2^+ - 5_1^+$	----	0.02575	$9_1^+ - 8_3^+$	----	0.03245
$6_2^+ - 4_3^+$	----	0.04709	$9_1^+ - 3_2^+$	----	0.04141
$6_2^+ - 2_5^+$	----	0.07527	$9_1^+ - 2_6^+$	----	0.06121

$6_2^+ - 0_3^+$	----	0.09398	$9_1^+ - 7_1^+$	----	0.06738
$6_2^+ - 2_4^+$	----	0.09889	$9_1^+ - 6_3^+$	----	0.17668
$6_2^+ - 2_3^+$	----	0.17906	$9_1^+ - 8_2^+$	----	0.32423
$6_2^+ - 0_2^+$	----	0.21989	$9_1^+ - 14_1^+$	----	0.33163
$6_2^+ - 4_2^+$	----	0.24053	$9_1^+ - 6_2^+$	----	0.38552
$6_2^+ - 3_1^+$	----	0.29616	$9_1^+ - 5_1^+$	----	0.41127
$6_2^+ - 12_1^+$	----	0.29709	$9_1^+ - 4_3^+$	----	0.43261
$6_2^+ - 2_2^+$	----	0.33553	$9_1^+ - 2_5^+$	----	0.46079
$6_2^+ - 10_1^+$	----	0.60273	$9_1^+ - 0_3^+$	----	0.47950
$6_2^+ - 8_1^+$	----	0.86274	$9_1^+ - 2_4^+$	----	0.48441
$6_2^+ - 6_1^+$	----	1.07640	$9_1^+ - 2_3^+$	----	0.56458
$6_2^+ - 4_1^+$	----	1.22817	$9_1^+ - 0_2^+$	----	0.60541
$6_2^+ - 2_1^+$	----	1.32750	$9_1^+ - 4_2^+$	----	0.62605
$6_2^+ - 0_1^+$	----	1.36989	$9_1^+ - 3_1^+$	----	0.68168
			$9_1^+ - 12_1^+$	----	0.68261
			$9_1^+ - 2_2^+$	----	0.72105
			$9_1^+ - 10_1^+$	----	0.98825
			$9_1^+ - 8_1^+$	----	1.24826
			$9_1^+ - 6_1^+$	----	1.46192
			$9_1^+ - 4_1^+$	----	1.61369
			$9_1^+ - 2_1^+$	----	1.71302
			$9_1^+ - 0_1^+$	----	1.75541

Figures 2-7 represent decay schematics, which are expressed as theoretically calculated energy transitions of $^{250}_{98}Cf_{152}$ isotope, for $(g, \gamma_1, \beta_1, \gamma_2, \beta_2, \gamma_3 - bands)$, respectively, where energy transitions were predicted for all levels and gradually according to their appearance in the beams and were very suitable for the given energy values.

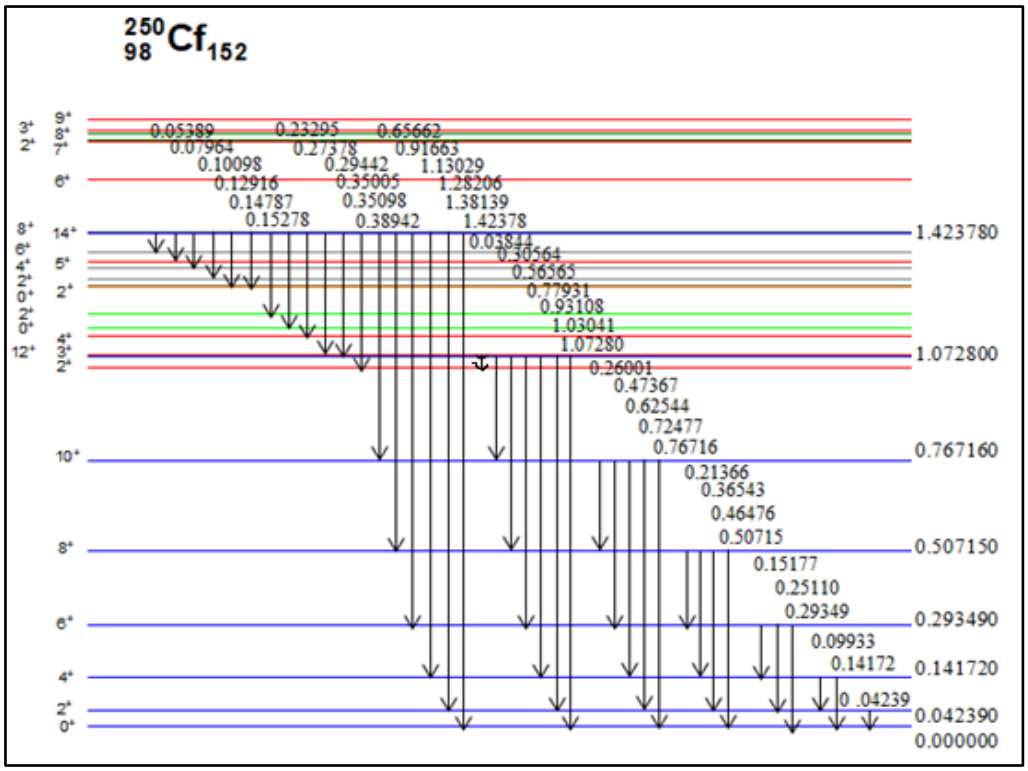

Figure 2. Decay scheme for ground state band of $^{250}_{98}Cf_{152}$ isotope using IBM-1

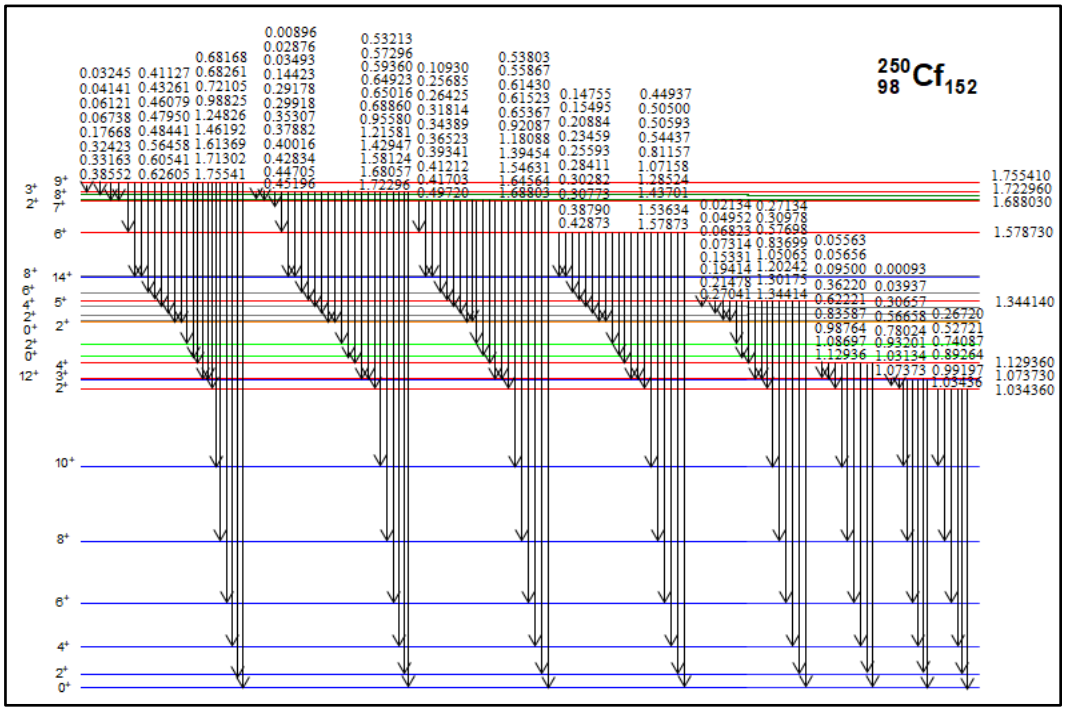

Figure 3. Decay scheme for γ_1 band of $^{250}_{98}Cf_{152}$ isotope using IBM-1

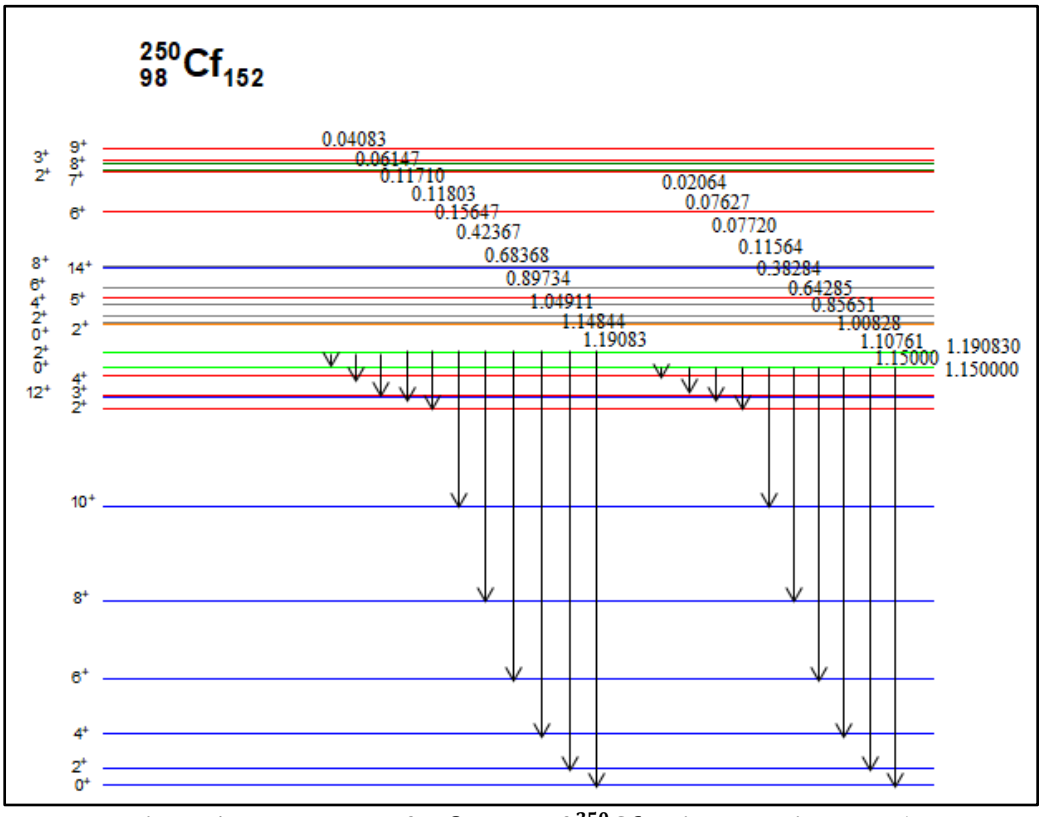

Figure 4. Decay scheme for β_1 band of $^{250}_{98}Cf_{152}$ isotope using IBM-1

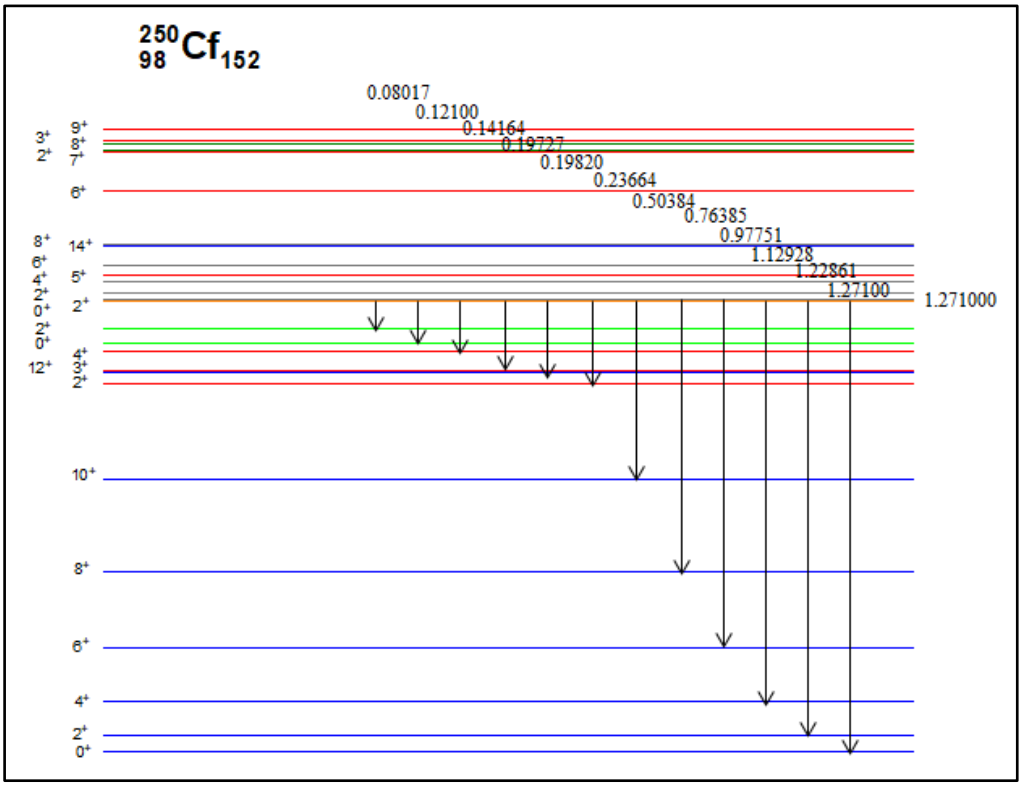

Figure 5. Decay scheme for γ_2 band of $^{250}_{98}Cf_{152}$ isotope using IBM-1

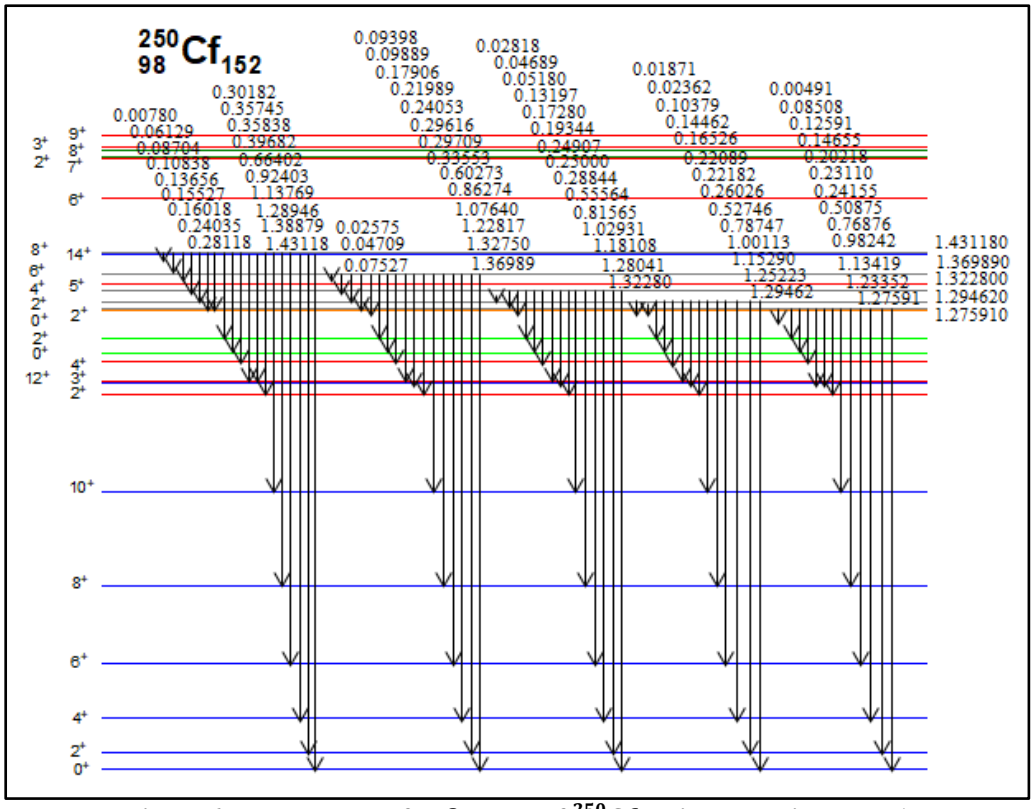

Figure 6. Decay scheme for β_2 band of $^{250}_{98}Cf_{152}$ isotope using IBM-1

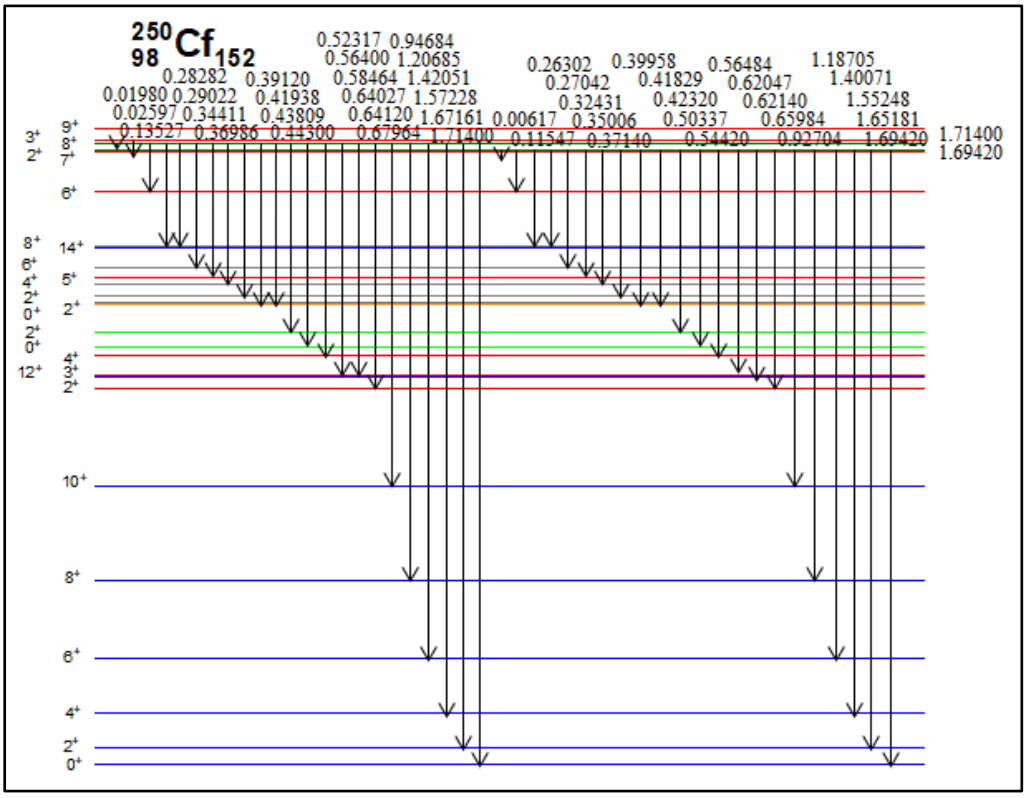

Figure 7. Decay scheme for γ_3 band of $^{250}_{98}Cf_{152}$ isotope using IBM-1

4. CONCLUSIONS

The nucleus of the californium isotope (A=250) belongs to the mixed dynamical symmetry SU(3)-U(5), it is located in the transition region between rotational and vibrational excitations, that is, one of the deformed nuclei, and therefore its Hamiltonian equation includes the variables of both symmetry. Through which, energy levels and their ratios were found, in addition to energy transitions, and some energy values were confirmed for γ_1, γ_3 bands of the spin and parity (4^+) to be (1.129360) and (3^+) to be (1.714000), respectively, and many energy values were predicted for some bands, and the energy values that were found were in good compatibility with the available experimental energy values. In addition, finding energy transitions for all levels through the IBM-1 program and the values were very suitable for the isotope under study.

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