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Mass-generation effects in a field theory comprising spins ranging from zero to one

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Abstract

The analysis of the consistent couplings that can be introduced between a massless real scalar field, a massless Majorana spinor, and a single massless Abelian 1-form is performed. This is done within the framework of the antifield-antibracket formalism by deformation of the solution to the classical master equation combined with specific techniques of BRST cohomology.

1 Introduction

Once the last piece of the Standard Model has shown its objective reality through the experimental evidence of the Higgs boson, the scientific community has known a renewed interest in mass-generation schemes in field theory, displayed by devising new methods [1, 2] that encompass the outputs of the standard Higgs mechanism [3, 4, 5, 6]. In all these procedures, the gauge fields acquire mass in the context of their interactions with some scalar matter fields. Whereas the standard Higgs mechanism [3, 4, 5, 6] requires a scalar field potential that displays a degenerate global minimum, the new methods [1, 2] do not assume such an ingredient from the outset, but only use the interaction as the sufficient condition for giving masses to gauge fields. When fermions are taken into account (e.g., spinor fields describing electrons in Weinberg-Salam electroweak theory), the Higgs mechanism requires, besides the scalar field potential, also a Yukawa-type potential necessary for the masses of spinors. At this point, the natural question arises: *Can new procedures [1, 2] be adapted by incorporating fermionic fields to produce a mass-generation mechanism that does not consider extra information concerning various potentials?* In this paper, by exploiting the idea [1], we show how a single Abelian 1-form gains mass in the context of its interactions with one scalar field and one Majorana spinor field.

In this paper, we analyse the consistent couplings that can be added to a massless free field theory comprising one real scalar field, one real spinor field, and one Abelian 1-form, by means of the deformation of the solution to the classical master equation [12, 13], supplemented with specific techniques of local BRST cohomology [14, 15, 16]. The procedure, supplemented with some reasonable hypotheses, standard in field theory, leads to some quadratic, derivative-free interaction vertices naturally interpreted as mass terms

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for various field spectrum components. Vis-a-vis this perspective, the obtained interacting models are parameterized by some real constants, which verify a set of consistency equations with a twofold 'dichotomy' in the behaviour of its solutions: i) the 1-form and the scalar field cannot be simultaneously massive, and ii) the 1-form and the spinor field are simultaneously massive unless there are cross-couplings between them.

The paper is organized into six sections as follows. In Section 2, employing the rules of antifield-antibracket BRST formalism, one derives the BRST symmetry corresponding to a massless free field theory, with field spectrum comprising one real scalar field, one Majorana spinor, and a single Abelian 1-form. Section 3 briefly points out how can be reformulated the problem of constructing consistent couplings mediated by gauge fields as a deformation problem for the solution to the classical master equation associated with a given (free) theory. In Section 4, one solves the equations that govern the deformation of the solution to the classical master equation corresponding to the considered free model. At this point, one proves that the most general interacting gauge theory, consistently constructed out of the considered starting model and subject to some standard hypotheses in field theory, depends on a degree four polynomial function in the scalar field and seven real constants, which are subject to some purely algebraic (consistency) equations. Section 5 exhibits the previously mentioned twofold 'dichotomy' by solving the consistency equations in terms of two complementary solutions. Section 6 ends the paper with the main conclusions.

2 Free model: the antifield-antibracket BRST symmetry

The starting free model evolves on a flat four-dimensional Minkowski spacetime of a mostly minus signature, $\mathbb{R}^{1|3}$, and consists of three non-interacting real massless fields: a scalar, a Majorana spinor, and a single Abelian 1-form. The Lagrangian dynamics is generated, via the variational principle, by the functional

$$S_0^{\mathrm{L}}[A,\varphi,\psi]=\int d^4x\Big[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+\tfrac{\mathrm{i}}{2}\bar{\psi}\partial\!\!\!/\psi\Big],\tag{1}$$

where $F_{\mu\nu}$ is the usual field-strength, $F_{\mu\nu}=\partial_{[\mu}A_{\nu]}$ and $\{\gamma^\mu\}$ are the generators of the Majorana representation of the Clifford algebra $\mathcal{C}(1,3)$. Remember that in this representation, all the γ-matrices are purely imaginary, γ^0 is Hermitian, and γ^j are anti-Hermitian. In the same context, the Dirac conjugation of the real (Majorana) spinors coincides with the charge conjugation,

$$\psi^{\mathcal{C}}\equiv(\mathcal{C}\psi)^\top=\bar{\psi}\equiv\psi^\dagger\gamma^0.$$

The action (1) is found to be gauge-invariant under the generating set of gauge transformations

$$\delta_\epsilon A^\mu=\partial^\mu\epsilon,\qquad\delta_\epsilon\varphi=0,\qquad\delta_\epsilon\psi=0,\tag{2}$$

which is manifestly irreducible and Abelian.

Implementing the general rules of the antifield-antibracket BRST formalism [7, 8, 9, 10, 11] to our situation, first, we introduce the BRST generators

$$\{A^\mu,\varphi,\psi\},\eta,\quad\{A^*_\mu,\varphi^*,\psi^*,\eta^*\},\tag{3}$$

whose degrees and Grassmann parities read

$$\mathrm{agh}(A^{\mu}) = \mathrm{agh}(\varphi) = 0, \qquad \mathrm{agh}(\psi) = 0, \qquad \mathrm{agh}(\eta) = 0, \tag{4}$$

$$\mathrm{agh}(A^{*}_{\mu}) = \mathrm{agh}(\varphi^{*}) = 1, \qquad \mathrm{agh}(\psi^{*}) = 1, \qquad \mathrm{agh}(\eta^{*}) = 2, \tag{5}$$

$$\mathrm{pgh}(A^{\mu}) = \mathrm{pgh}(\varphi) = 0, \qquad \mathrm{pgh}(\psi) = 0, \qquad \mathrm{pgh}(\eta) = 1, \tag{6}$$

$$\mathrm{pgh}(A^{*}_{\mu}) = \mathrm{pgh}(\varphi^{*}) = 0, \qquad \mathrm{pgh}(\psi^{*}) = 0, \qquad \mathrm{pgh}(\eta^{*}) = 0, \tag{7}$$

$$\varepsilon(A^{\mu}) = \varepsilon(\varphi) = 0, \qquad \varepsilon(\psi) = 1, \qquad \varepsilon(\eta) = 1, \tag{8}$$

$$\varepsilon(A^{*}_{\mu}) = \varepsilon(\varphi^{*}) = 0, \qquad \varepsilon(\psi^{*}) = 0, \qquad \varepsilon(\eta^{*}) = 0. \tag{9}$$

The BRST complex is also equipped with a natural involution according to which the fields are real and the antifields are purely imaginary, i.e.,

$$(A_{\mu})^{\star} = A_{\mu}, \qquad (\varphi)^{\star} = \varphi, \qquad (\psi)^{\star} = \psi, \qquad (\eta)^{\star} = \eta,$$
$$(A^{*}_{\mu})^{\star} = -A^{*}_{\mu}, \qquad (\varphi^{*})^{\star} = -\varphi^{*}, \qquad (\psi^{*})^{\star} = -\psi^{*}, \qquad (\eta^{*})^{\star} = -\eta^{*}.$$

Second, we construct the BRST differential associated with the theory (1)–(2). As the gauge generators from (2) are field-independent, it results that the BRST differential s simply reduces to

$$s = \delta + \gamma, \tag{10}$$

where δ signifies the Koszul–Tate differential, graded by the antighost number agh [$\mathrm{agh}(\delta) = -1$] and γ stands for the longitudinal exterior derivative [in this case a true differential], whose degree is named pure ghost number pgh [$\mathrm{pgh}(\gamma) = 1$]. These two degrees do not interfere [$\mathrm{agh}(\gamma) = 0$, $\mathrm{pgh}(\delta) = 0$]. The overall degree that grades the BRST algebra is known as the ghost number [gh] and is defined like the difference between the pure ghost number and the antifield number, such that $\mathrm{gh}(s) = \mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = 1$. The two differentials act on the BRST generators like

$$\delta A^{\mu} = 0, \qquad \delta\varphi = 0, \qquad \delta\psi = 0, \qquad \delta\eta = 0, \tag{11}$$

$$\delta A^{*}_{\mu} = \partial^{\nu} F_{\mu\nu}, \qquad \delta\varphi^{*} = \Box\varphi, \qquad \delta\psi^{*} = -\mathrm{i}\bar{\psi}\overleftarrow{\not{\partial}}, \qquad \delta\eta^{*} = -\partial^{\mu} A^{*}_{\mu}, \tag{12}$$

$$\gamma A^{\mu} = \partial^{\mu}\eta, \qquad \gamma\varphi = 0, \qquad \gamma\psi = 0, \qquad \gamma\eta = 0, \tag{13}$$

$$\gamma A^{*}_{\mu} = 0, \qquad \gamma\varphi^{*} = 0, \qquad \gamma\psi^{*} = 0, \qquad \gamma\eta^{*} = 0. \tag{14}$$

Third, we equip the BRST complex with a Gerstenhaber-like structure, the well-known antibracket, $(\cdot,\cdot)$, defined by decreeing the fields/ghosts conjugated with the corresponding antifields. Within this structure, the BRST differential s admits a canonical $s\cdot = (\cdot, S)$, with S the canonical generator. It is a real [i.e., invariant under the natural involution on the BRST algebra] bosonic functional of ghost number zero, involving both field/ghost and antifield spectra, that encodes the entire gauge structure of the associated theory and obeys the classical master equation

$$(S, S) = 0 \tag{15}$$

which is equivalent to the nilpotency of the BRST differential, $s^2 = 0$. For the model under study (1)–(2), the canonical generator of the BRST symmetry takes the simple form

$$S = S^{\mathrm{L}}_{0} + \int \mathrm{d}^4 x \left(A^{*}_{\mu} \partial^{\mu} \eta \right). \tag{16}$$

3 Consistent interactions: the BRST perspective

This section is dedicated to a brief review of the BRST approach to the problem of interacting vertices that can be added to a given gauge field theory so that the number of independent gauge symmetries is preserved. As the solution to the classical master equation completely captures the gauge structure of a given theory, it results that the problem can be reformulated as a deformation problem for the solution to the master equation corresponding to a given "free" theory [12, 13] in the framework of the local BRST cohomology [14, 15, 16]. This means that if an interacting theory can be consistently constructed, then the solution S to the master equation associated with the "free" theory can be deformed into a solution $\bar{S}$

$$S \rightarrow \bar{S} = S + gS_1 + g^2 S_2 + g^3 S_3 + g^4 S_4 + \cdots, \quad \varepsilon(\bar{S}) = 0, \quad \mathrm{gh}(\bar{S}) = 0 \tag{17}$$

of the master equation for the deformed theory that pertains to the original "free" BRST algebra, namely,

$$(\bar{S}, \bar{S}) = 0. \tag{18}$$

By projecting the equation (18) on various powers in the deformation parameter g, one obtains the equivalent tower of equations:

$$g^0 \ : \ (S, S) = 0, \tag{19}$$

$$g^1 \ : \ sS_1 = 0, \tag{20}$$

$$g^2 \ : \ \frac{1}{2}(S_1, S_1) + sS_2 = 0, \tag{21}$$

$$g^3 \ : \ (S_1, S_2) + sS_3 = 0, \tag{22}$$

$$g^4 \ : \ \frac{1}{2}(S_2, S_2) + (S_1, S_3) + sS_4 = 0, \tag{23}$$

$$\vdots$$

As S is nothing but the solution to the classical master equation corresponding to the "free" theory, it results that (19) is satisfied by construction. The remaining equations are to be solved recursively, from lower to higher orders, such that each equation corresponding to a given order of perturbation theory, say k ($k \geq 1$), contains a single unknown functional, namely, the deformation of order k, S_k. Once the deformation equations (20)–(23), etc., have been solved by means of specific cohomological techniques, from the consistent nontrivial deformed solution to the master equation (17) one can identify the entire gauge structure of the resulting interacting theory.

4 Consistent interactions between a collection of massless real fields of spins ranging from zero to one

In this section, we determine the consistent interactions that can be added to a massless "free" theory consisting of a real scalar field, a real (Majorana) spinor and a single Abelian 1-from. Precisely, we solve the tower of equations (19)–(23) with (16) as the initial term. Analysis is done within the framework delimited by specific hypotheses from field theory as analyticity in the coupling constant, Lorentz covariance, space-time locality, and Poincaré invariance. Moreover, the free parameters of strictly negative mass-dimension are not allowed. In this manner, there are avoided, at least, power-counting *non-renormalizable* [17] interacting field theories and also *higher-derivative* interaction vertices.

4.1 The deformed solution to the classical master equation

With the functional (16) as initial term at hand, we solve the tower of equations (20)–(23). As we are interested only in the local solutions, the first-order deformation S_1 can be expressed as

$$S_1 = \int \mathrm{d}^4x \, a, \tag{24}$$

where a is a local function. Inserting this realization into (20), one obtains the equivalent equation

$$sa = \partial_\mu j^\mu, \quad \mathrm{gh}(a) = 0, \quad \varepsilon(a) = 0, \tag{25}$$

with j^μ a local current. As we are looking only for true interacting vertices, i.e., those that do not come from some field redefinitions, we discard the solutions of the type $a = s\bar{a} + \partial_\mu \bar{j}^\mu$ as being trivial [14, 15, 16].

Using the structure of the considered field spectrum, it results that the firs-order deformation naturally decomposes into seven components

$$a = a^{(\varphi)} + a^{(\psi)} + a^{(A)} + a^{(\varphi-\psi)} + a^{(\varphi-A)} + a^{(A-\psi)} + a^{(\mathrm{int})}, \tag{26}$$

where $a^{(\varphi)}$, $a^{(\psi)}$, and $a^{(A)}$ govern the self-interactions of the scalar field φ, the spinor field ψ, and the vector field A^μ, respectively, $a^{(\varphi-\psi)}$, $a^{(\varphi-A)}$, and $a^{(A-\psi)}$ describe the cross-couplings scalar-spinor, scalar-vector, and vector-spinor, respectively, whereas a^{int} effectively mixes all the three sectors. The seven terms in decomposition (26) display different contents of BRST generators, such that equation (20) becomes equivalent to seven independent equations, one for each piece,

$$sa^{(\mathrm{sector})} = \partial_\mu j^\mu_{(\mathrm{sector})} \tag{27}$$

A simple analysis of the mass-dimension corresponding to field content reveals

$$[A] = M = [\varphi], \quad [\psi] = M^{3/2}, \tag{28}$$

which, further, exhibits the general real solutions to (27) that comply with our working hypotheses

$$a^{(\varphi)} = -\mathcal{V}(\varphi), \quad a^{(\psi)} = v\bar{\psi}\psi + \mathrm{i}\tilde{v}\bar{\psi}\gamma_5\psi, \quad a^{(A)} = \tfrac{\alpha}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}, \tag{29}$$

$$a^{(\varphi-\psi)} = (u\bar{\psi}\psi + \mathrm{i}\tilde{u}\bar{\psi}\gamma_5\psi)\varphi, \quad a^{(\varphi-A)} = n(\varphi^*\eta - A_\mu\partial^\mu\varphi), \tag{30}$$

$$a^{(A-\psi)} = \tilde{w}(\mathrm{i}\psi^*\gamma_5\psi\eta + \tfrac{1}{2}\bar{\psi}\gamma^\mu\gamma_5\psi A_\mu), \quad a^{(\mathrm{int})} = 0. \tag{31}$$

Previously, $\mathcal{V}$ is an arbitrary polynomial function in the scalar field, which is of degree at most equal to four, while v, α, u, $\tilde{u}$, n and $\tilde{w}$ are some real parameters.

As it has been synthesized in the previous section, the next step consists of finding the solution to (21). Direct computations based on (24) display

$$\begin{aligned}(S_1, S_1) = \int \mathrm{d}^4x \Big[& s(-n^2 A_\mu A^\mu) - n\frac{d\mathcal{V}}{d\varphi} + 4\tilde{w}(\mathrm{i}u\bar{\psi}\gamma_5\psi - \tilde{u}\bar{\psi}\psi)\varphi\eta \\ & +2(nu - 2\tilde{v}\tilde{w})\bar{\psi}\psi\eta + 2\mathrm{i}(n\tilde{u} + 2v\tilde{w})\bar{\psi}\gamma_5\psi\eta \Big],\end{aligned}$$

which further exhibit the second-order deformation

$$S_2 = \tfrac{1}{2}\int \mathrm{d}^4x (n^2 A_\mu A^\mu), \tag{32}$$

as well as the consistency equations

$$n\frac{d\mathcal{V}}{d\varphi}=0,\quad u\tilde{w}=0,\quad \tilde{u}\tilde{w}=0,\quad nu-2\tilde{v}\tilde{w}=0,\quad n\tilde{u}+2v\tilde{w}=0. \tag{33}$$

Inspecting the remaining equations, i.e., (22), (23), etc., direct computations based on the results (24) and (32) yield

$$(S_1,S_2)=0=(S_2,S_2),$$

which allow to conclude that higher-order deformations can be made trivial

$$S_k=0,\quad k\geq 2. \tag{34}$$

At this point we have determined the full deformed solution to the classical master equation

$$\bar{S}=S+gS_1+g^2S_2 \tag{35}$$

where the various order deformations S, S_1, and S_2 are given in (16), (24), and (32) respectively. This functional captures the whole Lagrangian gauge structure of an interacting field theories family parameterized by an arbitrary polynomial function of degree at most four in the undifferentiated scalar field and seven arbitrary real numbers subject to the consistency equations (33).

4.2 The Lagrangian gauge structure of the interacting field theories family

According to the general rules of the BRST formalism, the information about the gauge structure reads off by projecting (35) on various antighost numbers. Concretely, the antighost number zero component of (35)

$$\begin{aligned}S^{\mathrm{L}}\left[A,\varphi,\psi\right]=\int d^4x\,\big[&-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}(\partial_\mu\varphi-gnA_\mu)(\partial^\mu\varphi-gnA^\mu)+\tfrac{\mathrm{i}}{2}\bar{\psi}\not{D}\psi\\&-g\mathcal{V}+\tfrac{g\alpha}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}+g(v\bar{\psi}\psi+\mathrm{i}\tilde{v}\bar{\psi}\gamma_5\psi)+g(u\bar{\psi}\psi+\mathrm{i}\tilde{u}\bar{\psi}\gamma_5\psi)\varphi\big]\,,\end{aligned} \tag{36}$$

is nothing but the Lagrangian action of the interacting field theories family, while the antighost number one component of (35) displays

$$\bar{\delta}_\epsilon A^\mu=\partial^\mu\epsilon,\quad \bar{\delta}_\epsilon\varphi=gn\epsilon,\quad \bar{\delta}_\epsilon\psi=\mathrm{i}g\tilde{w}\gamma_5\psi\epsilon, \tag{37}$$

which is a generating set of gauge transformations for (36). As the functional (35) possesses a trivial antighost number two component, it results that the generating set of gauge transformation (37) is Abelian and irreducible. In (36) we employed the covariant derivatives of the spinor field

$$\not{D}\psi=\gamma^\mu(\partial_\mu-\mathrm{i}g\tilde{w}A_\mu)\psi.$$

5 Interacting models: Completion of the landscape

Now we are in the position to solve the consistency equations (33) and then to complete the Lagrangian gauge structure of the obtained interacting field models.

We solve the consistency conditions (33) starting with the first equation, which exhibits a dichotomy expressed by two main classes of solutions

$$n=0,\quad \mathcal{V}(\varphi)=\tfrac{\alpha_2}{2}\varphi^2+\tfrac{\alpha_3}{3!}\varphi^3+\tfrac{\alpha_4}{4!}\varphi^4,\quad \alpha_2,\alpha_3,\alpha_4\in\mathbb{R} \tag{38}$$

and

$$n\neq 0,\quad \mathcal{V}=0. \tag{39}$$

It should be noted that in (38) we could have added a linear term in the real scalar field, but this brings nothing new as it can always be removed by a shift redefinition of the scalar field.

Replacing (38) into the last four consistency conditions in (33), the following complementary solutions emerge:

$$\tilde{w}=0,\quad u,\tilde{u},v,\tilde{v}\in\mathbb{R}, \tag{40}$$

or

$$\tilde{w}\neq 0,\quad u=v=0=\tilde{u}=\tilde{v}. \tag{41}$$

According to the last three equations in (33), the second class of solutions (39) splits into two complementary types, namely

$$\tilde{w}=0=u=\tilde{u},\quad v,\tilde{v}\in\mathbb{R}, \tag{42}$$

or

$$\tilde{w}\neq 0,\quad u=v=0=\tilde{u}=\tilde{v}. \tag{43}$$

We conclude this section with the Lagrangian gauge structure of the obtained interacting field theories. The first class of models is associated with the solutions (38) and (40)/ (41). This displays one *massless* 1-form A_μ which does not interact with the possible massive (whenever $\alpha_2>0$) scalar field φ and also this is coupled with the spinor field φ if and only if the last remains massless. Concretely, the interacting models pertaining to the first class correspond to the complementary solutions (40)–(41) and are described by the Lagrangian actions

$$\begin{aligned} S^{\mathrm{Ia}}\left[A,\varphi,\psi\right]=\int d^4x\,\Big[&-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+\tfrac{\mathrm{i}}{2}\bar{\psi}\bar{\partial}\!\!\!/\psi-g\big(\tfrac{\alpha_2}{2}\varphi^2+\tfrac{\alpha_3}{3!}\varphi^3+\tfrac{\alpha_4}{4!}\varphi^4\big)\\ &+\tfrac{g\alpha}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}+g(v\bar{\psi}\psi+\mathrm{i}\tilde{v}\bar{\psi}\gamma_5\psi)+g(u\bar{\psi}\psi+\mathrm{i}\tilde{u}\bar{\psi}\gamma_5\psi)\varphi\Big]\,, \end{aligned} \tag{44}$$

and

$$\begin{aligned} S^{\mathrm{Ib}}\left[A,\varphi,\psi\right]=\int d^4x\,\Big[&-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+\tfrac{\mathrm{i}}{2}\bar{\psi}D\!\!\!\!/\,\psi\\ &+\tfrac{g\alpha}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}-g\big(\tfrac{\alpha_2}{2}\varphi^2+\tfrac{\alpha_3}{3!}\varphi^3+\tfrac{\alpha_4}{4!}\varphi^4\big)\Big]\,. \end{aligned} \tag{45}$$

Performing the same replacements in generating set of gauge transformations (37) show that the Lagrangian action (44) remains invariant under the original generating set of gauge transformations (2). It is worth noticing that the matter fields in this context become massive whenever $\alpha_2>0$ and $v>0$. Also, by inserting the solution (41) into (37), one obtains the infinitesimal gauge transformations

$$\delta_\epsilon^{\mathrm{Ib}}A^\mu=\partial^\mu\epsilon,\quad \delta_\epsilon^{\mathrm{Ib}}\varphi=0,\quad \delta_\epsilon^{\mathrm{Ib}}\psi=\mathrm{i}g\tilde{w}\gamma_5\psi\epsilon,$$

which leave (45) invariant.

It is worth noticing that the models in the first class of interacting theories exhibit five physical degrees of freedom, distributed as in the starting model, i.e., two associated with the 1-form A_μ, one corresponding to the scalar field φ, and two fermionic degrees of freedom coming from the spinor field ψ.

The second class of interacting models corresponds to the solutions (39) and (42)/ (43) to the consistency equations (33). In this context, the 1-form becomes *massive* while the scalar field remains *massless*. Also, solutions (42) and (43) allow concluding that the fermionic modes are massive only in the absence ($\tilde{w}=0$) of electromagnetic interaction. Inserting the solutions (39) and (42)/ (43) into (36), one gets the Lagrangian actions for the last two types of theories constituting the second class of interacting models

$$S^{\mathrm{IIa}}[A,\varphi,\psi]=\int d^4x\left[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}(\partial_\mu\varphi-gnA_\mu)(\partial^\mu\varphi-gnA^\mu)+\tfrac{\mathrm{i}}{2}\bar{\psi}\partial\!\!\!/\psi\right.$$
$$\left.+\tfrac{g\alpha}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}+g(v\bar{\psi}\psi+\mathrm{i}\tilde{v}\bar{\psi}\gamma_5\psi)\right], \tag{46}$$

and

$$S^{\mathrm{IIb}}[A,\varphi,\psi]=\int d^4x\left[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}(\partial_\mu\varphi-gnA_\mu)(\partial^\mu\varphi-gnA^\mu)+\tfrac{\mathrm{i}}{2}\bar{\psi}D\!\!\!\!/\,\psi\right.$$
$$\left.+\tfrac{g\alpha}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\right], \tag{47}$$

which are found to be invariant under the Abelian generating set of gauge transformations

$$\delta_\epsilon^{IIa}A^\mu=\partial^\mu\epsilon,\quad \delta_\epsilon^{IIa}\varphi=gn\epsilon,\quad \delta_\epsilon^{IIa}\psi=\mathrm{i}g\tilde{w}\gamma_5\psi\epsilon, \tag{48}$$

and respectively

$$\delta_\epsilon^{IIb}A^\mu=\partial^\mu\epsilon,\quad \delta_\epsilon^{IIb}\varphi=gn\epsilon,\quad \delta_\epsilon^{IIb}\psi=\mathrm{i}g\tilde{w}\gamma_5\psi\epsilon. \tag{49}$$

It is worth noticing that the Stuekelberg coupling between the scalar field and the Abelian 1-form present in the last two actions (46) and (47), combined with the shift gauge transformation of the scalar field in (48) and (49) respectively, show that the distribution of the physical modes is no longer as in the free model. Here, the five physical degrees of freedom come from the three of the now massive 1-form A_μ and the two of the Majorana spinor field ψ. At the same time, the scalar field is a purely gauge one as it can be seen, at the classical level, from the reparametrization

$$A_\mu\to\bar{A}_\mu\equiv A_\mu-\tfrac{1}{gn}\partial_\mu\varphi,\quad \varphi\to\bar{\varphi}\equiv\varphi,\quad \psi\to\bar{\psi}\equiv\psi$$

of the jet bundle corresponding to the considered field theory.

6 Conclusions

In this paper, we analysed the consistent couplings that can be added to a massless free field theory comprising one real scalar field, one real spinor field, and one Abelian 1-form, using the deformation of the solution to the classical master equation [12, 13], supplemented with specific techniques of local BRST cohomology [14, 15, 16]. The procedure, supplemented with some reasonable hypotheses, standard in field theory, led to two classes of interacting theories, each of them containing some quadratic, derivative-free interaction vertices naturally interpreted as mass terms for various field spectrum components. The first class exhibits a *massless* Abelian 1-form, a *massive* scalar field, and a spinor field that is *massive* unless this involves electromagnetic interaction. The five physical degrees of freedom labeling the models in the first class are distributed as in the starting model, i.e., two associated with the 1-form A_μ, one corresponding to the scalar field φ, and two fermionic degrees of freedom coming from the spinor field ψ. The second class displays a *massive* 1-form, a *massless* scalar field, and a spinor field that is *massive* unless this involves cross-couplings with the massive 1-form. The scalar field becomes purely gauge in this context, while the 1-form gets an extra physical degree of freedom.

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