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On $D = 8$ vertices in a collection of non-standard topological BF theories

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Abstract

Here, we address the construction of a special class of $D = 8$ self-interactions for a collection of topological BF models via the antifield-BRST deformation method based on the computation of the local BRST cohomology corresponding to the free limit under some standard "selection rules" from Quantum Field Theory. The interaction vertices provide a generalization of the famous BF self-couplings present in the $D = 2$ gravity formulation via topological BF theories.

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1 Introduction

One of the striking features of topological field theories [1] is the relationship of certain, interacting, non-Abelian versions to the Poisson algebra [2] present in various versions of Poisson sigma models [3]–[9], which are essential in the correct description of two-dimensional gravity [10]–[20]. Moreover, pure three-dimensional gravity is just a topological BF theory and, concerning the higher dimensional case, it is known that General Relativity and supergravity in Ashtekar formalism may also be formulated as topological BF models with some extra constraints [21]–[24]. This is why the construction of self-interacting BF theories may be crucial in understanding higher-dimensional gravity and possible supergravity theories.

This paper is devoted to the construction of consistent, non-trivial $D = 8$ self-interactions that can be added to a finite collection of free, topological BF models with a non-standard field spectrum, consisting in four sets of form fields with the form degree equal to 0, 1, 3, and 4, in the presence of several selection rules typical to gauge field theories, namely, analyticity in the coupling constant, space-time locality, Lorentz covariance, Poincaré invariance, and preservation of the differential order of each field equation with respect to its free limit. This is done by means of the antifield-BRST symmetry [25]–[28] and, more precisely, on the deformation of its canonical generator [29]–[31] by means of cohomological techniques adapted to the computation of specific sectors of the local BRST cohomology [32]–[34]. The results exposed here add to the previous ones obtained by the authors and related to various self-couplings in single or several topological

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BF models in various dimensions emerging from a Lagrangian or Hamiltonian approach based on the BRST symmetry [35]–[46].

Our paper is divided into introduction, three main sections, and conclusions. Section 2 analyzes both the Lagrangian formulation and BRST symmetry for the considered, finite collection of A free topological BF models evolving on a Minkowski $D = 8$ space-time of 'mostly positive' signature. Section 3 synthesizes the construction of the deformed, non-trivial solution to the master equation that complies with all the imposed selection rules via the detailed computation of the necessary cohomological ingredients. Finally, section 4 reveals the Lagrangian formulation of the resulting $D = 8$ self-interacting BF theory and a key interaction vertex that is quadratic in the BF 4-form fields and generalizes the BF self-couplings present in the $D = 2$ gravity formulation via topological BF theories. Succinctly, we only mention that all the components of the deformed gauge theory are modified through the deformation procedure with respect to their free limit and disclose a generating set of gauge symmetries for the $D = 8$ self-coupled model with an open gauge algebra and some on-shell reducibility relations.

2 Lagrangian formulation and BRST symmetry for a collection of $D = 8$ free topological BF models

The starting point is given by the Lagrangian action for a non-standard (finite) collection of Abelian topological BF models in $D = 8$

$$S^L\left[\overset{[0]}{\varphi}_a, \overset{[1]}{B}{}^a, \overset{[3]}{A}_a, \overset{[4]}{B}{}^a\right] = \int d^8x \left(B^a_\mu \partial^\mu \varphi_a + B^a_{\mu\nu\rho\tau}\partial^{[\mu} A^{\nu\rho\tau]}_a\right) \tag{1}$$

defined on a 8-dimensional Minkowski space-time manifold endowed with a metric of 'mostly positive' signature, $\sigma = (- + \cdots +)$. We assume a finite collection of BF fields in $D = 8$, namely the scalar–vector pairs $\left\{\overset{[0]}{\varphi}_a, \overset{[1]}{B}{}^a\right\}$ and the three-form–four-form pairs $\left\{\overset{[3]}{A}_a, \overset{[4]}{B}{}^a\right\}$, with $a = \overline{1, A}$ $(A \geq 2)$, whose coefficients are to be denoted without reference to their form degree simply by φ_a, B^a_μ, $A^{\mu\nu\rho}_a$, and $B^a_{\mu\nu\rho\tau}$, respectively. This BF field spectrum is non-standard in the sense that we discarded the vector–two-form and two-form–three-form pairs $\left\{\overset{[1]}{A}_a, \overset{[2]}{B}{}^a\right\}$ and $\left\{\overset{[2]}{A}_a, \overset{[3]}{B}{}^a\right\}$ due to our aim of exhibiting just a special class of self-interactions that generalize those from $D = 2$ BF-based gravity and therefore depend only on the four-form coefficients $B^a_{\mu\nu\rho\tau}$ in a background of the undifferentiated scalar fields $\{\varphi_a\}$. Everywhere in this paper the notation $[\mu_1 \cdots \mu_k]$ signifies the operation of fully antisymmetrization with respect to the (Lorentz) indices between brackets, defined via the next conventions: only the independent terms are taken once without further normalization factors, the expression $f_{\mu_1 \cdots \mu_0}$ is identified with a scalar $(f_{\mu_1 \cdots \mu_0} = f)$, the quantity $f_{\mu_1 \cdots \mu_1}$ with a 6-vector $(f_{\mu_1 \cdots \mu_1} = f_{\mu_1})$, and any negative label of a Lorentz index defines a vanishing term, $f_{\mu_1 \cdots \mu_{-1}} = 0$. For further computations, it is useful to denote the BF field spectrum in a collective manner by

$$\Phi^{\alpha_0} \equiv \left\{\varphi_a, A^{\mu\nu\rho}_a, B^a_{\mu\nu\rho\tau}, B^a_\mu\right\}. \tag{2}$$

The stationary surface of this free, non-interacting BF theory is defined via some linear field equations of derivative order equal to one

$$\Sigma : \frac{\delta S^L}{\delta \Phi^{\alpha_0}} \equiv \left\{ \begin{array}{c} \frac{\delta S^L}{\delta \varphi_a} = -\partial^\lambda B^a_\lambda \\ \frac{\delta S^L}{\delta A^{\mu\nu\rho}_a} = -4\partial^\lambda B^a_{\lambda\mu\nu\rho} \\ \frac{\delta S^L}{\delta B^a_{\mu\nu\rho\tau}} = \partial^{[\mu} A^{\nu\rho\tau]}_a \\ \frac{\delta S^L}{\delta B^a_\mu} = \partial^\mu \varphi_a \end{array} \right. \approx 0, \tag{3}$$

where "≈" is the symbol of weak equality.

We work with a generating set of (non-trivial) gauge symmetries of action (1) like

$$\delta_{\Omega^{\alpha_1}} \Phi^{\alpha_0} \equiv \left\{ \begin{array}{c} \delta_{\Omega^{\alpha_1}} \varphi_a = 0 \\ \delta_{\Omega^{\alpha_1}} A^{\mu\nu\rho}_a = \partial^{[\mu} \epsilon^{\nu\rho]}_{(3,0)a} \\ \delta_{\Omega^{\alpha_1}} B^a_{\mu\nu\rho\tau} = -5\partial^\lambda \xi^a_{(4,0)\lambda\mu\nu\rho\tau} \\ \delta_{\Omega^{\alpha_1}} B^a_\mu = -2\partial^\lambda \xi^a_{(1,0)\lambda\mu} \end{array} \right. , \tag{4}$$

where the gauge parameters were collectively denoted by

$$\Omega^{\alpha_1} \equiv \left\{ \epsilon^{\lambda\mu}_{(3,0)a}, \xi^a_{(4,0)\lambda\mu\nu\rho\tau}, \xi^a_{(1,0)\lambda\mu} \right\} \tag{5}$$

and represent the coefficients of some arbitrary form-fields of degrees 2, 5, and 2, respectively, defined on the chosen $D = 8$ Minkowski space-time manifold. The supplementary two-index pair $(m, 0)$ marks the form degree of the BF field whose gauge transformations depend on the corresponding gauge parameters (for instance, $m = 3$ in $\epsilon^{\lambda\mu}_{(3,0)a}$ signifies that these are precisely the coefficients of the two-forms $\overset{[2]}{\epsilon}_{(3,0)a}$ involved in the gauge transformations of the components of the three-forms $\overset{[3]}{A}_a$) and the fixed (second) label "0" refers to the reducibility level (the gauge parameters are also known as the zeroth order reducibility parameters). The above generating set of gauge transformations is *Abelian*

$$\left[\delta_{\Omega^{(1)\alpha_1}}, \delta_{\Omega^{(2)\alpha_1}} \right] \Phi^{\alpha_0} = 0 \tag{6}$$

for any two arbitrary sets of gauge parameters of the type (5) denoted by $\Omega^{(1)\alpha_1}$ and $\Omega^{(2)\alpha_1}$.

It is important to observe that the considered generating set of gauge transformations is also *reducible* (the gauge generators are not all independent), with the *maximum reducibility order equal to* 6. With this observation at hand, it can be shown that the non-trivial gauge variations from (4) vanish iff we perform the following transformations on the gauge parameters (5)

$$\delta_{\Omega^{\alpha_1}} \Phi^{\alpha_0}\big|_{\text{nontriv}} = 0 \Longleftrightarrow \Omega^{\alpha_1} \to \Omega^{\alpha_1}\left(\Omega^{\alpha_2}\right) \equiv \left\{ \begin{array}{c} \epsilon^{\lambda\mu}_{(3,0)a}\left(\Omega^{\alpha_2}\right) = \partial^{[\lambda} \epsilon^{\mu]}_{(3,1)a} \\ \xi^a_{(4,0)\mu\nu\rho\sigma\tau}\left(\Omega^{\alpha_2}\right) = -6\partial^\lambda \xi^a_{(4,1)\lambda\mu\nu\rho\sigma\tau} \\ \xi^a_{(1,0)\mu\nu}\left(\Omega^{\alpha_2}\right) = -3\partial^\lambda \xi^a_{(1,1)\lambda\mu\nu} \end{array} \right. , \tag{7}$$

where we used the compact notation

$$\Omega^{\alpha_2} \equiv \left\{ \epsilon^\mu_{(3,1)a}, \xi^a_{(4,1)\mu\nu\rho\sigma\tau\theta}, \xi^a_{(1,1)\mu\nu\rho} \right\}. \tag{8}$$

The variables (8) are called the *reducibility parameters of order one*, symbolized by the lower 2-index pair $(m,1)$, where m has the same meaning like in the case of notation (2) and the index "1" marks the reducibility order. Next, we notice that the transformed gauge parameters from (7) vanish (strongly) iff we realize the next transformations on the first-order reducibility parameters (8)

$$\Omega^{\alpha_1}\left(\Omega^{\alpha_2}\right)=0 \Longleftrightarrow \Omega^{\alpha_2}\rightarrow\Omega^{\alpha_2}\left(\Omega^{\alpha_3}\right)\equiv \left\{\begin{array}{c} \epsilon^{\mu}_{(3,1)a}\left(\Omega^{\alpha_3}\right)=\partial^{\mu}\epsilon_{(3,2)a} \\ \xi^{a}_{(4,1)\mu\nu\rho\sigma\tau\theta}\left(\Omega^{\alpha_3}\right)=-7\partial^{\lambda}\xi^{a}_{(4,2)\lambda\mu\nu\rho\sigma\tau\theta} \\ \xi^{a}_{(1,1)\mu\nu\rho}\left(\Omega^{\alpha_3}\right)=-4\partial^{\lambda}\xi^{a}_{(1,2)\lambda\mu\nu\rho} \end{array}\right. , \tag{9}$$

where

$$\Omega^{\alpha_3}\equiv\left\{\epsilon_{(3,2)a},\xi^{a}_{(4,2)\mu\nu\rho\sigma\tau\theta\eta},\xi^{a}_{(1,2)\mu\nu\rho\sigma}\right\} \tag{10}$$

are named the *reducibility parameters of order two* and are labeled by the lower index pair $(m,2)$. Now, we observe that the first-order reducibility parameters from (9) are (strongly) annihilated iff we enforce the next transformations on the second-order reducibility parameters (10)

$$\Omega^{\alpha_2}\left(\Omega^{\alpha_3}\right)\big|_{\text{nontriv}}=0 \Leftrightarrow \Omega^{\alpha_3}\rightarrow \Big\{\epsilon_{(3,2)a}=0,\\ \Omega^{\alpha_3}\left(\Omega^{\alpha_4}\right)\big|_{\text{nontriv}}\equiv\left\{\begin{array}{c} \xi^{a}_{(4,2)\mu\nu\rho\sigma\tau\theta\eta}\left(\Omega^{\alpha_4}\right)=-8\partial^{\lambda}\xi^{a}_{(4,3)\lambda\mu\nu\rho\sigma\tau\theta\eta} \\ \xi^{a}_{(1,2)\mu\nu\rho\sigma}\left(\Omega^{\alpha_4}\right)=-5\partial^{\lambda}\xi^{a}_{(1,3)\lambda\mu\nu\rho\sigma} \end{array}\right\}\Big\}, \tag{11}$$

where

$$\Omega^{\alpha_4}\equiv\left\{\xi^{a}_{(4,3)\lambda\mu\nu\rho\sigma\tau\theta\eta},\xi^{a}_{(1,3)\mu\nu\rho\sigma\tau}\right\} \tag{12}$$

represent the *reducibility parameters of order three*, labeled by the two-index pair $(m,3)$. Next, we notice that the non-trivially transformed second-order reducibility parameters from (11), $\Omega^{\alpha_3}\left(\Omega^{\alpha_4}\right)\big|_{\text{nontriv}}$, vanish strongly iff we transform the third-order reducibility parameters (12) into

$$\Omega^{\alpha_3}\left(\Omega^{\alpha_4}\right)\big|_{\text{nontriv}}=0 \Longleftrightarrow \Omega^{\alpha_4}\rightarrow\Big\{\xi^{a}_{(4,3)\lambda\mu\nu\rho\sigma\tau\theta\eta}=0,\\ \Omega^{\alpha_4}\left(\Omega^{\alpha_5}\right)\big|_{\text{nontriv}}\equiv\xi^{a}_{(1,3)\mu\nu\rho\sigma\tau}(\Omega^{\alpha_5})=-6\partial^{\lambda}\xi^{a}_{(1,4)\lambda\mu\nu\rho\sigma\tau}\Big\}, \tag{13}$$

with

$$\Omega^{\alpha_5}\equiv\left\{\xi^{a}_{(1,4)\mu\nu\rho\sigma\tau\theta}\right\} \tag{14}$$

the *reducibility parameters of order four*. Similarly, the non-trivially transformed third-order reducibility parameters from (13), $\Omega^{\alpha_4}\left(\Omega^{\alpha_5}\right)\big|_{\text{nontriv}}\equiv\xi^{a}_{(1,3)\mu\nu\rho\sigma\tau}(\Omega^{\alpha_5})$, vanish strongly iff we transform the fourth-order reducibility parameters (14) into

$$\Omega^{\alpha_4}\left(\Omega^{\alpha_5}\right)\big|_{\text{nontriv}}\equiv\xi^{a}_{(1,3)\mu\nu\rho\sigma\tau}(\Omega^{\alpha_5})=0 \Longleftrightarrow \\ \Omega^{\alpha_5}\rightarrow\left\{\Omega^{\alpha_5}\left(\Omega^{\alpha_6}\right)\equiv\xi^{a}_{(1,4)\mu\nu\rho\sigma\tau\theta}(\Omega^{\alpha_6})=-7\partial^{\lambda}\xi^{a}_{(1,5)\lambda\mu\nu\rho\sigma\tau\theta}\right\}, \tag{15}$$

with

$$\Omega^{\alpha_6}\equiv\left\{\xi^{a}_{(1,5)\mu\nu\rho\sigma\tau\theta\eta}\right\} \tag{16}$$

the *reducibility parameters of order five*. Finally, the transformed fourth-order reducibility parameters from (15), $\Omega^{\alpha_5}\left(\Omega^{\alpha_6}\right)\equiv\xi^{a}_{(1,4)\mu\nu\rho\sigma\tau\theta}(\Omega^{\alpha_6})$, vanish strongly iff we transform the fifth-order reducibility parameters (16) into

$$\Omega^{\alpha_5}\left(\Omega^{\alpha_6}\right)\equiv\xi^{a}_{(1,4)\mu\nu\rho\sigma\tau\theta}(\Omega^{\alpha_6})=0 \Longleftrightarrow$$

$$\Omega^{\alpha_6} \to \left\{\Omega^{\alpha_6}\left(\Omega^{\alpha_7}\right) \equiv \xi^a_{(1,5)\mu\nu\rho\sigma\tau\theta\eta}(\Omega^{\alpha_7}) = -8\partial^\lambda \xi^a_{(1,6)\lambda\mu\nu\rho\sigma\tau\theta\eta}\right\}, \tag{17}$$

with

$$\Omega^{\alpha_7} \equiv \left\{\xi^a_{(1,6)\lambda\mu\nu\rho\sigma\tau\theta\eta}\right\} \tag{18}$$

the *reducibility parameters of order six.* The reducibility of (4) stops in order 6 since $\xi^a_{(1,5)\mu\nu\rho\sigma\tau\theta\eta}(\Omega^{\alpha_7})$ in (17) vanish iff all the reducibility parameters of order six from (18) also vanish.

In conclusion, the considered collection of free topological BF models in $D = 8$ is described at the Lagrangian level by a set of linear field equations and an Abelian generating set of gauge transformations that is reducible of order six or, in other words, by a *normal gauge theory of Cauchy order equal to* 8.

In the final part of this section we construct the antifield-BRST symmetry for this model, which can be shown to decompose into

$$s = \delta + \gamma, \tag{19}$$

where s signifies the BRST differential, δ the Koszul–Tate (co)differential, and γ the exterior longitudinal differential (which may be just a differential modulo delta in more general cases)

$$s^2 = 0 \Leftrightarrow \left\{\delta^2 = 0,\ \delta\gamma + \gamma\delta = 0,\ \gamma^2 = 0\right\}, \tag{20}$$

In what follows, ε denotes the Grassmann parity, ant and pgh stand for the two different $\mathbb{N}$-graduations of the BRST algebra on which the operators δ, γ, and s act (ant, known as the antifield number, is specific to the Koszul–Tate differential and pgh — the pure ghost number — to the exterior longitudinal differential), while their difference, $\mathrm{pgh} - \mathrm{ant} \equiv \mathrm{gh}$, is named the ghost number and provides a $\mathbb{Z}$-graduation of the BRST algebra.

In order to construct the differential BRST algebra $(\mathcal{A}, s)$, we initially introduce the BRST generators, which are of two kinds: fields/ghosts and their antifields. Related to the first kind, we associate ghost fields with all the gauge and reducibility parameters of various orders, (5), (8), (10), (12), (14), (16), and (18)

$$\Omega^{\alpha_1} \equiv \left\{\epsilon^{\lambda\mu}_{(3,0)a}, \xi^a_{(4,0)\lambda\mu\nu\rho\sigma}, \xi^a_{(1,0)\lambda\mu}\right\} \to \eta^{\alpha_1} \equiv \left\{\eta^{\lambda\mu}_{(3,0)a}, C^a_{(4,0)\lambda\mu\nu\rho\sigma}, C^a_{(1,0)\lambda\mu}\right\}, \tag{21}$$

$$\Omega^{\alpha_2} \equiv \left\{\epsilon^{\mu}_{(3,1)a}, \xi^a_{(4,1)\mu\nu\rho\sigma\tau\theta}, \xi^a_{(1,1)\mu\nu\rho}\right\} \to \eta^{\alpha_2} \equiv \left\{\eta^{\mu}_{(3,1)a}, C^a_{(4,1)\mu\nu\rho\sigma\tau\theta}, C^a_{(1,1)\mu\nu\rho}\right\}, \tag{22}$$

$$\Omega^{\alpha_3} \equiv \left\{\epsilon_{(3,2)a}, \xi^a_{(4,2)\lambda\mu\nu\rho\sigma\tau\theta}, \xi^a_{(1,2)\mu\nu\rho\sigma}\right\} \to \eta^{\alpha_3} \equiv \left\{\eta_{(3,2)a}, C^a_{(4,2)\lambda\mu\nu\rho\sigma\tau\theta}, C^a_{(1,2)\mu\nu\rho\sigma}\right\}, \tag{23}$$

$$\Omega^{\alpha_4} \equiv \left\{\xi^a_{(4,3)\lambda\mu\nu\rho\sigma\tau\theta\eta}, \xi^a_{(1,3)\mu\nu\rho\sigma\tau}\right\} \to \eta^{\alpha_4} \equiv \left\{C^a_{(4,3)\lambda\mu\nu\rho\sigma\tau\theta\eta}, C^a_{(1,3)\mu\nu\rho\sigma\tau}\right\}, \tag{24}$$

$$\Omega^{\alpha_5} \equiv \left\{\xi^a_{(1,4)\lambda\mu\nu\rho\sigma\tau}\right\} \to \eta^{\alpha_5} \equiv \left\{C^a_{(1,4)\lambda\mu\nu\rho\sigma\tau}\right\}, \tag{25}$$

$$\Omega^{\alpha_6} \equiv \left\{\xi^a_{(1,5)\lambda\mu\nu\rho\sigma\tau\theta}\right\} \to \eta^{\alpha_6} \equiv \left\{C^a_{(1,5)\lambda\mu\nu\rho\sigma\tau\theta}\right\}, \tag{26}$$

$$\Omega^{\alpha_7} \equiv \left\{\xi^a_{(1,6)\lambda\mu\nu\rho\sigma\tau\theta\eta}\right\} \to \eta^{\alpha_7} \equiv \left\{C^a_{(1,6)\lambda\mu\nu\rho\sigma\tau\theta\eta}\right\}, \tag{27}$$

such that the generators of the BRST algebra are precisely

$$\Phi^A \equiv \{\Phi^{\alpha_0}, \eta^{\alpha_1}, \eta^{\alpha_2}, \eta^{\alpha_3}, \eta^{\alpha_4}, \eta^{\alpha_5}, \eta^{\alpha_6}, \eta^{\alpha_7}\}, \tag{28}$$

where Φ^{α_0} are the original BF fields (2). The second type of BRST generators are the antifields respectively corresponding to the field and ghost spectra

$$\Phi^*_A \equiv \left\{\Phi^*_{\alpha_0}, \eta^*_{\alpha_1}, \eta^*_{\alpha_2}, \eta^*_{\alpha_3}, \eta^*_{\alpha_4}, \eta^*_{\alpha_5}, \eta^*_{\alpha_6}, \eta^*_{\alpha_7}\right\}, \tag{29}$$

with

$$\Phi^*_{\alpha_0} \equiv \left\{\varphi^{*a}, A^{*a}_{\mu\nu\rho}, B^{*\mu\nu\rho\sigma}_a, B^{*\mu}_a\right\}, \quad \eta^*_{\alpha_1} \equiv \left\{\eta^{*a}_{(3,0)\lambda\mu}, C^{*\lambda\mu\nu\rho\sigma}_{(4,0)a}, C^{*\lambda\mu}_{(1,0)a}\right\}, \tag{30}$$

$$\eta^*_{\alpha_2} \equiv \left\{\eta^{*a}_{(3,1)\mu}, C^{*\mu\nu\rho\sigma\tau\theta}_{(4,1)a}, C^{*\mu\nu\rho}_{(1,1)a}\right\}, \quad \eta^*_{\alpha_3} \equiv \left\{\eta^{*a}_{(3,2)}, C^{*\lambda\mu\nu\rho\sigma\tau\theta}_{(4,2)a}, C^{*\mu\nu\rho\sigma}_{(1,2)a}\right\}, \tag{31}$$

$$\eta^*_{\alpha_4} \equiv \left\{C^{*\lambda\mu\nu\rho\sigma\tau\theta\eta}_{(4,3)a}, C^{*\mu\nu\rho\sigma\tau}_{(1,3)a}\right\}, \quad \eta^*_{\alpha_5} \equiv \left\{C^{*\lambda\mu\nu\rho\sigma\tau}_{(1,4)a}\right\}, \tag{32}$$

$$\eta^*_{\alpha_6} \equiv \left\{C^{*\lambda\mu\nu\rho\sigma\tau\theta}_{(1,5)a}\right\}, \quad \eta^*_{\alpha_7} \equiv \left\{C^{*\lambda\mu\nu\rho\sigma\tau\theta\eta}_{(1,6)a}\right\}. \tag{33}$$

Meanwhile, according to the antifield-BRST method, we endow the field/ghost spectrum with the following properties

$$\varepsilon\left(\eta^{\alpha_k}\right) = k \bmod 2, \quad \mathrm{pgh}\left(\Phi^{\alpha_0}\right) = 0, \quad \mathrm{pgh}\left(\eta^{\alpha_k}\right) = k, \tag{34}$$

$$\varepsilon\left(\Phi^*_A\right) = \left(\varepsilon\left(\Phi^A\right) + 1\right) \bmod 2, \quad \mathrm{ant}\left(\Phi^*_{\alpha_0}\right) = 1, \quad \mathrm{ant}\left(\eta^*_{\alpha_k}\right) = k + 1, \tag{35}$$

$$\mathrm{ant}\left(\Phi^A\right) = 0, \quad \mathrm{pgh}\left(\Phi^*_A\right) = 0, \tag{36}$$

with $k = \overline{1,7}$.

The actions of the operators δ and γ on the BRST generators (28) and (29) that implement the required properties are defined by

$$\delta\Phi^A = 0, \quad \gamma\Phi^*_A = 0 \tag{37}$$

together with

$$\delta\varphi^{*a} \equiv -\frac{\delta S^L}{\delta\varphi_a} = \partial^\lambda B^a_\lambda, \ \delta\eta^{*a}_{(3,-1)\mu_1\mu_2\mu_3} \equiv -\frac{\delta S^L}{\delta A^{\mu_1\mu_2\mu_3}_a} = 4\partial^\lambda B^a_{\lambda\mu_1\mu_2\mu_3}, \tag{38}$$

$$\delta\eta^{*a}_{(3,l(3))\mu_1\cdots\mu_{2-l(3)}} = (-)^{l(3)+1}(3 - l(3))\partial^\lambda\eta^{*a}_{(3,l(3)-1)\lambda\mu_1\cdots\mu_{2-l(3)}}, l(3) = \overline{0,2}, \tag{39}$$

$$\delta C^{*\mu_1\cdots\mu_4}_{(4,-1)a} \equiv -\frac{\delta S^L}{\delta B^a_{\mu_1\cdots\mu_4}} = -\partial^{[\mu_1}A^{\mu_2\mu_3\mu_4]}_a, \ \delta C^{*\mu_1}_{(1,-1)a} \equiv -\frac{\delta S^L}{\delta B^a_{\mu_1}} = -\partial^{\mu_1}\varphi_a, \tag{40}$$

$$\delta C^{*\mu_1\cdots\mu_{m+l(m)+1}}_{(m,l(m))a} = (-)^{l(m)}\partial^{[\mu_1}C^{*\mu_2\cdots\mu_{m+l(m)+1}]}_{(m,l(m)-1)a}, \ l(m) = \overline{0,7-m}, \ m = 1,4 \tag{41}$$

and respectively

$$\gamma\varphi_a = 0, \ \gamma\eta^{\mu_1\cdots\mu_{2-l(3)}}_{(3,l(3))a} = \partial^{[\mu_1}\eta^{\mu_2\cdots\mu_{2-l(3)}]}_{(3,l(3)+1)a}, \ l(3) = \overline{-1,1}, \ \gamma\eta_{(3,2)a} = 0, \tag{42}$$

$$\gamma C^a_{(m,l(m))\mu_1\cdots\mu_{m+l(m)+1}} = -(m + l(m) + 2)\partial^\lambda C^a_{(m,l(m)+1)\lambda\mu_1\cdots\mu_{m+l(m)+1}}, \tag{43}$$

$$l(m) = \overline{-1,6-m}, \ m = 1,4, \ \gamma C^a_{(m,7-m)\mu_1\cdots\mu_8} = 0, \ m = 1,4, \tag{44}$$

where we employed the notations

$$A^{\mu\nu\rho}_a \equiv \eta^{\mu\nu\rho}_{(3,-1)a}, \ B^a_{\mu\nu\rho\sigma} \equiv C^a_{(4,-1)\mu\nu\rho\sigma}, \ B^a_\mu \equiv C^a_{(1,-1)\mu}, \tag{45}$$

$$A^{*a}_{\mu\nu\rho} \equiv \eta^{*a}_{(3,-1)\mu\nu\rho}, \ B^{*\mu\nu\rho\sigma}_a \equiv C^{*\mu\nu\rho\sigma}_{(4,-1)a}, \ B^{*\mu}_a \equiv C^{*\mu}_{(1,-1)a}. \tag{46}$$

Obviously, the actions of the BRST differential on the BRST generators follow from (37)–(44) via expansion (19). We mention that all the operators ((co)-differentials) from (19) are assumed to act like right derivations.

A major feature of the antifield-BRST formalism is given by its *canonical action* in a structure named *antibracket*, which is denoted by $(,)$ and is defined by decreeing the fields/ghosts respectively conjugated with the corresponding antifields

$$\left(\Phi^A, \Phi^*_B\right) = \delta^A_B. \tag{47}$$

The *canonical generator of the antifield-BRST symmetry*, S, is a bosonic functional depending on the fields/ghosts and antifields, of ghost number 0, in terms of which the (right derivation) action of the BRST operator s is recovered precisely via the antibracket

$$\forall F \in \mathcal{A},\ sF = (F, S)\,, \quad \varepsilon(S) = 0, \quad \mathrm{gh}(S) = 0 \tag{48}$$

and the second-order nilpotency of s is equivalent to the famous classical master equation satisfied by S

$$s^2 = 0 \Leftrightarrow (S, S) = 0. \tag{49}$$

In view of this, S is usually referred to as the solution to the classical master equation. In the case of the model under study, the solution to the classical master equation can be taken as

$$\begin{aligned} S \ =\ & \int d^8x \left(B_\mu^a \partial^\mu \varphi_a + B^a_{\mu\nu\rho\sigma} \partial^{[\mu} A_a^{\nu\rho\sigma]} + A^{*a}_{\mu\nu\rho} \partial^{[\mu} \eta^{\nu\rho]}_{(3,0)a} - 5B_a^{*\mu\nu\rho\sigma} \partial^\lambda C^a_{(4,0)\lambda\mu\nu\rho\sigma} \right. \\ & -2B_a^{*\mu} \partial^\lambda C^a_{(1,0)\lambda\mu} + \eta^{*a}_{(3,0)\mu\nu} \partial^{[\mu} \eta^{\nu]}_{(3,1)a} - 6C^{*\mu\nu\rho\sigma\tau}_{(4,0)a} \partial^\lambda C^a_{(4,1)\lambda\mu\nu\rho\sigma\tau} \\ & -3C^{*\mu\nu}_{(1,0)a} \partial^\lambda C^a_{(1,1)\lambda\mu\nu} + \eta^{*a}_{(3,1)\mu} \partial^\mu \eta_{(3,2)a} - 7C^{*\mu\nu\rho\sigma\tau\theta}_{(4,1)a} \partial^\lambda C^a_{(4,2)\lambda\mu\nu\rho\sigma\tau\theta} \\ & -4C^{*\mu\nu\rho}_{(1,1)a} \partial^\lambda C^a_{(1,2)\lambda\mu\nu\rho} - 8C^{*\mu\nu\rho\sigma\tau\theta\kappa}_{(4,2)a} \partial^\lambda C^a_{(4,3)\lambda\mu\nu\rho\sigma\tau\theta\kappa} - 5C^{*\mu\nu\rho\sigma}_{(1,2)a} \partial^\lambda C^a_{(1,3)\lambda\mu\nu\rho\sigma} \\ & -6C^{*\mu\nu\rho\sigma\tau}_{(1,3)a} \partial^\lambda C^a_{(1,4)\lambda\mu\nu\rho\sigma\tau} - 7C^{*\mu\nu\rho\sigma\tau\theta}_{(1,4)a} \partial^\lambda C^a_{(1,5)\lambda\mu\nu\rho\sigma\tau\theta} \\ & \left. -8C^{*\mu\nu\rho\sigma\tau\theta\kappa}_{(1,5)a} \partial^\lambda C^a_{(1,6)\lambda\mu\nu\rho\sigma\tau\theta\kappa} \right). \end{aligned} \tag{50}$$

We organized S according to the increasing values of the antifield number of its components and thus it contains pieces of ant ranging from 0 to 7. The component of antifield number zero always reduces to the Lagrangian action of the considered gauge theory (the first two terms from (50) provide precisely (1)). The elements of antifield number one are always written as the antifields of the original fields times the gauge transformations of the corresponding fields where the gauge parameters are replaced with the associated ghosts of pure ghost number 1. The components of antifield numbers strictly greater than 1 from the solution to the classical master equation (if any) are present only if the chosen generating set of gauge transformations for the theory under study is reducible and/or generates a non-Abelian gauge algebra. The terms related to the reducibility functions and relations of various orders specific to the generating set are always linear in the ghosts of pure ghost numbers strictly greater than 1. In the case of our $D = 8$ BF model, this type of components covers the remaining, last eleven elements from (50), of antifield number ranging between 2 and 7.

3 Antifield-BRST deformation method and its application to a collection of $D = 6$ BF models

3.1 Brief review to the antifield-BRST deformation method

It is possible to reformulate the long standing problem of generating consistent interactions in gauge field theories via the antifield-BRST deformation method [29]–[31] based on the observation that, if consistent couplings can be added, then the solution to the classical master equation of the original gauge theory, S, may be deformed into a solution to the classical master equation for the coupled gauge theory, $\bar{S}$,

$$\bar{S} = S + \lambda S_1 + \lambda^2 S_2 + \cdots\,, \quad \tfrac{1}{2}(\bar{S}, \bar{S}) = 0, \tag{51}$$

with λ the coupling constant or deformation parameter. The projection of the key equation $\frac{1}{2}(\bar{S},\bar{S})=0$ on the various, increasing powers in the coupling constant λ is equivalent to the chain of equations

$$\begin{aligned}
\lambda^0 &: \tfrac{1}{2}(S,S)=0, && (52)\\
\lambda^1 &: (S_1,S)=0, && (53)\\
\lambda^2 &: (S_2,S)+\tfrac{1}{2}(S_1,S_1)=0, && (54)\\
\lambda^3 &: (S_3,S)+(S_1,S_2)=0, && (55)\\
&\vdots
\end{aligned}$$

known as the *equations of the antifield-BRST deformation method*. The functionals S_i, $i \geq 1$, are known as the *deformations of order* i of the solution to the classical master equation. The first equation is fulfilled by assumption, while the remaining ones may be expressed (via the canonical action $s\cdot=(\cdot,S)$) like

$$\begin{aligned}
\lambda^1 &: sS_1=0, && (56)\\
\lambda^2 &: sS_2+\tfrac{1}{2}(S_1,S_1)=0, && (57)\\
\lambda^3 &: sS_3+(S_1,S_2)=0, && (58)\\
&\vdots
\end{aligned}$$

The solutions to (56) always exist as long as they pertain to the cohomology of the BRST differential s in ghost number 0 computed in the space of *all functionals* (local and nonlocal) of fields, ghosts, and antifields, $H^0(s)$, which is generically non-empty. All *trivial first-order deformations*, defined via s-exact elements of $H^0(s)$, must be *discarded* since they produce trivial interactions. The existence of solutions to the remaining deformation equations, (57), (58), etc., has been proved to exist [29] if we enforce *no restrictions on the interactions* (such as the space-time locality).

On the other hand, if we impose some restrictions on the deformations, like for instance that $\bar{S}$ should be a local functional, then the construction of consistent interactions via the antifield-BRST method must be approached differently. Assuming the space-time locality of deformations, if we make the notations

$$S_1=\int d^8x\ a,\quad S_2=\int d^8x\ b,\quad S_3=\int d^8x\ c, \tag{59}$$

$$\tfrac{1}{2}(S_1,S_1)=\int d^8x\ \Delta,\quad (S_1,S_2)=\int d^8x\ \Gamma, \tag{60}$$

then equations (56)–(58), etc. take the local form

$$\begin{aligned}
sa &= \partial^\mu j_\mu, && (61)\\
sb+\Delta &= \partial^\mu k_\mu, && (62)\\
sc+\Gamma &= \partial^\mu l_\mu, && (63)\\
&\vdots
\end{aligned}$$

Thus, equation (61), which is now responsible for the non-integrated density of the first-order deformation, is equivalent to the fact that a should be a (non-trivial) element of the local cohomology of the BRST differential at ghost number 0, $a \in H^0(s|d)$. In the next subsection we will construct the general, non-trivial solution to the first-order deformation equation, (61), but in an even more restricted BRST algebra than $\mathcal{A}_{\mathrm{local}}$ such that to comply with all the standard "selection rules" imposed on field theories.

3.2 Deformed solution to the master equation

The goal of the present paper is to generate all non-trivial, consistent self-interactions that can be added to the free model exposed in Section 2 with the help of the antifield-BRST deformation method briefly reviewed in the previous subsection. We adopt the standard selection rules from field theory on the deformed solution to the classical master equation, (51), namely, *analyticity in the coupling constant, space-time locality, Lorentz covariance, Poincaré invariance, and conservation of the differential order of the interacting field equations with respect to their free limit* ($\lambda \to 0$). Due to the space-time locality hypothesis and based on the first notation from (59) and on equation (61), it follows that the non-integrated density of the first-order deformation, a, should be a non-trivial element of the local BRST cohomology $H^0(s|d)$. The last cohomology space will be computed in the BRST algebra of local "functions", which, in addition, must comply with all the other selection rules.

Due to the fact that the starting $D = 8$ collection of Abelian BF models is a normal linear gauge theory of Cauchy order equal to 8, some standard results from the literature [32] adapted to this case stipulates that one can take the first-order deformation to stop at antifield number 8. Moreover, it can be shown (see, for instance [44]) that the last component, a_8, can be taken as a non-trivial element of the cohomology of the longitudinal exterior differential $H(\gamma)$, such that we can write

$$a = \sum_{k=0}^{8} a_k, \tag{64}$$

so equation (61) becomes equivalent to the tower of equations

$$\gamma a_8 = 0, \tag{65}$$

$$\delta a_k + \gamma a_{k-1} = \partial^\mu j_{k-1,\mu}, \quad k = \overline{1,8}, \tag{66}$$

where the components of a satisfy the properties

$$\varepsilon\left(a_k\right) = 0, \quad \mathrm{gh}(a_k) = 0, \quad \mathrm{ant}(a_k) \equiv k, \quad \mathrm{pgh}(a_k) = k. \tag{67}$$

If we manipulate the previous equations, we reach the conclusion that the non-trivial solution to the equation (65) satisfied by the component of maximum antifield number from (64) can be generated, without loss of non-trivial terms, by 'gluing' the ghost basis of pure ghost number equal to 8 from $H_0^8(\gamma)$ to the non-trivial elements of $H_8^{\mathrm{inv}}(\delta|d)$

$$a_8 : e^8 \in H_0^8(\gamma) \longleftrightarrow \alpha_8^{\mathrm{inv}} \in H_8^{\mathrm{inv}}(\delta|d), \tag{68}$$

where $H_8^{\mathrm{inv}}(\delta|d)$ signifies the local homology space of the Koszul–Tate differential at antifield number 8 and pure ghost number 0 computed in the space of gauge-invariant functions. It i easy to see that the ghost basis of pure ghost number equal to 8 from $H_0^8(\gamma)$ is generated only by the monomials

$$\left\{C^a_{(4,3)\mu_1\cdots\mu_8} C^b_{(4,3)\nu_1\cdots\nu_8}\right\}. \tag{69}$$

On the other hand, it can be shown [43] that $H_8^{\mathrm{inv}}(\delta|d)$ is generated by the elements

$$\alpha_8^{\mathrm{inv}} \to P_8^{\Delta|\mu_1\cdots\mu_8}(f) \equiv \frac{\partial f^\Delta}{\partial \varphi_{d_1}} C^{*\mu_1\cdots\mu_8}_{(1,6)d_1}$$

$$+\frac{\partial^2 f^\Delta}{\partial\varphi_{d_1}\partial\varphi_{d_2}}\left(C^{*[\mu_1\cdots\mu_7}_{(1,5)d_1}C^{*\mu_8]}_{(1,-1)d_2}+C^{*[\mu_1\cdots\mu_6}_{(1,4)d_1}C^{*\mu_7\mu_8]}_{(1,0)d_2}+C^{*[\mu_1\cdots\mu_5}_{(1,3)d_1}C^{*\mu_6\mu_7\mu_8]}_{(1,1)d_2}\right.$$
$$\left.+C^{*[\mu_1\cdots\mu_4}_{(1,2)d_1}C^{*\mu_5\cdots\mu_8]}_{(1,2)d_2}\right)+\frac{\partial^3 f^\Delta}{\partial\varphi_{d_1}\partial\varphi_{d_2}\partial\varphi_{d_3}}\left(C^{*[\mu_1\cdots\mu_6}_{(1,4)d_1}C^{*\mu_7}_{(1,-1)d_2}C^{*\mu_8]}_{(1,-1)d_3}\right.$$
$$+C^{*[\mu_1\cdots\mu_5}_{(1,3)d_1}C^{*\mu_6\mu_7}_{(1,0)d_2}C^{*\mu_8]}_{(1,-1)d_3}+C^{*[\mu_1\cdots\mu_4}_{(1,2)d_1}C^{*\mu_5\mu_6\mu_7}_{(1,1)d_2}C^{*\mu_8]}_{(1,-1)d_3}+C^{*[\mu_1\cdots\mu_4}_{(1,2)d_1}C^{*\mu_5\mu_6}_{(1,0)d_2}C^{*\mu_7\mu_8]}_{(1,0)d_3}$$
$$\left.+C^{*[\mu_1\mu_2\mu_3}_{(1,1)d_1}C^{*\mu_4\mu_5\mu_6}_{(1,1)d_2}C^{*\mu_7\mu_8]}_{(1,0)d_3}\right)+\frac{\partial^4 f^\Delta}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_4}}\left(C^{*[\mu_1\cdots\mu_5}_{(1,3)d_1}C^{*\mu_6}_{(1,-1)d_2}C^{*\mu_7}_{(1,-1)d_3}C^{*\mu_8]}_{(1,-1)d_4}\right.$$
$$+C^{*[\mu_1\cdots\mu_4}_{(1,2)d_1}C^{*\mu_5\mu_6}_{(1,0)d_2}C^{*\mu_7}_{(1,-1)d_3}C^{*\mu_8]}_{(1,-1)d_4}+C^{*[\mu_1\mu_2\mu_3}_{(1,1)d_1}C^{*\mu_4\mu_5\mu_6}_{(1,1)d_2}C^{*\mu_7}_{(1,-1)d_3}C^{*\mu_8]}_{(1,-1)d_4}$$
$$\left.+C^{*[\mu_1\mu_2\mu_3}_{(1,1)d_1}C^{*\mu_4\mu_5}_{(1,0)d_2}C^{*\mu_6\mu_7}_{(1,0)d_3}C^{*\mu_8]}_{(1,-1)d_4}+C^{*[\mu_1\mu_2}_{(1,0)d_1}C^{*\mu_3\mu_4}_{(1,0)d_2}C^{*\mu_5\mu_6}_{(1,0)d_3}C^{*\mu_7\mu_8]}_{(1,0)d_4}\right)$$
$$+\frac{\partial^5 f^\Delta}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_5}}\left(C^{*[\mu_1\cdots\mu_4}_{(1,2)d_1}C^{*\mu_5}_{(1,-1)d_2}\cdots C^{*\mu_8]}_{(1,-1)d_5}+C^{*[\mu_1\mu_2\mu_3}_{(1,1)d_1}C^{*\mu_4\mu_5}_{(1,0)d_2}C^{*\mu_6}_{(1,-1)d_3}C^{*\mu_7}_{(1,-1)d_4}C^{*\mu_8]}_{(1,-1)d_5}\right.$$
$$\left.+C^{*[\mu_1\mu_2}_{(1,0)d_1}C^{*\mu_3\mu_4}_{(1,0)d_2}C^{*\mu_5\mu_6}_{(1,0)d_3}C^{*\mu_7}_{(1,-1)d_4}C^{*\mu_8]}_{(1,-1)d_5}\right)+\frac{\partial^6 f^\Delta}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_6}}\left(C^{*[\mu_1\mu_2\mu_3}_{(1,1)d_1}C^{*\mu_4}_{(1,-1)d_2}\cdots C^{*\mu_8]}_{(1,-1)d_6}\right.$$
$$\left.+C^{*[\mu_1\mu_2}_{(1,0)d_1}C^{*\mu_3\mu_4}_{(1,0)d_2}C^{*\mu_5}_{(1,-1)d_3}\cdots C^{*\mu_8]}_{(1,-1)d_6}\right)+\frac{\partial^7 f^\Delta}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_7}}C^{*[\mu_1\mu_2}_{(1,0)d_1}C^{*\mu_3}_{(1,-1)d_2}\cdots C^{*\mu_8]}_{(1,-1)d_7}$$
$$+\frac{\partial^8 f^\Delta}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_8}}C^{*\mu_1}_{(1,-1)d_1}\cdots C^{*\mu_8}_{(1,-1)d_8}, \tag{70}$$

where $f^\Delta = f^\Delta(\varphi)$ stand for some arbitrary, smooth functions allowed to depend only on the undifferentiated scalar fields $\{\varphi_a\}$. Inserting results (69) and (70) into (68), it follows that a_8 reduces to

$$a_8(Z) = \frac{1}{2}\varepsilon^{\mu_1\cdots\mu_8}P^{\rho_1\cdots\rho_8}_{8,ab}(Z)C^a_{(4,3)\rho_1\cdots\rho_8}C^b_{(4,3)\mu_1\cdots\mu_8}, \tag{71}$$

where elements $P^{\rho_1\cdots\rho_8}_{8,ab}$ read as in (70), with

$$f^\Delta(\varphi)\to Z_{ab}(\varphi), \quad Z_{ab}=Z_{ba}. \tag{72}$$

The remaining pieces from (64) as solutions to equations (66) follow by direct computation and will be given below. We observe that (71), so actually the entire first-order deformation (64), is parameterized in terms of a single set of symmetric, smooth functions depending on the undifferentiated scalar fields, $\{Z_{ab}(\varphi)\}_{a,b=\overline{1,A}}$.

Once we have completed the construction of the first-order deformation, it can be shown by direct computation that $(S_1, S_1) = 0$, so we can take all the higher-order deformations, as solutions to equations (57), (58), etc., to vanish

$$S_2 = S_3 = \cdots = 0. \tag{73}$$

Putting together the results deduced until now via (51), we conclude that *the non-trivial deformation of the solution to the master equation, which is consistent, complies with all the working hypotheses, and provides all $D = 8$ self-interactions among a non-standard collection of topological BF models ends at order one in the deformation parameter*

$$\bar{S} = S + \lambda S_1, \tag{74}$$

where S is the solution to the master equation for the starting free model, (50). Assembling (74) according to its components organized along the increasing values of the antifield number, we can write that

$$\bar{S}=\sum_{j=0}^{8}\left(\int d^{8}x\ \mathcal{L}_{j}\right),\quad \varepsilon\left(\mathcal{L}_{j}\right)=0,\quad \mathrm{gh}\left(\mathcal{L}_{j}\right)=0,\quad \mathrm{ant}\left(\mathcal{L}_{j}\right)=j. \tag{75}$$

The pieces of antifield number 0 and respectively 1 read

$$\mathcal{L}_{0}=B_{\mu_{1}}^{a}\partial^{\mu_{1}}\varphi_{a}+B_{\mu_{1}\cdots\mu_{4}}^{a}\left(\partial^{[\mu_{1}}A_{a}^{\mu_{2}\mu_{3}\mu_{4}]}+\frac{\lambda}{2}\varepsilon^{\mu_{1}\cdots\mu_{8}}Z_{ab}\left(\varphi\right)B_{\mu_{4}\cdots\mu_{8}}^{b}\right), \tag{76}$$

$$\begin{aligned}\mathcal{L}_{1} = {} & A_{\mu_{1}\mu_{2}\mu_{3}}^{*a}\left(\partial^{[\mu_{1}}\eta_{(3,0)a}^{\mu_{2}\mu_{3}]}-\lambda\varepsilon^{\mu_{1}\cdots\mu_{8}}Z_{ab}\left(\varphi\right)C_{(4,0)\mu_{4}\cdots\mu_{8}}^{b}\right)-5B_{a}^{*\mu_{1}\cdots\mu_{4}}\partial^{\rho}C_{(4,0)\rho\mu_{1}\cdots\mu_{4}}^{a}\\ & -B_{a}^{*\mu_{1}}\left(2\partial^{\rho}C_{(1,0)\rho\mu_{1}}^{a}+5\lambda\varepsilon^{\rho_{1}\cdots\rho_{8}}\frac{\partial Z_{bc}}{\partial\varphi_{a}}B_{\rho_{1}\cdots\rho_{4}}^{b}C_{(4,0)\mu_{1}\rho_{5}\cdots\rho_{8}}^{c}\right).\end{aligned} \tag{77}$$

The terms of antifield number 2 are structured as follows

$$\begin{aligned}\mathcal{L}_{2}= {} & \eta_{(3,0)\mu_{1}\mu_{2}}^{*a}\left(\partial^{[\mu_{1}}\eta_{(3,1)a}^{\mu_{2}]}-\lambda\varepsilon^{\mu_{1}\cdots\mu_{8}}Z_{ab}\left(\varphi\right)C_{(4,1)\mu_{3}\cdots\mu_{8}}^{b}\right)-6C_{(4,0)a}^{*\mu_{1}\cdots\mu_{5}}\partial^{\rho}C_{(4,1)\rho\mu_{1}\cdots\mu_{5}}^{a}\\ & +3C_{(1,0)a}^{*\mu_{1}\mu_{2}}\left(-\partial^{\rho}C_{(1,1)\rho\mu_{1}\mu_{2}}^{a}+5\lambda\varepsilon^{\rho_{1}\cdots\rho_{8}}\frac{\partial Z_{bc}}{\partial\varphi_{a}}B_{\rho_{1}\cdots\rho_{4}}^{b}C_{(4,1)\mu_{1}\mu_{2}\rho_{5}\cdots\rho_{8}}^{c}\right)\\ & +3\lambda\varepsilon^{\mu_{1}\cdots\mu_{8}}B_{d_{1}}^{*\rho_{1}}\left(\frac{\partial Z_{ab}}{\partial\varphi_{d_{1}}}A_{\rho_{1}\mu_{1}\mu_{2}}^{*a}+2\frac{\partial^{2}Z_{ab}}{\partial\varphi_{d_{1}}\partial\varphi_{d_{2}}}B_{d_{2}}^{*\rho_{2}}B_{\rho_{1}\rho_{2}\mu_{1}\mu_{2}}^{a}\right)C_{(4,1)\mu_{3}\cdots\mu_{8}}^{b}\\ & +5\lambda\varepsilon^{\mu_{1}\cdots\mu_{8}}\left(\frac{\partial Z_{ab}}{\partial\varphi_{d_{1}}}C_{(1,0)d_{1}}^{*\rho_{1}\rho_{2}}+\frac{\partial^{2}Z_{ab}}{\partial\varphi_{d_{1}}\partial\varphi_{d_{2}}}B_{d_{1}}^{*\rho_{1}}B_{d_{2}}^{*\rho_{2}}\right)C_{(4,0)\rho_{1}\rho_{2}\mu_{1}\mu_{2}\mu_{3}}^{a}C_{(4,0)\mu_{4}\cdots\mu_{8}}^{b}.\end{aligned} \tag{78}$$

The non-integrated density with the antifield number equal to 3 is given by

$$\begin{aligned}\mathcal{L}_{3}= {} & \eta_{(3,1)\mu_{1}}^{*a}\left(\partial^{\mu_{1}}\eta_{(3,2)a}+\lambda\varepsilon^{\mu_{1}\cdots\mu_{8}}Z_{ab}\left(\varphi\right)C_{(4,2)\mu_{2}\cdots\mu_{8}}^{b}\right)-7C_{(4,1)a}^{*\mu_{1}\cdots\mu_{6}}\partial^{\rho}C_{(4,2)\rho\mu_{1}\cdots\mu_{6}}^{a}\\ & -C_{(1,1)a}^{*\mu_{1}\mu_{2}\mu_{3}}\left(4\partial^{\rho}C_{(1,2)\rho\mu_{1}\mu_{2}\mu_{3}}^{a}+35\lambda\varepsilon^{\rho_{1}\cdots\rho_{8}}\frac{\partial Z_{bc}}{\partial\varphi_{a}}B_{\rho_{1}\cdots\rho_{4}}^{b}C_{(4,2)\mu_{1}\mu_{2}\mu_{3}\rho_{5}\cdots\rho_{8}}^{c}\right)\\ & +\lambda\varepsilon^{\mu_{1}\cdots\mu_{8}}\left[4\left(3\frac{\partial^{2}Z_{ab}}{\partial\varphi_{d_{1}}\partial\varphi_{d_{2}}}C_{(1,0)d_{1}}^{*\rho_{1}\rho_{2}}+\frac{\partial^{3}Z_{ab}}{\partial\varphi_{d_{1}}\partial\varphi_{d_{2}}\partial\varphi_{d_{3}}}B_{d_{1}}^{*\rho_{1}}B_{d_{2}}^{*\rho_{2}}\right)B_{d_{3}}^{*\rho_{3}}B_{\rho_{1}\rho_{2}\rho_{3}\mu_{1}}^{a}\right.\\ & \left.-3\left(\frac{\partial Z_{ab}}{\partial\varphi_{d_{1}}}C_{(1,0)d_{1}}^{*\rho_{1}\rho_{2}}+\frac{\partial^{2}Z_{ab}}{\partial\varphi_{d_{1}}\partial\varphi_{d_{2}}}B_{d_{1}}^{*\rho_{1}}B_{d_{2}}^{*\rho_{2}}\right)A_{\rho_{1}\rho_{2}\mu_{1}}^{*a}-2\frac{\partial Z_{ab}}{\partial\varphi_{d_{1}}}B_{d_{1}}^{*\rho}\eta_{(3,0)\rho\mu_{1}}^{*a}\right]C_{(4,2)\mu_{2}\cdots\mu_{8}}^{b}\\ & -10\lambda\varepsilon^{\mu_{1}\cdots\mu_{8}}\left(\frac{\partial Z_{ab}}{\partial\varphi_{d_{1}}}C_{(1,1)d_{1}}^{*\rho_{1}\rho_{2}\rho_{3}}+3\frac{\partial^{2}Z_{ab}}{\partial\varphi_{d_{1}}\partial\varphi_{d_{2}}}C_{(1,0)d_{1}}^{*\rho_{1}\rho_{2}}B_{d_{2}}^{*\rho_{3}}\right.\\ & \left.+\frac{\partial^{3}Z_{ab}}{\partial\varphi_{d_{1}}\partial\varphi_{d_{2}}\partial\varphi_{d_{3}}}B_{d_{1}}^{*\rho_{1}}B_{d_{2}}^{*\rho_{2}}B_{d_{3}}^{*\rho_{3}}\right)C_{(4,0)\rho_{1}\rho_{2}\rho_{3}\mu_{1}\mu_{2}}^{a}C_{(4,1)\mu_{3}\cdots\mu_{8}}^{b}.\end{aligned} \tag{79}$$

Related to the piece of antifield number 4, we have that

$$\begin{aligned}\mathcal{L}_{4}= {} & \lambda\varepsilon^{\mu_{1}\cdots\mu_{8}}Z_{ab}\left(\varphi\right)\eta_{(3,2)}^{*a}C_{(4,3)\mu_{1}\cdots\mu_{8}}^{b}-8C_{(4,2)a}^{*\mu_{1}\cdots\mu_{7}}\partial^{\rho}C_{(4,3)\rho\mu_{1}\cdots\mu_{7}}^{a}\\ & +5C_{(1,2)a}^{*\mu_{1}\cdots\mu_{4}}\left(-\partial^{\rho}C_{(1,3)\rho\mu_{1}\cdots\mu_{4}}^{a}+14\lambda\varepsilon^{\rho_{1}\cdots\rho_{8}}\frac{\partial Z_{bc}}{\partial\varphi_{a}}B_{\rho_{1}\cdots\rho_{4}}^{b}C_{(4,3)\mu_{1}\cdots\mu_{4}\rho_{5}\cdots\rho_{8}}^{c}\right)\\ & +\lambda\varepsilon^{\mu_{1}\cdots\mu_{8}}\left\{\left[\frac{\partial^{2}Z_{ab}}{\partial\varphi_{d_{1}}\partial\varphi_{d_{2}}}\left(4C_{(1,1)d_{1}}^{*\rho_{1}\rho_{2}\rho_{3}}B_{d_{2}}^{*\rho_{4}}+3C_{(1,0)d_{1}}^{*\rho_{1}\rho_{2}}C_{(1,0)d_{2}}^{*\rho_{3}\rho_{4}}\right)\right.\right.\end{aligned}$$

$$+\left(6\frac{\partial^3 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}\partial\varphi_{d_3}}C^{*\rho_1\rho_2}_{(1,0)d_1}+\frac{\partial^4 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_4}}B^{*\rho_1}_{d_1}B^{*\rho_2}_{d_2}\right)B^{*\rho_3}_{d_2}B^{*\rho_4}_{d_3}\Bigg]B^a_{\rho_1\cdots\rho_4}$$
$$+\left[\frac{\partial Z_{ab}}{\partial\varphi_{d_1}}C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}+\left(3\frac{\partial^2 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}}C^{*\rho_1\rho_2}_{(1,0)d_1}+\frac{\partial^3 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}\partial\varphi_{d_3}}B^{*\rho_1}_{d_1}B^{*\rho_2}_{d_2}\right)B^{*\rho_3}_{d_3}\right]A^{*a}_{\rho_1\rho_2\rho_3}$$
$$-\left(\frac{\partial Z_{ab}}{\partial\varphi_{d_1}}C^{*\rho_1\rho_2}_{(1,0)d_1}+\frac{\partial^2 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}}B^{*\rho_1}_{d_1}B^{*\rho_2}_{d_2}\right)\eta^{*a}_{(3,0)\rho_1\rho_2}-\frac{\partial Z_{ab}}{\partial\varphi_{d_1}}B^{*\rho_1}_{d_1}\eta^{*a}_{(3,1)\rho_1}\Bigg\}C^b_{(4,3)\mu_1\cdots\mu_8}$$
$$+5\lambda\varepsilon^{\mu_1\cdots\mu_8}\left[\frac{\partial Z_{ab}}{\partial\varphi_{d_1}}C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}+\frac{\partial^2 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}}\left(4C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}B^{*\rho_4}_{d_2}+3C^{*\rho_1\rho_2}_{(1,0)d_1}C^{*\rho_3\rho_4}_{(1,0)d_2}\right)\right.$$
$$\left.+6\frac{\partial^3 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}\partial\varphi_{d_3}}C^{*\rho_1\rho_2}_{(1,0)d_1}B^{*\rho_3}_{d_2}B^{*\rho_4}_{d_3}+\frac{\partial^4 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_4}}B^{*\rho_1}_{d_1}\cdots B^{*\rho_4}_{d_4}\right]\times$$
$$\times\left(C^a_{(4,0)\rho_1\cdots\rho_4\mu_1}C^b_{(4,2)\mu_2\cdots\mu_8}+\frac{3}{2}C^a_{(4,1)\rho_1\cdots\rho_4\mu_1\mu_2}C^b_{(4,1)\mu_3\cdots\mu_8}\right). \tag{80}$$

Along the same line, we can organize the terms of antifield number 5 and respectively 6 like

$$\mathcal{L}_5=-6C^{*\mu_1\cdots\mu_5}_{(1,3)a}\partial^\rho C^a_{(1,4)\rho\mu_1\cdots\mu_5}+\lambda\varepsilon^{\mu_1\cdots\mu_8}\left\{\frac{\partial Z_{ab}}{\partial\varphi_{d_1}}C^{*\rho_1\cdots\rho_5}_{(1,3)d_1}+5\left[\frac{\partial^2 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}}\left(C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}B^{*\rho_5}_{d_2}\right.\right.\right.$$
$$\left.+2C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5}_{(1,0)d_2}\right)+\frac{\partial^3 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}\partial\varphi_{d_3}}\left(2C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}B^{*\rho_4}_{d_2}+3C^{*\rho_1\rho_2}_{(1,0)d_1}C^{*\rho_3\rho_4}_{(1,0)d_2}\right)B^{*\rho_5}_{d_3}$$
$$\left.\left.+2\frac{\partial^4 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_4}}C^{*\rho_1\rho_2}_{(1,0)d_1}B^{*\rho_3}_{d_2}B^{*\rho_4}_{d_3}B^{*\rho_5}_{d_4}\right]+\frac{\partial^5 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_5}}B^{*\rho_1}_{d_1}\cdots B^{*\rho_5 a}_{d_5(3,1)\rho_1\cdots\rho_5}\right\}\times$$
$$\times\left(-C^a_{(4,0)\rho_1\cdots\rho_5}C^b_{(4,3)\mu_1\cdots\mu_8}+6C^a_{(4,1)\rho_1\cdots\rho_5\mu_1}C^b_{(4,2)\mu_2\cdots\mu_8}\right), \tag{81}$$

$$\mathcal{L}_6=-7C^{*\mu_1\cdots\mu_6}_{(1,4)a}\partial^\rho C^a_{(1,5)\rho\mu_1\cdots\mu_6}+\lambda\varepsilon^{\mu_1\cdots\mu_8}\left\{\frac{\partial Z_{ab}}{\partial\varphi_{d_1}}C^{*\rho_1\cdots\rho_6}_{(1,4)d_1}+\frac{\partial^2 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}}\left(6C^{*\rho_1\cdots\rho_5}_{(1,3)d_1}B^{*\rho_6}_{d_2}\right.\right.$$
$$\left.+15C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}C^{*\rho_5\rho_6}_{(1,0)d_2}+10C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5\rho_6}_{(1,1)d_2}\right)+15\frac{\partial^3 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}\partial\varphi_{d_3}}\left[\left(C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}B^{*\rho_5}_{d_2}\right.\right.$$
$$\left.\left.+4C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5}_{(1,0)d_2}\right)B^{*\rho_6}_{d_3}+C^{*\rho_1\rho_2}_{(1,0)d_1}C^{*\rho_3\rho_4}_{(1,0)d_2}C^{*\rho_5\rho_6}_{(1,0)d_3}\right]+5\frac{\partial^4 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_4}}\left(4C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}B^{*\rho_4}_{d_2}\right.$$
$$\left.+9C^{*\rho_1\rho_2}_{(1,0)d_1}C^{*\rho_3\rho_4}_{(1,0)d_2}\right)B^{*\rho_5}_{d_3}B^{*\rho_6}_{d_4}+15\frac{\partial^5 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_5}}C^{*\rho_1\rho_2}_{(1,0)d_1}B^{*\rho_3}_{d_2}\cdots B^{*\rho_6}_{d_5}$$
$$\left.+\frac{\partial^6 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_6}}B^{*\rho_1}_{d_1}\cdots B^{*\rho_6}_{d_6}\right\}\left(C^a_{(4,1)\rho_1\cdots\rho_6}C^b_{(4,3)\mu_1\cdots\mu_8}+\frac{7}{2}C^a_{(4,2)\rho_1\cdots\rho_6\mu_1}C^b_{(4,2)\mu_2\cdots\mu_8}\right). \tag{82}$$

Finally, the terms of antifield number 7 and respectively 8 present in (75) display the expressions

$$\mathcal{L}_7=-8C^{*\mu_1\cdots\mu_7}_{(1,5)a}\partial^\rho C^a_{(1,6)\rho\mu_1\cdots\mu_7}-\lambda\varepsilon^{\mu_1\cdots\mu_8}\left\{\frac{\partial Z_{ab}}{\partial\varphi_{d_1}}C^{*\rho_1\cdots\rho_7}_{(1,5)d_1}+7\left\{\frac{\partial^2 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}}\left(C^{*\rho_1\cdots\rho_6}_{(1,4)d_1}B^{*\rho_7}_{d_2}\right.\right.\right.$$
$$\left.+3C^{*\rho_1\cdots\rho_5}_{(1,3)d_1}C^{*\rho_6\rho_7}_{(1,0)d_2}+5C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}C^{*\rho_5\rho_6\rho_7}_{(1,1)d_2}\right)+\frac{\partial^3 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}\partial\varphi_{d_3}}\left\{3C^{*\rho_1\cdots\rho_5}_{(1,3)d_1}B^{*\rho_6}_{d_2}B^{*\rho_7}_{d_3}\right.$$
$$\left.\left.+5\left[\left(3C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}C^{*\rho_5\rho_6}_{(1,0)d_2}+2C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5\rho_6}_{(1,1)d_2}\right)B^{*\rho_7}_{d_3}+3C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5}_{(1,0)d_2}C^{*\rho_6\rho_7}_{(1,0)d_3}\right]\right\}$$
$$+5\left\{\frac{\partial^4 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_4}}\left[\left(C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}B^{*\rho_5}_{d_2}+6C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5}_{(1,0)d_2}\right)B^{*\rho_6}_{d_3}+3C^{*\rho_1\rho_2}_{(1,0)d_1}C^{*\rho_3\rho_4}_{(1,0)d_2}C^{*\rho_5\rho_6}_{(1,0)d_3}\right]B^{*\rho_7}_{d_4}\right.$$

$$+\frac{\partial^5 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_5}}\left(C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}B^{*\rho_4}_{d_2}+3C^{*\rho_1\rho_2}_{(1,0)d_1}C^{*\rho_3\rho_4}_{(1,0)d_2}\right)B^{*\rho_5}_{d_3}B^{*\rho_6}_{d_4}B^{*\rho_7}_{d_5}\Bigg\}$$
$$+3\frac{\partial^6 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_6}}C^{*\rho_1\rho_2}_{(1,0)d_1}B^{*\rho_3}_{d_2}\cdots B^{*\rho_7}_{d_6}\Bigg\}+\frac{\partial^7 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_7}}B^{*\rho_1}_{d_1}\cdots B^{*\rho_7}_{d_7}\Bigg\}\times$$
$$\times C^a_{(4,2)\rho_1\cdots\rho_7}C^b_{(4,3)\mu_1\cdots\mu_8}, \tag{83}$$

$$\mathcal{L}_8=\frac{\lambda}{2}\varepsilon^{\mu_1\cdots\mu_8}\left\{\frac{\partial Z_{ab}}{\partial\varphi_{d_1}}C^{*\rho_1\cdots\rho_8}_{(1,6)d_1}+\frac{\partial^2 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}}\left[8C^{*\rho_1\cdots\rho_7}_{(1,5)d_1}B^{*\rho_8}_{d_2}+7\left(4C^{*\rho_1\cdots\rho_6}_{(1,4)d_1}C^{*\rho_7\rho_8}_{(1,0)d_2}\right.\right.\right.$$
$$\left.\left.+8C^{*\rho_1\cdots\rho_5}_{(1,3)d_1}C^{*\rho_6\rho_7\rho_8}_{(1,1)d_2}+5C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}C^{*\rho_5\cdots\rho_8}_{(1,2)d_2}\right)\right]+7\left\{2\frac{\partial^3 Z_{ab}}{\partial\varphi_{d_1}\partial\varphi_{d_2}\partial\varphi_{d_3}}\left[2\left(C^{*\rho_1\cdots\rho_6}_{(1,4)d_1}B^{*\rho_7}_{d_2}\right.\right.\right.$$
$$\left.+6C^{*\rho_1\cdots\rho_5}_{(1,3)d_1}C^{*\rho_6\rho_7}_{(1,0)d_2}+10C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}C^{*\rho_5\rho_6\rho_7}_{(1,1)d_2}\right)B^{*\rho_8}_{d_3}+5\left(3C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}C^{*\rho_5\rho_6}_{(1,0)d_2}\right.$$
$$\left.\left.+4C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5\rho_6}_{(1,1)d_2}\right)C^{*\rho_7\rho_8}_{(1,0)d_3}\right]+\frac{\partial^4 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_4}}\left\{4\left[\left(2C^{*\rho_1\cdots\rho_5}_{(1,3)d_1}B^{*\rho_6}_{d_2}+15C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}C^{*\rho_5\rho_6}_{(1,0)d_2}\right.\right.\right.$$
$$\left.\left.+10C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5\rho_6}_{(1,1)d_2}\right)B^{*\rho_7}_{d_3}+30C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5}_{(1,0)d_2}C^{*\rho_6\rho_7}_{(1,0)d_3}\right]B^{*\rho_8}_{d_4}+15C^{*\rho_1\rho_2}_{(1,0)d_1}C^{*\rho_3\rho_4}_{(1,0)d_2}C^{*\rho_5\rho_6}_{(1,0)d_3}C^{*\rho_7\rho_8}_{(1,0)d_4}\Bigg\}$$
$$+10\frac{\partial^5 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_5}}\left[\left(\left(C^{*\rho_1\cdots\rho_4}_{(1,2)d_1}B^{*\rho_5}_{d_2}+8C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}C^{*\rho_4\rho_5}_{(1,0)d_2}\right)B^{*\rho_6}_{d_3}+6C^{*\rho_1\rho_2}_{(1,0)d_1}C^{*\rho_3\rho_4}_{(1,0)d_2}C^{*\rho_5\rho_6}_{(1,0)d_3}\right]B^{*\rho_7}_{d_4}B^{*\rho_8}_{d_5}\right.$$
$$+2\frac{\partial^6 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_6}}\left(4C^{*\rho_1\rho_2\rho_3}_{(1,1)d_1}B^{*\rho_4}_{d_2}+15C^{*\rho_1\rho_2}_{(1,0)d_1}C^{*\rho_3\rho_4}_{(1,0)d_2}\right)B^{*\rho_5}_{d_3}\cdots B^{*\rho_8}_{d_6}$$
$$+4\frac{\partial^7 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_7}}C^{*\rho_1\rho_2}_{(1,0)d_1}B^{*\rho_3}_{d_2}\cdots B^{*\rho_8}_{d_7}\Bigg\}+\frac{\partial^8 Z_{ab}}{\partial\varphi_{d_1}\cdots\partial\varphi_{d_8}}B^{*\rho_1}_{d_1}\cdots B^{*\rho_8}_{d_8}\Bigg\}C^a_{(4,3)\rho_1\cdots\rho_8}C^b_{(4,3)\mu_1\cdots\mu_8}. \tag{84}$$

With all the above results at hand, in the sequel we address the defining properties of the Lagrangian formulation of the self-interacting $D=8$ BF model behind the deformed solution to the master equation expressed by (74), whose various components introduced in expansion (75) are listed in formulas (76)–(84).

4 Lagrangian formulation of the self-interacting model

Once the deformed solution to the master equation has been completed, (74), from its sectors of fixed antifield number $j=\overline{0,8}$, provided by (76)–(84), we read all the information regarding the gauge structure of the associated self-interacting $D=8$ BF theory.

The piece of antifield number 0 from (75), namely (76), is both antifield- and ghost-independent, so it involves only the BF field spectrum and its space-time derivatives. Moreover, it defines a crucial ingredient of the $D=8$ BF self-coupled model, namely, its Lagrangian action

$$\bar{S}^L\left[\overset{[0]}{\varphi}_a,\overset{[1]}{B}{}^a,\overset{[3]}{A}_a,\overset{[4]}{B}{}^a\right]=\int d^8x\left[B^a_{\mu_1}\partial^{\mu_1}\varphi_a+B^a_{\mu_1\mu_2\mu_3\mu_4}\left(\partial^{[\mu_1}A_a^{\mu_2\mu_3\mu_4]}\right.\right.$$
$$\left.\left.+\frac{\lambda}{2}\varepsilon^{\mu_1\cdots\mu_8}Z_{ab}(\varphi)B^b_{\mu_5\mu_6\mu_7\mu_8}\right)\right]. \tag{85}$$

We notice that the only vertices due to the self-interactions couple the two components of the BF 4-forms from each term via the elements of some symmetric functions $Z_{ab}(\varphi)$

$$\frac{\lambda}{2}\varepsilon^{\mu_1\cdots\mu_8}Z_{ab}(\varphi)B^a_{\mu_1\mu_2\mu_3\mu_4}B^b_{\mu_5\mu_6\mu_7\mu_8},\quad Z_{ab}=Z_{ba} \tag{86}$$

and provides the $D=8$ generalization of the well-known BF self-couplings present in the $D=2$ gravity formulation via topological BF theories [17]

$$D=2:\frac{\lambda}{2}\varepsilon^{\mu_1\mu_2}Z_{ab}\left(\varphi\right)B^a_{\mu_1}B^b_{\mu_2},\quad Z_{ab}=-Z_{ba}, \tag{87}$$

$$Z_{ad}\frac{\partial Z_{bc}}{\partial\varphi_d}+Z_{bd}\frac{\partial Z_{ca}}{\partial\varphi_d}+Z_{cd}\frac{\partial Z_{ab}}{\partial\varphi_d}=0,\quad a,b,c=\overline{1,A}. \tag{88}$$

We mention that the starting free Lagrangian action in $D=2$ has the simplest field spectrum, consisting only in two kinds of BF forms, namely, $\overset{[0]}{\varphi}_a$ and $\overset{[1]}{B}{}^a$, while its Lagrangian density is similar to the first term from (85). Relations (88), obtained as the consistency conditions specific to the two-dimensional case, together with the antisymmetry of the Z's, allow for an interpretation of the functions $Z_{ab}\left(\varphi\right)$ as the components of the (Poisson) two-tensor corresponding to a Poisson manifold, $[\varphi_a,\varphi_b]=Z_{ab}\left(\varphi\right)$, where (88) play the role of the associated Jacobi identities. Here, the vertices (86) still generalize those from the $D=2$ case, (87), but the Z's are symmetric by contrast, so $Z_{ab}\left(\varphi\right)$ no longer have a definite geometric interpretation. The above vertices in $D=8$ can be interpreted as a mass-term for the generalized tensor field $B^{a\mu_1\mu_2\mu_3\mu_4}$.

The stationary surface of the self-interacting BF model (85) is defined by the equations

$$\bar{\Sigma}:\frac{\delta\bar{S}^L}{\delta\Phi^{\alpha_0}}\equiv\left\{\begin{array}{c}\frac{\delta\bar{S}^L}{\delta\varphi_a}=-\partial^\rho B^a_\rho+\frac{\lambda}{2}\varepsilon^{\mu_1\cdots\mu_8}\frac{\partial Z_{bc}}{\partial\varphi_a}B^b_{\mu_1\mu_2\mu_3\mu_4}B^c_{\mu_5\mu_6\mu_7\mu_8}\\ \frac{\delta\bar{S}^L}{\delta A_a^{\mu_1\mu_2\mu_3}}=-4\partial^\rho B^a_{\rho\mu_1\mu_2\mu_3}\\ \frac{\delta\bar{S}^L}{\delta B^a_{\mu_1\mu_2\mu_3\mu_4}}=\partial^{[\mu_1}A_a^{\mu_2\mu_3\mu_4]}+\lambda\varepsilon^{\mu_1\cdots\mu_8}Z_{ab}B^b_{\mu_5\mu_6\mu_7\mu_8}\\ \frac{\delta\bar{S}^L}{\delta B^a_{\mu_1}}=\frac{\delta S^L}{\delta B^a_{\mu_1}}=\partial^{\mu_1}\varphi_a\end{array}\right.\approx 0. \tag{89}$$

Comparing (89) with (3), we observe that the self-interacting BF theory possesses some non-linear field equations with respect to some of the fields, by contrast to their free limit, meanwhile preserving their differential order being equal to one.

From the elements of antifield number 1 in (75), given by (77), we read the deformed set of generating gauge transformations corresponding to the self-coupled action (85) by detaching the antifields and replacing the ghosts with the corresponding gauge parameters from (5)

$$\bar{\delta}_{\Omega^{\alpha_1}}\Phi^{\alpha_0}\equiv\left\{\begin{array}{c}\bar{\delta}_{\Omega^{\alpha_1}}\varphi_a=0\\ \bar{\delta}_{\Omega^{\alpha_1}}A_a^{\mu_1\mu_2\mu_3}=\partial^{[\mu_1}\epsilon^{\mu_2\mu_3]}_{(3,0)a}-\lambda\varepsilon^{\mu_1\cdots\mu_8}Z_{ab}\left(\varphi\right)\xi^b_{(4,0)\mu_4\cdots\mu_8}\\ \bar{\delta}_{\Omega^{\alpha_1}}B^a_{\mu_1\mu_2\mu_3\mu_4}=-5\partial^\rho\xi^a_{(4,0)\rho\mu_1\mu_2\mu_3\mu_4}\\ \bar{\delta}_{\Omega^{\alpha_1}}B^a_{\mu_1}=-2\partial^\rho\xi^a_{(1,0)\rho\mu_1}-5\lambda\varepsilon^{\rho_1\cdots\rho_8}\frac{\partial Z_{bc}}{\partial\varphi_a}B^b_{\rho_1\cdots\rho_4}\xi^c_{(4,0)\mu_1\rho_5\cdots\rho_8}\end{array}\right.. \tag{90}$$

We observe that all the BF scalar fields remain gauge-invariant, like in the free limit, while the gauge transformations of the 4-forms are not deformed by the added self-interactions.

The components of antifield number strictly greater that 1 from (75), collected in formulas (78)–(84), provide all the information on the deformed gauge algebra and reducibility of the generating set (90) of gauge transformations.

Regarding the deformed guge algebra, the concrete expressions of the (non-trivial) commutators among the gauge transformations (90), $\left[\bar{\delta}_{\Omega^{(1)\alpha_1}},\bar{\delta}_{\Omega^{(2)\alpha_1}}\right]\Phi^{\alpha_0}$, where Φ^{α_0} are introduced in (2) and $\Omega^{(1)\alpha_1}$ and $\Omega^{(2)\alpha_1}$ are two different sets of gauge parameters as in (5), read

$$\left[\bar{\delta}_{\Omega^{(1)\alpha_1}},\bar{\delta}_{\Omega^{(2)\alpha_1}}\right]\Phi^{\alpha_0}\Big|_{\text{nontriv}}\equiv$$

$$\equiv \left\{ \begin{array}{c} \left[\bar{\delta}_{\Omega^{(1)\alpha_1}}, \bar{\delta}_{\Omega^{(2)\alpha_1}}\right] A_a^{\mu_1\mu_2\mu_3} = 0 \\ \left[\bar{\delta}_{\Omega^{(1)\alpha_1}}, \bar{\delta}_{\Omega^{(2)\alpha_1}}\right] B^a_{\mu_1\mu_2\mu_3\mu_4} = 0 \\ \left[\bar{\delta}_{\Omega^{(1)\alpha_1}}, \bar{\delta}_{\Omega^{(2)\alpha_1}}\right] B^a_{\mu_1} = -2\partial^\rho \bar{\xi}^a_{(1,0)\rho\mu_1} + \lambda M^{ab}_{\mu_1\rho_1} \frac{\delta \bar{S}^L}{\delta B^b_{\rho_1}} \approx -2\partial^\rho \bar{\xi}^a_{(1,0)\rho\mu_1} \end{array} \right. , \tag{91}$$

where

$$\bar{\Omega}^{\alpha_1} \equiv \left\{ \bar{\epsilon}^{\mu_1\mu_2}_{(3,0)a}, \bar{\xi}^a_{(4,0)\mu_1\cdots\mu_5}, \bar{\xi}^a_{(1,0)\mu_1\mu_2} \right\}, \tag{92}$$

$$\bar{\epsilon}^{\mu_1\mu_2}_{(3,0)a} = 0, \quad \bar{\xi}^a_{(4,0)\mu_1\cdots\mu_5} = 0, \tag{93}$$

$$\bar{\xi}^a_{(1,0)\mu_1\mu_2} = -200\lambda\varepsilon_{\mu_1\mu_2\nu_1\nu_2\nu_3\nu_4\nu_5\nu_6} \frac{\partial Z_{bc}}{\partial\varphi_a} \tilde{\xi}^{(1)b\nu_1\nu_2\nu_3}_{(4,0)} \tilde{\xi}^{(2)c\nu_4\nu_5\nu_6}_{(4,0)}, \tag{94}$$

$$M^{ab}_{\mu_1\rho_1} = 400\varepsilon_{\mu_1\rho_1\nu_1\nu_2\nu_3\nu_4\nu_5\nu_6} \frac{\partial^2 Z_{cd}}{\partial\varphi_a\partial\varphi_b} \tilde{\xi}^{(1)c\nu_1\nu_2\nu_3}_{(4,0)} \tilde{\xi}^{(2)d\nu_4\nu_5\nu_6}_{(4,0)} = -M^{ba}_{\rho_1\mu_1}. \tag{95}$$

The notations $\left\{ \tilde{\xi}^{(i)a\nu_1\nu_2\nu_3}_{(4,0)} \right\}_{i=1,2}$ from (94) and (95) respectively denote the Hodge duals of the gauge parameters $\left\{ \xi^{(i)a}_{(4,0)\mu_1\cdots\mu_5} \right\}_{i=1,2}$ from the two different sets $\Omega^{(1)\alpha_1}$ and $\Omega^{(2)\alpha_1}$ taken at the evaluation of the commutators among the deformed gauge transformations

$$\tilde{\xi}^{(i)a\nu_1\nu_2\nu_3}_{(4,0)} \equiv \frac{1}{5!}\varepsilon^{\nu_1\cdots\nu_8} \xi^{(i)a}_{(4,0)\nu_4\cdots\nu_8}. \tag{96}$$

Thus, we conclude that the deformed gauge algebra corresponding to the self-interacting BF model in $D = 8$ is now open, in contrast to the initial, Abelian one, since the commutators among the gauge transformations of the 1-forms $\overset{[1]}{B}{}^a$ only close on-shell.

Next, we analyze the main ingredients connected to the reducibility of the deformed gauge transformations (90) via the terms linear in the ghosts of pure ghost number greater or equal to 2 present in (78)–(84). Consequently, the deformed non-vanishing reducibility functions of orders between 1 and 6 follow from the transformations

$$\Omega^{\alpha_1}(\Omega^{\alpha_2}) \equiv \left\{ \begin{array}{c} \epsilon^{\mu_1\mu_2}_{(3,0)a}(\Omega^{\alpha_2}) = \partial^{[\mu_1}\epsilon^{\mu_2]}_{(3,1)a} - \lambda\varepsilon^{\mu_1\cdots\mu_8} Z_{ab}(\varphi)\, \xi^b_{(4,1)\mu_3\cdots\mu_8} \\ \xi^a_{(4,0)\mu_1\cdots\mu_5}(\Omega^{\alpha_2}) = -6\partial^\rho \xi^a_{(4,1)\rho\mu_1\cdots\mu_5} \\ \xi^a_{(1,0)\mu_1\mu_2}(\Omega^{\alpha_2}) = -3\partial^\rho \xi^a_{(1,1)\rho\mu_1\mu_2} \\ +\lambda C^2_6 \varepsilon^{\rho_1\cdots\rho_8} \frac{\partial Z_{bc}}{\partial\varphi_a} B^b_{\rho_1\cdots\rho_4} \xi^c_{(4,1)\mu_1\mu_2\rho_5\cdots\rho_8} \end{array} \right. , \tag{97}$$

$$\Omega^{\alpha_2}(\Omega^{\alpha_3}) \equiv \left\{ \begin{array}{c} \epsilon^{\mu_1}_{(3,1)a}(\Omega^{\alpha_3}) = \partial^{\mu_1}\epsilon_{(3,2)a} + \lambda\varepsilon^{\mu_1\cdots\mu_8} Z_{ab}(\varphi)\, \xi^b_{(4,2)\mu_2\cdots\mu_8} \\ \xi^a_{(4,1)\mu_1\cdots\mu_6}(\Omega^{\alpha_3}) = -7\partial^\rho \xi^a_{(4,2)\rho\mu_1\cdots\mu_6} \\ \xi^a_{(1,1)\mu_1\mu_2\mu_3}(\Omega^{\alpha_3}) = -4\partial^\rho \xi^a_{(1,2)\rho\mu_1\mu_2\mu_3} \\ -\lambda C^3_7 \varepsilon^{\rho_1\cdots\rho_8} \frac{\partial Z_{bc}}{\partial\varphi_a} B^b_{\rho_1\cdots\rho_4} \xi^c_{(4,2)\mu_1\mu_2\mu_3\rho_5\cdots\rho_8} \end{array} \right. , \tag{98}$$

$$\Omega^{\alpha_3}(\Omega^{\alpha_4}) \equiv \left\{ \begin{array}{c} \epsilon_{(3,2)a}(\Omega^{\alpha_4}) = \lambda\varepsilon^{\mu_1\cdots\mu_8} Z_{ab}(\varphi)\, \xi^b_{(4,3)\mu_1\cdots\mu_8} \\ \xi^a_{(4,2)\mu_1\cdots\mu_7}(\Omega^{\alpha_4}) = -8\partial^\rho \xi^a_{(4,3)\rho\mu_1\cdots\mu_7} \\ \xi^a_{(1,2)\mu_1\cdots\mu_4}(\Omega^{\alpha_4}) = -5\partial^\rho \xi^a_{(1,3)\rho\mu_1\cdots\mu_4} \\ +\lambda C^4_8 \varepsilon^{\rho_1\cdots\rho_8} \frac{\partial Z_{bc}}{\partial\varphi_a} B^b_{\rho_1\cdots\rho_4} \xi^c_{(4,3)\mu_1\cdots\mu_4\rho_5\cdots\rho_8} \end{array} \right. , \tag{99}$$

$$\Omega^{\alpha_4}(\Omega^{\alpha_5})\big|_{\text{nontriv}} \equiv \xi^a_{(1,3)\mu_1\cdots\mu_5}(\Omega^{\alpha_5}) = -6\partial^\rho \xi^a_{(1,4)\rho\mu_1\cdots\mu_5}, \tag{100}$$

$$\Omega^{\alpha_5}(\Omega^{\alpha_6}) \equiv \xi^a_{(1,4)\mu_1\cdots\mu_6}(\Omega^{\alpha_6}) = -7\partial^\rho \xi^a_{(1,5)\rho\mu_1\cdots\mu_6}, \tag{101}$$

$$\Omega^{\alpha_6}(\Omega^{\alpha_7}) \equiv \xi^a_{(1,5)\mu_1\cdots\mu_7}(\Omega^{\alpha_7}) = -8\partial^\rho \xi^a_{(1,6)\rho\mu_1\cdots\mu_7}. \tag{102}$$

Inspecting the previous relations, we notice that the reducibility functions corresponding to the gauge transformations of the 4-forms $\overset{[4]}{B}{}^a$ are not affected by the deformation procedure, while the others are modified only at the first three stages by terms of order one in the coupling constant. It is also interesting to observe that the partial reducibility order of the gauge transformations of the 3-forms $\overset{[3]}{A}_a$ is lifted by one unit, from two to three, while the overall reducibility order of the self-interacting $D=8$ BF model is of course preserved with respect to the free limit. Moreover, some of the associated reducibility relations of order ranging between one and three hold on-shell, by contrast to what happens in the free limit

$$\bar{\delta}_{\Omega^{\alpha_1}(\Omega^{\alpha_2})}\Phi^{\alpha_0}\Big|_{\text{nontriv}} \equiv \left\{ \begin{array}{c} \bar{\delta}_{\Omega^{\alpha_1}(\Omega^{\alpha_2})}A_a^{\mu_1\mu_2\mu_3} = \lambda\bar{M}_{a\rho}^{\mu_1\mu_2\mu_3 b}\frac{\delta\bar{S}^L}{\delta B_\rho^b} \approx 0 \\ \bar{\delta}_{\Omega^{\alpha_1}(\Omega^{\alpha_2})}B^a_{\mu_1\mu_2\mu_3\mu_4} = 0 \\ \bar{\delta}_{\Omega^{\alpha_1}(\Omega^{\alpha_2})}B^a_\mu = \lambda\bar{M}^{ab}_{\mu\rho}\frac{\delta\bar{S}^L}{\delta B_\rho^b} + \lambda\bar{M}'^{a\rho_1\rho_2\rho_3}_{\mu b}\frac{\delta\bar{S}^L}{\delta A_b^{\rho_1\rho_2\rho_3}} \approx 0 \end{array} \right. , \tag{103}$$

$$\Omega^{\alpha_1}\left(\Omega^{\alpha_2}\left(\Omega^{\alpha_3}\right)\right) \equiv \left\{ \begin{array}{c} \epsilon^{\mu_1\mu_2}_{(3,0)a}\left(\Omega^{\alpha_2}\left(\Omega^{\alpha_3}\right)\right) = \bar{M}^{\mu_1\mu_2 b}_{a\rho}\frac{\delta\bar{S}^L}{\delta B^b_\rho} \approx 0 \approx 0 \\ \xi^a_{(4,0)\mu_1\cdots\mu_5}\left(\Omega^{\alpha_2}\left(\Omega^{\alpha_3}\right)\right) = 0 \\ \xi^a_{(1,0)\mu_1\mu_2}\left(\Omega^{\alpha_2}\left(\Omega^{\alpha_3}\right)\right) = \lambda\bar{M}^{ab}_{\mu_1\mu_2\rho}\frac{\delta\bar{S}^L}{\delta B^b_\rho} \\ +\lambda\bar{M}'^{a\rho_1\rho_2\rho_3}_{\mu_1\mu_2 b}\frac{\delta\bar{S}^L}{\delta A_b^{\rho_1\rho_2\rho_3}} \approx 0 \end{array} \right. , \tag{104}$$

$$\Omega^{\alpha_2}\left(\Omega^{\alpha_3}\left(\Omega^{\alpha_4}\right)\right) \equiv \left\{ \begin{array}{c} \epsilon^{\mu_1}_{(3,1)a}\left(\Omega^{\alpha_3}\left(\Omega^{\alpha_4}\right)\right) = \bar{M}^{\mu_1 b}_{a\rho}\frac{\delta\bar{S}^L}{\delta B^b_\rho} \approx 0 \\ \xi^a_{(4,1)\mu_1\cdots\mu_6}\left(\Omega^{\alpha_3}\left(\Omega^{\alpha_4}\right)\right) = 0 \\ \xi^a_{(1,1)\mu_1\mu_2\mu_3}\left(\Omega^{\alpha_3}\left(\Omega^{\alpha_4}\right)\right) = \lambda\bar{M}^{ab}_{\mu_1\mu_2\mu_3\rho}\frac{\delta\bar{S}^L}{\delta B^b_\rho} \\ +\lambda\bar{M}'^{a\rho_1\rho_2\rho_3}_{\mu_1\mu_2\mu_3 b}\frac{\delta\bar{S}^L}{\delta A_b^{\rho_1\rho_2\rho_3}} \approx 0 \end{array} \right. . \tag{105}$$

The various coefficients implied in the previous formulas and their antisymmetry properties are

$$\left\{ \begin{array}{c} \bar{M}^{\mu_1\mu_2\mu_3 b}_{a\rho} = -C^1_6\varepsilon^{\mu_1\mu_2\mu_3\mu_4\cdots\mu_8}\frac{\partial Z_{ac}}{\partial\varphi_b}\xi^c_{(4,1)\rho\mu_4\cdots\mu_8} \\ \bar{M}^{ab}_{\mu\rho} = 2C^2_6\varepsilon^{\rho_1\cdots\rho_8}\frac{\partial^2 Z_{cd}}{\partial\varphi_a\partial\varphi_b}B^c_{\rho_1\cdots\rho_4}\xi^d_{(4,1)\rho_5\cdots\rho_8\mu\rho} \\ \bar{M}'^{a\rho_1\rho_2\rho_3}_{\mu b} = C^1_6\frac{\partial Z_{bc}}{\partial\varphi_a}\varepsilon^{\rho_1\rho_2\rho_3\rho_4\cdots\rho_8}\xi^c_{(4,1)\mu\rho_4\cdots\rho_8} \end{array} \right. , \quad \left\{ \begin{array}{c} \bar{M}'^{b\mu_1\mu_2\mu_3}_{\mu a} = -\bar{M}^{\mu_1\mu_2\mu_3 b}_{a\mu} \\ \bar{M}^{ab}_{\mu\rho} = -\bar{M}^{ba}_{\rho\mu} \end{array} \right. , \tag{106}$$

$$\left\{ \begin{array}{c} \bar{M}^{\mu_1\mu_2 b}_{a\rho} = -C^1_7\varepsilon^{\mu_1\mu_2\rho_1\cdots\rho_6}\frac{\partial Z_{ac}}{\partial\varphi_b}\xi^c_{(4,2)\rho_1\cdots\rho_6\rho} \\ \bar{M}^{ab}_{\mu_1\mu_2\rho} = 3C^3_7\varepsilon^{\rho_1\cdots\rho_8}\frac{\partial^2 Z_{cd}}{\partial\varphi_a\partial\varphi_b}B^c_{\rho_1\cdots\rho_4}\xi^d_{(4,2)\rho_5\cdots\rho_8\mu_1\mu_2\rho} \\ \bar{M}'^{a\rho_1\rho_2\rho_3}_{\mu_1\mu_2 b} = C^2_7\frac{\partial Z_{bc}}{\partial\varphi_a}\varepsilon^{\rho_1\rho_2\rho_3\rho_4\cdots\rho_8}\xi^c_{(4,2)\mu_1\mu_2\rho_4\cdots\rho_8} \end{array} \right. , \tag{107}$$

$$\left\{ \begin{array}{c} \bar{M}^{\mu_1 b}_{a\rho} = -C^1_8\varepsilon^{\mu_1\rho_1\cdots\rho_7}\frac{\partial Z_{ac}}{\partial\varphi_b}\xi^c_{(4,3)\rho_1\cdots\rho_7\rho} \\ \bar{M}^{ab}_{\mu_1\mu_2\mu_3\rho} = 4C^4_8\varepsilon^{\rho_1\cdots\rho_8}\frac{\partial^2 Z_{cd}}{\partial\varphi_a\partial\varphi_b}B^c_{\rho_1\cdots\rho_4}\xi^d_{(4,3)\rho_5\cdots\rho_8\mu_1\mu_2\mu_3\rho} \\ \bar{M}'^{a\rho_1\rho_2\rho_3}_{\mu_1\mu_2\mu_3 b} = C^3_8\frac{\partial Z_{bc}}{\partial\varphi_a}\varepsilon^{\rho_1\rho_2\rho_3\rho_4\cdots\rho_8}\xi^c_{(4,3)\mu_1\mu_2\mu_3\rho_4\cdots\rho_8} \end{array} \right. . \tag{108}$$

All the higher-order reducibility relations hold off-shell and actually coincide with the initial ones due to the fact that transformations (100), (101), and (102) are nothing but (13), (15), and (17) respectively, so all the reducibility functions of order four and higher are not affected by the deformation procedure.

5 Conclusions

The main conclusion of this work is that there exist consistent, non-trivial self-interactions that can be added to a special collection of free topological BF models in $D = 8$ space-time dimensions, whose field spectrum comprises four sets of form fields, of form degrees 0, 1, 3, and 4. The couplings are deduced within the cohomological framework of the antifield-BRST deformation method and in the presence of several usual selection rules employed in gauge field theory, namely, analyticity in the coupling constant, space-time locality, Lorentz covariance, Poincaré invariance, and conservation of the differential order of each interacting field equation with respect to its free limit.

The deformed solution to the classical master equation stops at order one in the coupling constant, comprises components of antifield number valued between 0 and 8, and is parameterized by a set of symmetric functions depending on the undifferentiated BF 0-forms, $\{Z_{ab}(\varphi)\}$. The self-coupled Lagrangian density adds to the free Lagrangian some vertices quadratic in the 4-forms $\overset{[4]}{B}{}^a$ and having $Z_{ab}(\varphi)$ as background, which generalize the well-known vertices present in the BF formulation of $D = 2$ gravity. Still, the similar, generalized vertices are different and less restricted here, since the Z's are now symmetric and otherwise arbitrary, while the similar functions in $D = 2$ are antisymmetric and, in addition, satisfy $D = 2$ Jacobi identities corresponding to a Poisson two-tensor of a certain Poisson manifold. The structure of the deformed solution to the classical master equation emphasizes a self-coupled $D = 8$ topological BF theory with several, new features compared to the starting, free limit: some of the gauge transformations of the BF-forms of strictly positive form degrees are modified, the associated gauge algebra becomes open, in contrast to the original, Abelian one, and some of the reducibility functions and relations are deformed, some of the latter holding on-shell, in opposition to the original ones, which take place only off-shell.

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