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Some aspects of the solutions of a Navier Stokes equation for a charged particle in a magnetic field

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Abstract

In this paper we have calculated some coordinates and velocities of a charged fluid moving through a rectangular hole in a time dependent magnetic field.

Different expression for the magnetic field and initial conditions are used.

1 Introduction

In this paper we study a particular case of a charged fluid that moves in a specific region under the action of the Lorentz force (without taking into consideration of the electric field). We calculate the time-dependence of the spatial coordinates and the corresponding velocities. The magnetic field is considered to depend only on time and is oriented along the Ox axis. The fluid is incompressible i.e. with a divergenceless velocity. The gravitational effects are neglected and this study is based on [1] and other papers related to the study of the mahnetorheological fluids, e.g. [2]-[4].

2 Model equation

The equation of Navier Stokes used here is [1]:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\,\mathbf{v} = -\frac{\boldsymbol{\nabla} p}{\rho} + \nu \Delta \mathbf{v} + \mathbf{f} \tag{1}$$

where

$$\mathbf{f} = q/m\,(\mathbf{v} \times \mathbf{B}) \tag{2}$$

is the volumetric Lorentz force. We neglect the spatial dependence of the velocity components, the viscosity ν and we also consider a mass equal to unity. Because the magnetic field is oriented along the Ox axis, the components of the Lorentz force are:

$$f_x = 0, \quad f_y = qB_0 v_z, \quad f_z = -qB_0 v_y \tag{3}$$

where the general expression of magnetic field considered in this paper is

$$\mathbf{B} = B_0\,(t)\,\mathbf{i} \tag{4}$$

where $B_0(t) = B_0 t^\alpha$, with α a variable exponent. We consider a quadratic spatial dependence of the pressure p of the following form

$$p(y,z) = p_0\left(y^2 + z^2\right) \tag{5}$$

In this case, from equation (1) in the direction Ox the velocity is constant and we will no longer refer to. We also consider that the velocities depend only on time. The remaining two equations are

$$\frac{dv_y}{dt} = -\frac{\nabla_y p}{\rho} + f_x \equiv -\frac{\nabla_y p}{\rho} + qB_0(t)\, v_z$$

and

$$\frac{dv_z}{dt} = -\frac{\nabla_z p}{\rho} + f_z \equiv -\frac{\nabla_z p}{\rho} - qB_0(t)\, v_y \tag{6}$$

The system (6) is the main ingredient of our analysis and the solutions are the time-dependent velocities, $v_z = v_z(t)$ and $v_y = v_y(t)$.

We introduce the following dimensionless variables, T, Y, Z, V_y, V_z as:

$$T = \frac{t}{t_0},\; Y = \frac{y}{l_0},\; Z = \frac{z}{l_0},\; V_y = \frac{v_y}{v_0},\; V_z = \frac{v_z}{v_0} \tag{7}$$

where t_0, l_0, v_0 are typical values specific to the fluid. Introducing these variables we obtain the system

$$\frac{dV_y}{dT} = -K_1 Y + K_2 V_z \tag{8}$$

$$\frac{dV_z}{dT} = -K_1 Z + K_2 V_y \tag{9}$$

where the following parameters are defined

$$K_1 = \frac{2 t_0 l_0 p_0}{\rho v_0}$$

and

$$K_2(T) = q t_0 B_0(t_0 T) \equiv q t_0^{\alpha+1} B_0 T^\alpha \equiv K_{20} T^\alpha$$

If we choose $B_0(t) = B_0 t^\alpha$,the dimensionless magnetic field is $B_0(t_0 T) = B_0(t_0 T)^\alpha \equiv B_0 t_0^\alpha T^\alpha$.

3 Comments

In this section we used some solutions of the system (8-9) in order to represent the time dependence of the coordinates and of the velocities. These pictures are obtained for different values of the power α in the relation for the magnetic field and also different initial conditions for the spatial coordinates. In figure (1), where the parameters are $K_1 = 0.05$ and $K_{20} = 0.1$ it is obviously that the coordinates increases in time more rapidly if the exponent α also increases. The same behaviour for the velocities is observed. The initial conditions used here are:

$$y_0 = 0.01, z_0 = 0.001, v_{y0} = 0.01, v_{z0} = 0.001$$

In figure (2) we only changed the initial conditions for the spatial coordinates, i.e. $y_0 = 0.01, z_0 = 0.01$. Only small chaanges in the behaviour of the coordinates and the velocities

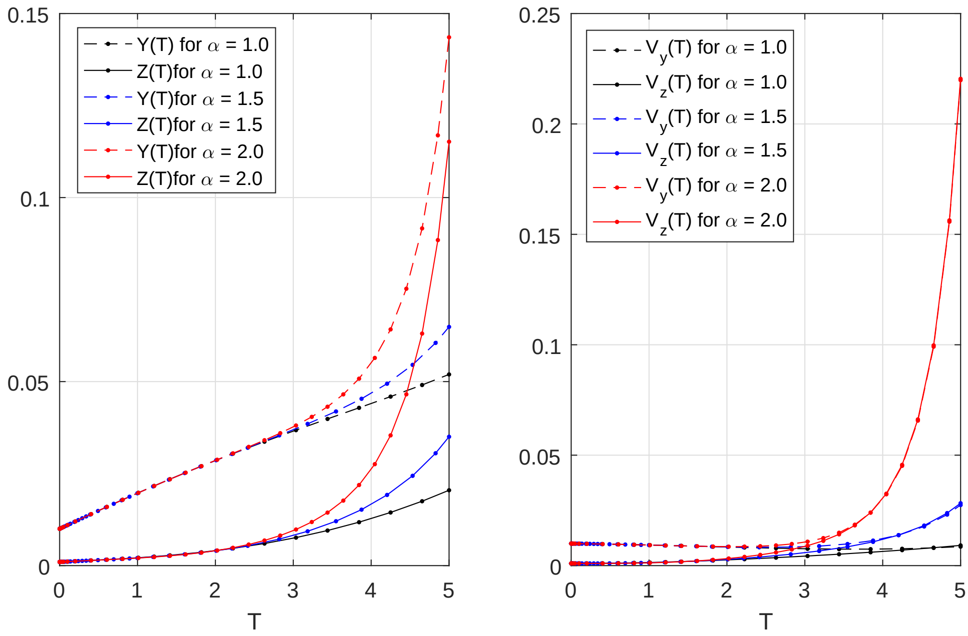

Figure 1: The solution of the system 8-9 for the following numerical parameters: $K_1 = 0.05$, $K_{20} = 0.1$

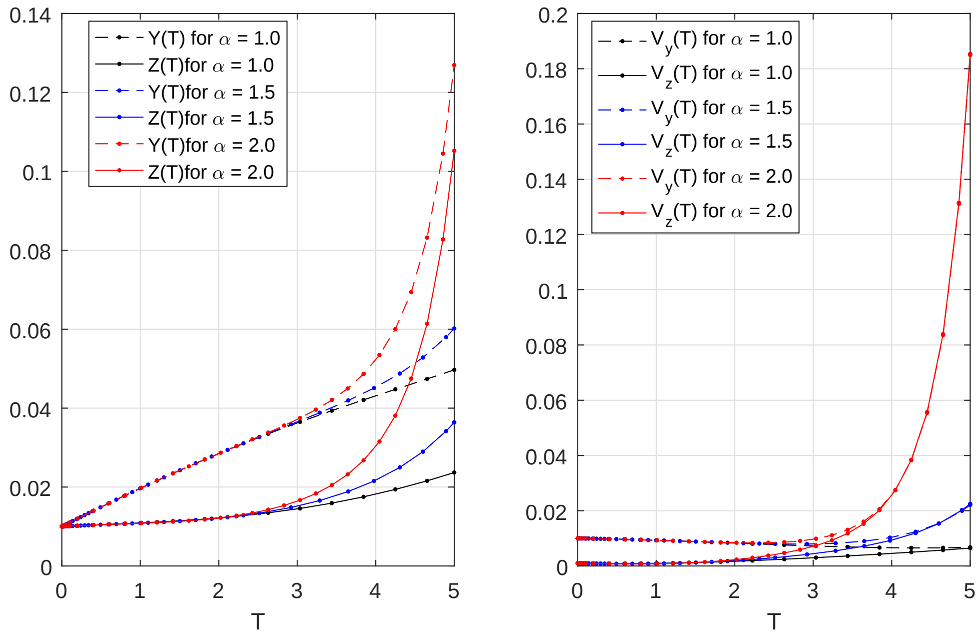

Figure 2: Idem like in figure 1, with other initial conditions ($y_0 = 0.01, z_0 = 0.01$)

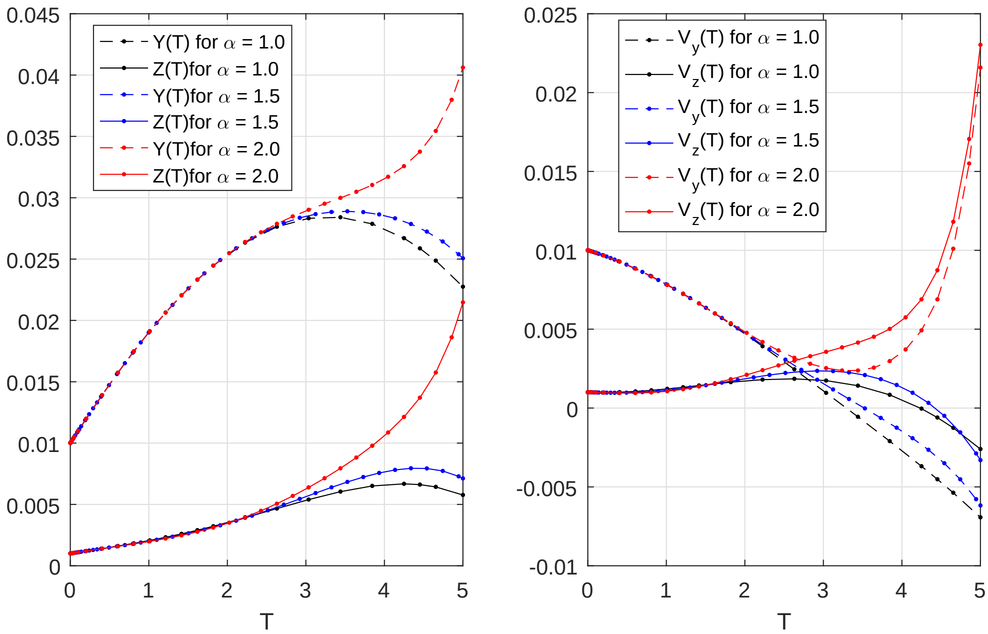

Figure 3: Idem like in figure 1, with $K_1 = 0.15$.

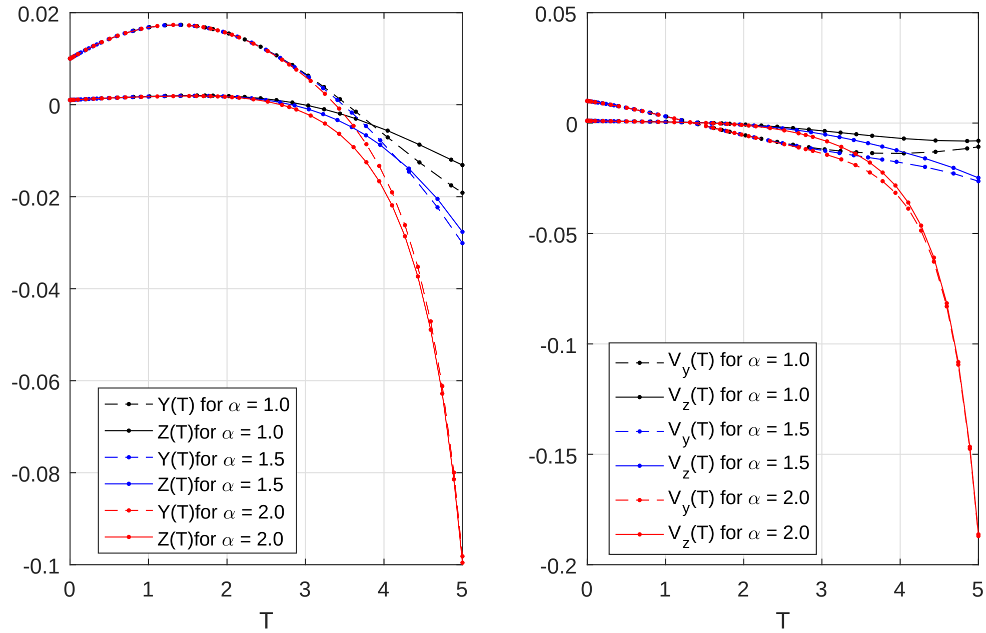

Figure 4: Idem like in figure 1, with $K_1 = 0.5$.

are observed. In figure (3) we only changed the parameter $K_1 = 0.15$ and the behaviour becomes different with a sort of maxima for $\alpha = 1$ and $\alpha = 1.5$ and increases for $\alpha = 2$. In figure (4) for $K_1 = 0.5$ the behaviour is practically the same with the former one except for a rapid decrease for $\alpha = 2$. The initial conditions are chosen considering a rectangular hole. In all figures (1) and (4) we represented the same quantities for different values of the initial conditions.

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