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SmeftFR – Automatising calculations in the SM EFT*

Lampros Trifyllis†

Department of Physics, Division of Theoretical Physics,
University of Ioannina, GR 45110 Ioannina, Greece

Abstract

Under the assumption that the Standard Model (SM) is an effective field theory (EFT) of a more complete high-energy theory, the SM Lagrangian is augmented with non-renormalisable operators encoding the remnants of the UV completion. We discuss the need of using computer software to perform calculations in the SM EFT due to the high complexity of the model, and we present the Mathematica package `SmeftFR` which is used to generate the Feynman rules for the SM EFT in linear R_ξ-gauges with dimension 6 operators and produce outputs to be used with other computer programs for calculations in particle physics.

1 Introduction

There are two general approaches in the search for physics beyond the Standard Model (SM) of elementary particle physics. One is to search directly for new particles and forces and the other is to look for deviations from the SM predictions in the experimental data. Currently no new particles have been discovered, and the SM predictions are in good agreement with the experimental data, which creates the need of high-precision computations in a UV model independent framework. The use of the Effective Field Theory (EFT) approach [1] seems to fit perfectly in this situation, since we can use EFT to systematically parameterise corrections to the theoretical SM predictions.

The EFT description of the SM, abbreviated as SMEFT, extends the (renormalisable) SM Lagrangian using a bottom-up approach, i.e. by adding all possible independent and gauge invariant operators with dimension greater than 4 (effective operators). These operators are constructed from the SM fields and have to respect the SM gauge symmetry. The most general SMEFT Lagrangian can be schematically presented as

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{p=1}^{\infty} \sum_{i} \frac{C_{p,i}}{\Lambda^p} Q_i^{(4+p)}, \tag{1}$$

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†e-mail: `l.trifyllis@uoi.gr`

where $\mathcal{L}_{\rm SM}$ is the renormalisable SM Lagrangian, Λ is the typical UV scale of the new physics and $C_{p,i}$ are the dimensionless Wilson coefficients multiplying the gauge invariant effective operators of mass dimension $(4+p)$, $Q_i^{(4+p)}$. For practical computations the EFT series has to be truncated at some dimension d and, for most applications, it is sufficient to consider the case $d = 6$.

SMEFT is, by construction, a highly complicated model. While only 1 operator class exists at dimension 5, at dimension 6 there are 59 baryon-number conserving and 4 baryon-number violating independent operator classes [2, 3]. Taking into account fermion generations, flavour structure and Hermitian conjugated operators, there are in total 2499 independent free parameters. Due to the large number and complicated structure of the new terms in the Lagrangian, theoretical calculations of physical processes within the SMEFT can be very challenging — for some examples of analytical computations at NLO in SMEFT see [4, 5] and references therein. Therefore, it is evident that we need to develop methods and technical tools in order to promote the automatisation of the various steps in a calculation in SMEFT.

The classification of all independent $d \leq 6$ operators in the SMEFT was given in ref. [3] and is referred to as the "Warsaw basis". In table 1 we present, in the Warsaw basis, all the $d = 6$ operator classes other than the 4-fermion ones. The quantisation of the dimension 6 SMEFT in the Warsaw basis[1] and the production of the complete set of Feynman rules after spontaneous symmetry breaking was presented for the first time in ref. [7]. There, an initial version of the `SmeftFR` code was briefly introduced and was successfully used to produce all relevant Feynman vertices.

2 The `SmeftFR` package

The package `SmeftFR` [8] is a Mathematica package designed to generate the Feynman rules for the SMEFT, consistently truncated at dimension 6. The complete set of Feynman rules is generated using the Mathematica package `FeynRules` [9], upon which the `SmeftFR` code is written as an overlay. The Feynman rules are directly exported in the physical (mass eigenstates) basis for all fields. For maximum utility there are many user options provided. The options allow the user to include only a subset of SMEFT operators or the complete set, to choose between unitary gauge or general R_ξ-gauges for the gauge fixing of the model, to initialise the numerical values for the Wilson coefficients using WCxf format [10], to treat the neutrino fields as massless Weyl or (in the case of non-vanishing dimension 5 operator) as massive Dirac fields, etc.

As already mentioned in the introduction, the ultimate goal is to provide tools capable of partly (or even completely) automatising calculations in the SMEFT framework. There are various approaches when performing theoretical calculations, for example one can choose between handmade or (semi-)automatised calculations, numerical or analytical calculations etc, depending upon the desired outcome and personal preference. It would be therefore very convenient to be able to export the output of `SmeftFR` in different formats. That way, we would end up with a divergent arsenal of tools in our disposal.

[1] For the gauge fixing of SMEFT to all orders in the EFT expansion in linear R_ξ-gauges see ref. [6].

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^* \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi \widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi \widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$i(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi \widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Table 1: Dimension 6 effective operators other than the 4-fermion ones in the Warsaw basis [3].

Thankfully, there is already a large number of computer tools to achieve this goal: the mass basis Lagrangian derived with `SmeftFR` can be exported in various formats supported by `FeynRules`, such as `UFO` [11] and `FeynArts` [12], or even in plain LaTeX, using the dedicated generator provided with the `SmeftFR` code.

Having given a synopsis of the basic usage and scope of `SmeftFR`, let us explain in this paragraph the structure and outputs of the package in more detail. The complete set of Feynman rules for the chosen (sub)set of SMEFT operators and gauge fixing is calculated in `FeynRules` format. The physical fields are rotated in the mass basis and the Goldstone and ghost fields are canonically normalised. In each step of the calculation, expressions for interaction vertices are analytically expanded in powers of inverse Λ, with all terms of dimension higher than 6 consistently truncated. The initialisation of numerical values of $d = 6$ Wilson coefficients used by `SmeftFR` is interfaced to WCxf format [10]. After generating them, the Feynman rules can be exported in other formats, for instance:

- `UFO` [11] format, which is then importable to Monte Carlo generators (e.g. `Sherpa` [13], `MadGraph5_aMC@NLO` [14], `CalcHEP` [15], `Whizard` [16]),
- `FeynArts` [12] format, which generates the relevant Feynman diagrams for a process and can be used as an input for tree- and loop-level amplitude calculators, like `FeynCalc` [17] and `FormCalc` [18],

or any other output types supported by `FeynRules`. A dedicated LaTeX generator allows to print the Feynman rules in human-readable form, best suited for handmade calculations and easy reference. The structure and possible outputs of the `SmeftFR` code are summarised in fig. 1.

The most up to date version of SmeftFR and the complete manual can be downloaded from the website www.fuw.edu.pl/smeft.

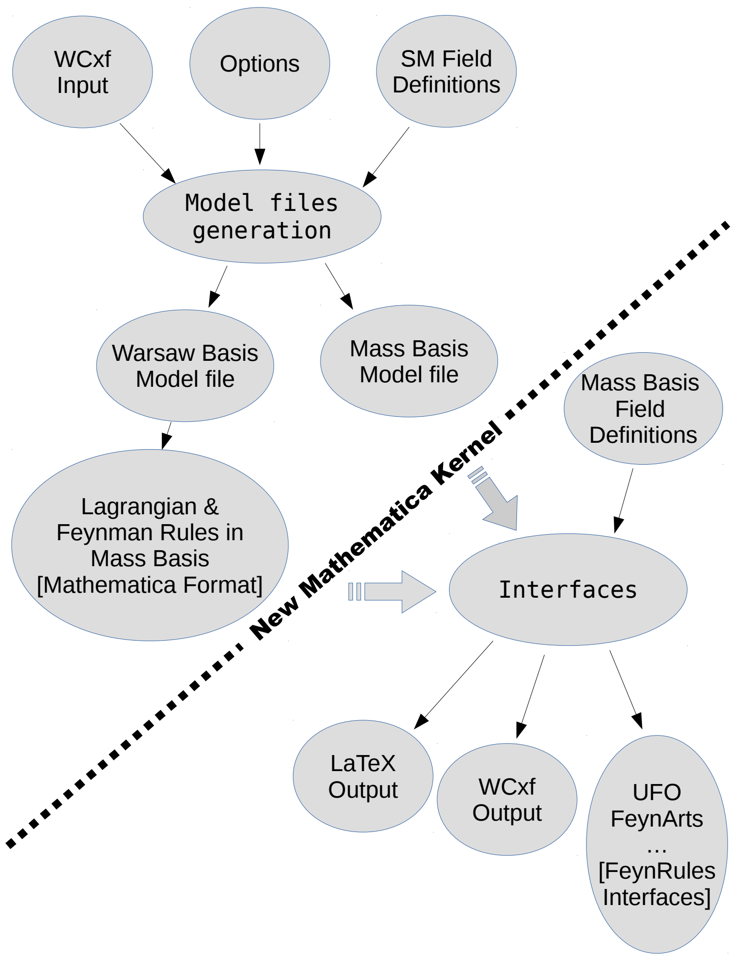

Figure 1: A flowchart depicting the code structure and outputs of the SmeftFR package [8].

3 A simple example: W scattering in the high-energy limit

To conclude the presentation of SmeftFR, in this section we briefly present the results for the elastic W^+W^- scattering in the CP conserving $d = 6$ SMEFT at leading order in the $\hbar$-expansion and in the high-energy limit, and explain how one could reproduce the results by utilising the SmeftFR package. On each amplitude we denote with a subscript 0 the longitudinal and with $\pm$ the transverse degrees of freedom for the gauge boson polarisations. In our notation θ is the scattering angle, s is the square of the centre of mass energy, λ is the Higgs quartic coupling and $\bar{g}$ and $\bar{g}'$ are the $SU(2)$ and $U(1)$ gauge couplings, respectively.

The results for the amplitudes, produced using the Feynman rules output from `SmeftFR`, are

$$\begin{aligned}
\mathcal{M}_{\pm\pm\pm\pm} &= \frac{4\bar{g}^2}{1-\cos\theta}, \\
\mathcal{M}_{\pm\mp\mp\pm} &= \bar{g}^2(1-\cos\theta), \\
\mathcal{M}_{\pm\mp\pm\mp} &= \bar{g}^2\frac{(1+\cos\theta)^2}{(1-\cos\theta)}, \\
\mathcal{M}_{0\pm0\pm} = \mathcal{M}_{\pm0\pm0} &= \bar{g}^2\frac{1+\cos\theta}{1-\cos\theta}, \\
\mathcal{M}_{00\pm\mp} = \mathcal{M}_{\pm\mp00} &= \frac{\bar{g}^2}{2}(1+\cos\theta), \\
\mathcal{M}_{0000} &= -\frac{1}{4}\left(\bar{g}^2+\bar{g}'^2\right)\left[\cos\theta - \frac{3+\cos\theta}{1-\cos\theta}\right] - 2\lambda,
\end{aligned} \tag{2}$$

for the SM-like operators, and

$$\begin{aligned}
\mathcal{M}_{\pm\pm\mp\mp} &= -6\bar{g}(1+\cos\theta)\frac{s}{\Lambda^2}C^W, \\
\mathcal{M}_{\pm\mp\mp\mp} = \mathcal{M}_{\mp\pm\mp\mp} = \mathcal{M}_{\mp\mp\pm\mp} = \mathcal{M}_{\mp\mp\mp\pm} &= 3\bar{g}(1+\cos\theta)\frac{s}{\Lambda^2}C^W, \\
\mathcal{M}_{0\pm0\mp} = \mathcal{M}_{\pm0\mp0} &= \frac{3}{4}\bar{g}(3+\cos\theta)\frac{s}{\Lambda^2}C^W + (1-\cos\theta)\frac{s}{\Lambda^2}C^{\varphi W}, \\
\mathcal{M}_{00\pm\pm} = \mathcal{M}_{\pm\pm00} &= -\frac{3}{2}\bar{g}\cos\theta\frac{s}{\Lambda^2}C^W + 2\frac{s}{\Lambda^2}C^{\varphi W}, \\
\mathcal{M}_{0000} &= -(1+\cos\theta)\frac{s}{\Lambda^2}\left[C^{\varphi\Box} + \frac{1}{2}C^{\varphi D}\right],
\end{aligned} \tag{3}$$

for the effective operators in $d = 6$ SMEFT.

A possible routine to achieve the analytic results in a semi-automatic fashion could, for example, involve passing the vertices in the `FeynRules` or `FormCalc` package and then use the package to calculate the relevant diagrams given by the user. A fully automatic approach on the other hand could involve the `FeynArts` package to create the relevant diagrams using the `SmeftFR` model file, and then calculate the generated amplitudes by passing them to the `FormCalc` or `FeynRules` package. Of course one could use the `UFO` output and a Monte Carlo generator like `MadGraph` and proceed to calculate the numerical cross sections for this process. For a complete analysis of the *same sign* W-boson scattering at the LHC, involving both analytical calculations and Monte Carlo simulations using the `UFO` output from `SmeftFR`, see ref. [19].

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