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On the flow model of a rheological fluid

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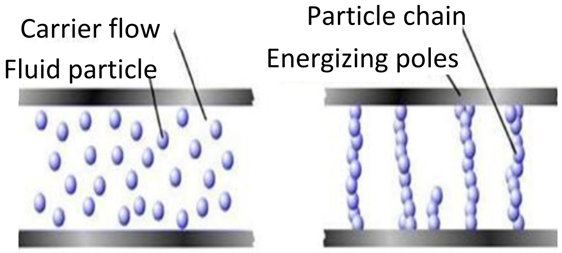
Abstract

In order to design an experimental device that involve a rheological (R) or magneto-rheological (MR) fluid, is useful to make the evaluation of a quantitative behavior of the fluid (i.e. the calculation of the stress and/or viscosity) in order to obtain correct information about the following issues: the volume fraction of the particles, the magnetic properties and the intensity of the external applied fields. Thereafter, is important to develop the study of simple viscous fluid flows [1] in order to have correct predictions in both the numerical simulation of MR fluids and experimental design of devices that involve MR fluids [2-4]. In this paper, we focused on some numerical models of flows, starting with a 2D plane flow of a fluid, and progressively prepare the way to obtain a general model (a semi-analytical one, with some coefficients gained from the experiments).

1. Introduction to simple flow patterns

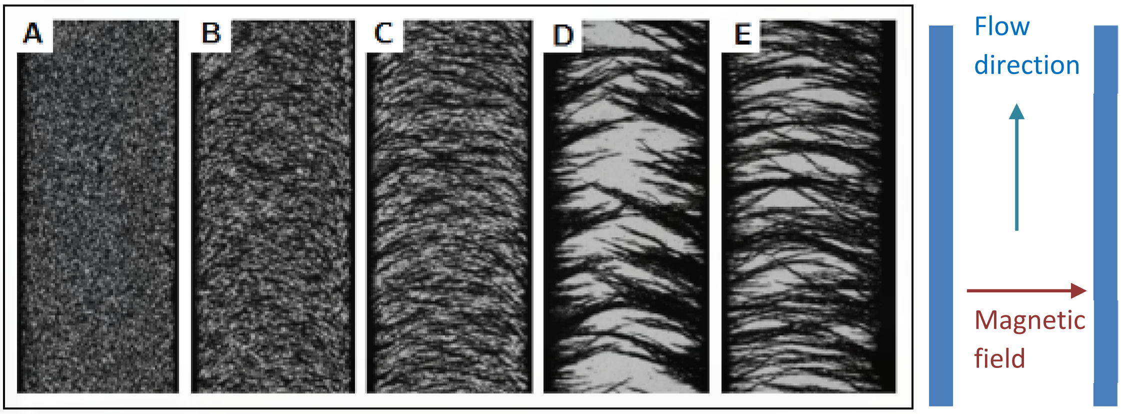
A. Rheological fluids are part of intelligent materials and are fluids that change their viscosity under the action of an energizing field. They consist of a suspension of solid micrometric particles dispersed in a fluid. Depending on the type of energy field (electric or magnetic) we have electrorheological and magnetorheological fluids, respectively.

When energizing the fluid by applying an energizing field, the particles are grouped into elongated groups of particles that are aligned along the field lines between the poles of the field generator. When applying an energizing field, transverse to the flow direction, these groups (chains) of particles slow down, until blocking, the flow, presented in the next figure.



The use of these fluids in robotic or mechatronic fields has expanded especially motion control elements. Basic applications refer to brakes, clutches, shock absorbers, and stop valves. The magneto-rheological stop valve (VMR) is one of the main applications of MR fluids. For this type of application, the energization area is fixed, the fluid flow being influenced by changing the size of the magnetic field applied to the area. VMR is a robust structure, easy to control, and simple from a constructive point of view.

VMR is a simple construction, which allows the flow of MR fluid through a channel inside it, as well as its exposure to a magnetic field. Constructively the flow channel can have different sections and different shapes. For a large pressure difference between the valve ends, the fluid flow breaks the chains of particles. The speed of their recovery depends on the intensity of the magnetic field, presented in the following figure.



However, its mathematical modeling is difficult to achieve because the MR fluid is a viscous fluid with a non-Newtonian flow. None of the mathematical models that have prevailed over time are satisfactory for accurately modeling flow. For this reason, a large part of the parameters of the mathematical model must be determined experimentally, depending on the type of application, the properties of the fluid used, and the geometry of the elements.

The mathematical model presented in the paper models the flow of a rheological fluid, eliminating some of the deficiencies mentioned.

B. In the case of the simple model considered here, specific to a rheological fluid, we are interested in the behavior of the latter during the flow. Flow is considered as a system subject to constraints and we are interested in the mathematical modeling of the forces acting on and the type of response induced by them. Rheological fluids are usually divided into two classes: Newtonian fluids and non-Newtonian fluids.

The simplest mathematical model [1], the Newtonian fluid, introduces the viscosity and the applied force (F) tangent on the surface of the liquid (S) generates the displacement of the liquid in layers and the flow velocity is proportional to the applied force:

$$\tau = \frac{F}{S} = \eta\dot{\gamma}$$

where: τ is the shear stress, η is the (plastic) viscosity and $\dot{\gamma}$ is the flow velocity gradient or the shear rate.

Shortly, in the case of Newtonian fluid the flow occurs in layers, it is a laminar flow (non-turbulent) and the layers have different speeds. The viscosity is constant with increasing shear stress.

In the case of the non-Newtonian fluid:

$$\tau = \tau_0 + \eta\dot{\gamma}$$

In addition to the plastic viscosity η, two other types of viscosities exist, i.e., the elastic viscosity ($\frac{\tau_0}{\dot{\gamma}}$) and the apparent viscosity ($\frac{\tau}{\dot{\gamma}}$).

Plastic or pseudo-plastic non-Newtonian rheological fluids are the fluids that begin to flow only when a shear stress (an application force) that exceeds a certain threshold value acts on the fluid. We can exemplify here the case of suspensions (from solutions) and of some viscous liquids (honey, ketchup). The threshold (or the flow) value is very important if it is desired that the fluid does not flow (e.g., from the container or through a coil) until it receives an external force (by shaking or by crushing) or other type of disturbance. (e.g., electrical or magnetic field). The viscosity in this case is not constant if the shear stress increases. In the case of pseudo-plastic non-Newtonian liquids, the rheogram is linear and we practically observe a liquid flow as soon as we apply the shear stress and the viscosity of the liquid decrease with the increasing of the shear stress, which leads to an accelerated fluid flow correlated with a decreasing viscosity. In our model, we study a magneto-rheological fluid that is characterized by two components: the basic fluid and the (metallic) particles. In general, magneto-rheological fluids are e.g., synthetic oils, silicone oils, etc. where iron particles are used for magnetic particles (in the literature carbonyliron). A common approach to magnetic fluid is the Bingham plastic model [2-3], where:

$$\tau = \tau_0(H) + \eta\dot{\gamma}^n$$

and n < 1.

2. A fluid flow pattern

2.1. Setup:

We will analyze the case of a viscous fluid, which flows in the direction *Ox* between two planes parallel to the plane *xOz* ($y = \pm h$). We will estimate the flow velocity as a function of η, h and the pressure gradient on the direction Ox. It is assumed that there are no variations in the direction Oz and, in a first approximation, the gravitational effects are neglected and there is no flow on the walls $y = \pm h$.

2.2. The 2D case

We consider the flow being in the Oxdirection, $v \equiv (v_x, 0,0)$. Because we consider that the fluid is incompressible: $\nabla \cdot v(x,t) = 0$ from which results $\frac{\partial v_x(x,t)}{\partial x} = 0$or $v_x = v_x(y,z)$, the Navier-Stokes equation (N-S) will be of the form:

$$-\rho\frac{\partial v_x(y,z,t)}{\partial t} + \eta\left(\nabla_z^2 + \nabla_y^2\right)v_x(y,z,t) = \nabla_x p(x,t)$$

and

$$\nabla_y p(x,t) = 0, \quad \nabla_z p(x,t) = 0$$

that is, the pressure depends only on time and coordinate x. In equation (N-S) it results that the left hand side does not depend on x, and the right hand side does not depend on $y,\ z$. So the pressure gradient depends only on time, i.e.: $\nabla_x p(x,t) = -G(t)$.

A stationary flow assumes a constant function, i.e.:

$$\nabla_x p(x) = -k = const$$

After integration we have:

$$p(x) = -kx + p_0$$

From the hypothesis, we have stationary flow and the flow rate does not depend on z, i.e.:

$$\nabla_x p(x) = \eta \nabla_y^2 v_x(y) = -k$$

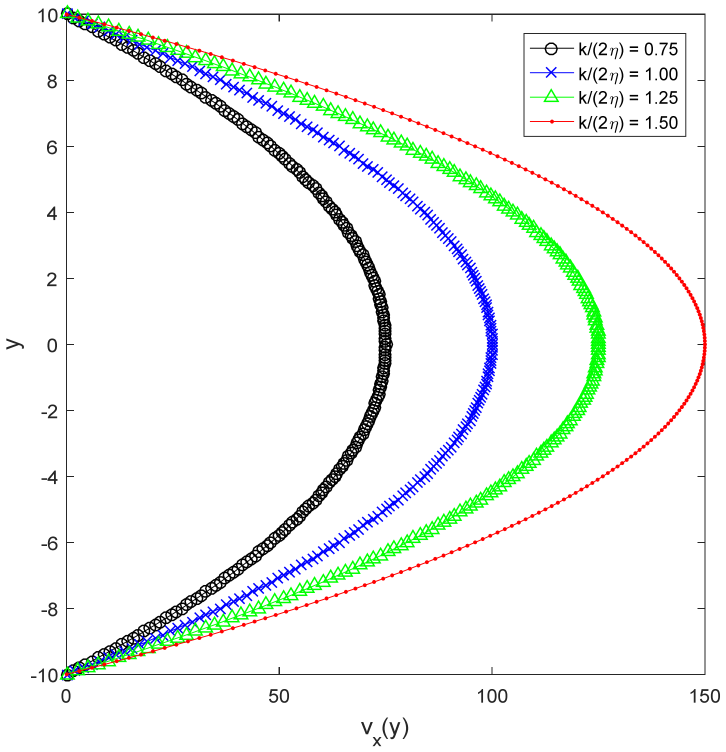
The solution of the previous equation is of the form:

$$v_x(y) = -\frac{k}{2\eta} y^2 + Cy + D$$

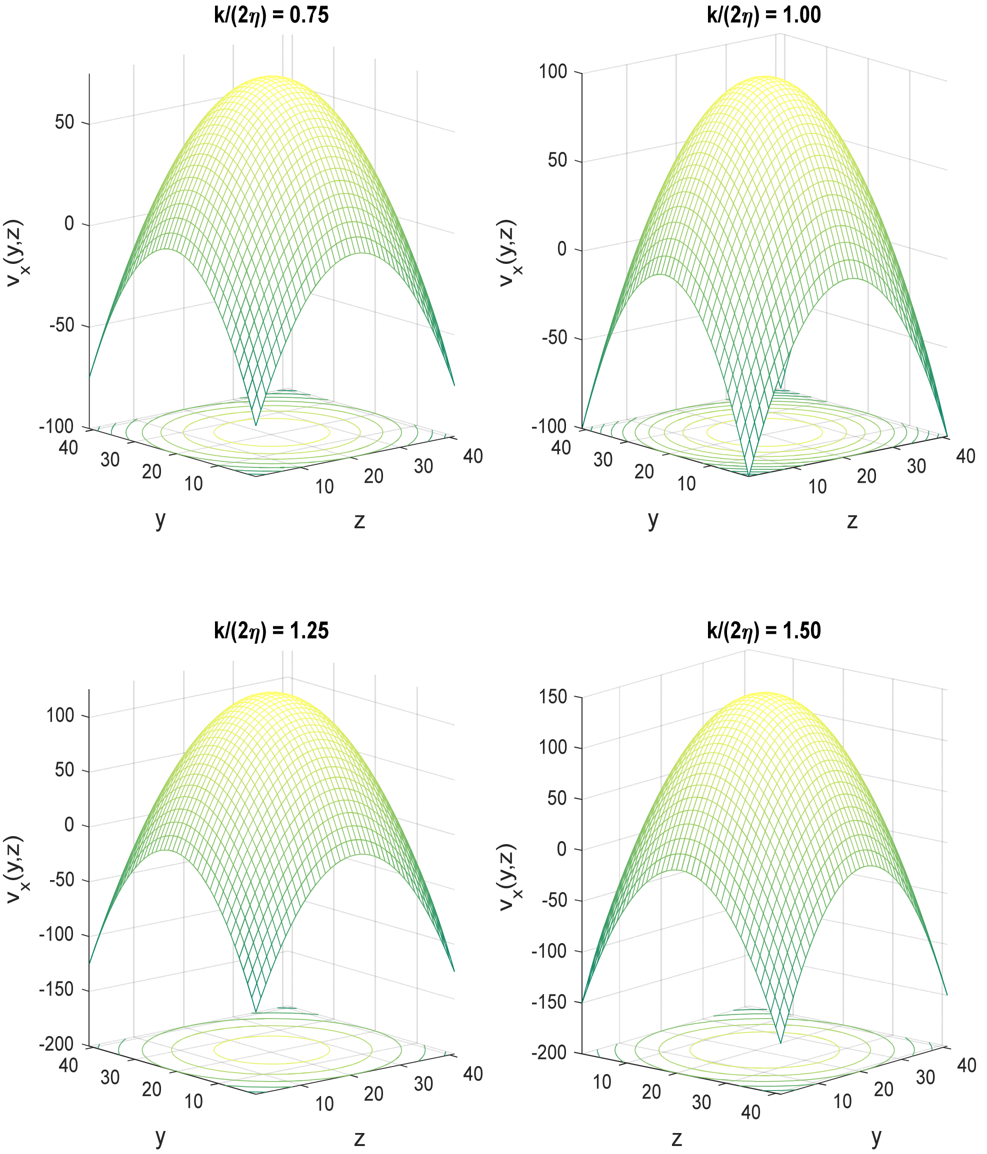
The constants are determined from the boundary conditions in the statement: $v_x(\pm h) = 0$. The general solution is:

$$v_x(y) = \frac{k}{2\eta}\left(h^2 - y^2\right)$$

that is, there is a flow with a parabolic profile, presented in the next figure.



Parabolic profile of the boundary layer flow



$k/(2\eta) = 0.75$

$k/(2\eta) = 1.00$

$k/(2\eta) = 1.25$

$k/(2\eta) = 1.50$

Parabolic profile of the boundary layer flow in the direction of the x-axis, with circular symmetry in the Oyz plane

The transition to a horizontal flow model 3D, in the direction of the x-axis, with circular symmetry in the *Oyz* plane on the y and z axis, leads to a flow profile of the most normal shape with vertical axis speed (see above figure).

Conclusions

The mathematical model presented can be extended and adapted for magnetorheological or electrorheological fluids, including the terms of fluid energization and correspondence with the change in viscosity. Also using an experimental platform to determine the characteristics of different stop valves, the terms of the model correlated with the types of stop valves can be adjusted [6-7].

Acknowledgment

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