Evolution of geometric quantum discord in a squeezed thermal bath

Alina Dobre[1 2 a], Aurelian Isar[1 b]

[1]"Horia Hulubei" National Institute for Physics and Nuclear Engineering, Măgurele-Bucharest, Romania,
[2]Faculty of Physics, University of Bucharest, Romania

[a]dobre.alina@theory.nipne.ro
[b]isar@theory.nipne.ro

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Abstract

We analyze the time evolution of the geometric quantum discord for a system composed of two uncoupled bosonic modes in interaction with a squeezed thermal bath. We work in the framework of the theory of open quantum systems, based on completely positive quantum dynamical semi-groups, using the geometric quantification of the total non-classical correlations for Gaussian states, considering a continuous variable system. We use the Hellinger measure for the geometric quantum discord, taking the initial squeezed thermal states and we show that discord decays in time as we increase the parameters of the bath, tending asymptotically to zero.

1 Introduction

At the core of quantum information theory there are quantum correlations which represent an indispensable physical resource for the description and performance of quantum information processing tasks [1, 2]. One of the most known and used resources is entanglement, however it does not describe all the existing quantum correlations, because there are separable mixed states which cannot be simulated by a classical probability distribution [3, 4]. In this mindset, Zurek [5, 3] proposed a quantification for the total amount of quantum correlations between a bipartite system, called quantum discord, which can have non-zero values for separable state.

In this paper, we use a geometric approach through Hellinger metric [6], to investigate the irreversible time evolution of quantum discord for a system composed of two bosonic modes, uncoupled, with equal masses and frequencies, in interaction with a squeezed thermal bath. We take the initial state of the modes to be a squeezed thermal one. We show the loss of quantum correlations, in time, due to the interaction with the thermal bath, showing the dependency of quantum discord on different parameters.

2 Framework

We describe the irreversible time evolution of an open quantum system, based on completely positive dynamical semi-groups, by using the Kossakowski-Lindblad quantum Markovian master equation for the density operator $\rho(t)$ in the Schrödinger representation:

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \frac{1}{2\hbar}\sum_j \left(2V_j\rho(t)V_j^\dagger - \left\{\rho(t), V_j^\dagger V_j\right\}_+\right), \quad (1)$$

where we used H to denote the Hamiltonian of the system, V_j and $V_j^\dagger$ are operators on the Hilbert space used, describing the interaction between the environment and the open system composed of the two bosonic modes, $V_j^\dagger$ being the Hermitian conjugate of V_j. The Hamiltonian of the system, considering that $m_1 = m_2 = 1$ and $\omega_1 = \omega_2 = 1$, is given by:

$$H = \frac{1}{2}\left(p_x^2 + p_y^2 + q_x^2 + q_y^2\right), \quad (2)$$

where x, y are the coordinates and p_x, p_y the momenta of the two bosonic modes. V_j and $V_j^\dagger$ are first degree polynomials of the canonical observables:

$$\begin{aligned} V_j &= a_{xj}p_x + a_{yj}p_y + b_{xj}x + b_{yj}y, \\ V_j^+ &= a_{xj}^*p_x + a_{yj}^*p_y + b_{xj}^*x + b_{yj}^*y \end{aligned} \quad (3)$$

Under local unitary operations we can assume that the first canonical moments are set to zero and we can write again the bimodal covariance matrix:

$$\sigma(t) = \begin{pmatrix} \sigma_{xx}(t) & \sigma_{xp_x}(t) & \sigma_{xy}(t) & \sigma_{xp_y}(t) \\ \sigma_{xp_x}(t) & \sigma_{p_xp_x}(t) & \sigma_{yp_x}(t) & \sigma_{p_xp_y}(t) \\ \sigma_{xy}(t) & \sigma_{yp_x}(t) & \sigma_{yy}(t) & \sigma_{yp_y}(t) \\ \sigma_{xp_y}(t) & \sigma_{p_xp_y}(t) & \sigma_{yp_y}(t) & \sigma_{p_yp_y}(t) \end{pmatrix} \equiv \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix} \quad (4)$$

A, B are the covariance matrices for the modes, and C is the matrix which describe the correlations between them. The elements are given by $\sigma_{ij} = \frac{1}{2}\mathrm{Tr}\left(\rho_{AB}\left\{R_i, R_j\right\}_+\right)$, $i, j = 1, \ldots, 4$, R_i and R_j being vectors of operator which satisfy the canonical commutation relation:

$$[R_i, R_j] = i\Omega_{ij}, \quad (5)$$

where Ω_{ij} is the $2N \times 2N$ symplectic matrix

$$\Omega = \bigoplus_{k=1}^{N}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (6)$$

3 Hellinger quantum discord for two bosonic modes in interaction with a squeezed thermal bath

The concept of quantum discord was introduced by Zurek and Ollivier [5] as a quatification for the total amount of non-classical correlations between the subsystems of a composed system. The quantum correlations of a system, described by geometric quantum discord, are given by the distance between the state of the system of interest and a state which has zero discord [9].

In a previous paper, the Hellinger geometric discord was evaluated for a system composed of two non-interacting, non-resonant bosonic modes in interaction with a thermal environment [10]. As a follow up, in this paper we use the Hellinger measure in order to calculate the quantum discord for two non-interacting, resonant bosonic modes, in interaction with a squeezed thermal bath, also considering the case in which each bosonic mode interacts with its own reservoir. Choosing a state ρ with a covariance matrix of the form:

$$\sigma = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix}, \tag{7}$$

the geometric quantum discord will be expressed as follows:

$$D_H(\sigma) = 1 - \frac{2P\left(\det \sigma^{\frac{1}{4}}\right)}{\left(P^2\sqrt{\det \sigma} + P(\det \sigma)^{\frac{1}{4}}\sqrt{Q} + \sqrt{B_1 B_2}\right)^{\frac{1}{2}}}, \tag{8}$$

where

$$\begin{aligned} B_1 &= abP^2 + \tfrac{1}{4}(ac+bd)^2, \\ B_2 &= abP^2 + \tfrac{1}{4}(bc+ad)^2. \end{aligned} \tag{9}$$

Denoting k_1, k_2 the symplectic eigenvalues of the covariance matrix:

$$\sigma = \begin{pmatrix} a & 0 & c & 0 \\ 0 & a & 0 & d \\ c & 0 & b & 0 \\ 0 & d & 0 & b \end{pmatrix}, \tag{10}$$

$$\begin{aligned} 2k_1^2 &= \Delta - \sqrt{\Delta^2 - 4\det \sigma}, \\ 2k_2^2 &= \Delta + \sqrt{\Delta^2 - 4\det \sigma}, \end{aligned} \tag{11}$$

where $\Delta = \det A + \det B + 2\det C$, we obtain the following expressions:

$$Q = (\sqrt{ab-c^2} + \sqrt{ab-d^2})^2 \left[ab(\sqrt{M_1} + \sqrt{M_2})^2 - \frac{1}{4}(a-b)^2\right] - (\sqrt{B_1} - \sqrt{B_2})^2, \tag{12}$$

$$P = \frac{1}{2}(\sqrt{M_1} + \sqrt{M_2})(\sqrt{N_1} + \sqrt{N_2}). \tag{13}$$

and

$$\begin{aligned} M_1 &= \left(k_1 - \tfrac{1}{2}\right)\left(k_2 + \tfrac{1}{2}\right), M_2 = \left(k_1 + \tfrac{1}{2}\right)\left(k_2 - \tfrac{1}{2}\right), \\ N_1 &= \left(k_1 + \tfrac{1}{2}\right)\left(k_2 + \tfrac{1}{2}\right), N_2 = \left(k_1 - \tfrac{1}{2}\right)\left(k_2 - \tfrac{1}{2}\right). \end{aligned} \tag{14}$$

In order to evaluate the irreversible time evolution of the geometric quantum discord we take initial squeezed states for the bosonic modes, described by the following covariance matrix [11]:

$$\sigma(0) = \begin{pmatrix} a(0) & 0 & c(0) & 0 \\ 0 & a(0) & 0 & -c(0) \\ c(0) & 0 & b(0) & 0 \\ 0 & -c(0) & 0 & b(0) \end{pmatrix}, \tag{15}$$

with

$$\begin{aligned} a(0) &= n_1 \cosh^2 r + n_2 \sinh^2 r + \tfrac{1}{2}\cosh 2r, \\ b(0) &= n_1 \sinh^2 r + n_2 \cosh^2 r + \tfrac{1}{2}\cosh 2r, \\ c(0) &= \tfrac{1}{2}(n_1 + n_2 + 1)\sinh 2r. \end{aligned} \tag{16}$$

where n_1 and n_2 are the average photon numbers of the modes and r represents the squeezing parameter.

From the master Eq. (1) we obtain a differential equation for the covariance matrix:

$$\frac{d\sigma(t)}{dt} = Y\sigma(t) + \sigma(t)Y^{\mathrm{T}} + 2\mathcal{D}, \tag{17}$$

with

$$Y = \begin{pmatrix} -\lambda & 1 & 0 & 0 \\ -\omega_1^2 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & -\omega_2^2 & -\lambda \end{pmatrix}, \tag{18}$$

$$\mathcal{D} = \begin{pmatrix} D_{xx} & D_{xp_x} & D_{xy} & D_{xp_y} \\ D_{xp_x} & D_{p_xp_x} & D_{yp_x} & D_{p_xp_y} \\ D_{xy} & D_{yp_x} & D_{yy} & D_{yp_y} \\ D_{xp_y} & D_{p_xp_y} & D_{yp_y} & D_{p_yp_y} \end{pmatrix}, \tag{19}$$

where $\mathcal{D}$ is the diffusion matrix.
Eq. (17) has the following solution:

$$\sigma(t) = Z(t)\left(\sigma_0 - \sigma_\infty\right) Z^{\mathrm{T}}(t) + \sigma_\infty, \tag{20}$$

where

$$Z(t) = \exp(Yt) \tag{21}$$

and σ_∞ is the asymptotic covariance matrix:

$$\sigma_\infty = \begin{pmatrix} 1 + 2(N + M_R) & 2M_I & 0 & 0 \\ 2M_I & 1 + 2(N - M_R) & 0 & 0 \\ 0 & 0 & 1 + 2(N + M_R) & 2M_I \\ 0 & 0 & 2M_I & 1 + 2(N - M_R) \end{pmatrix}. \tag{22}$$

Here M_R is the real part of the squeezing of the bath, and M_I the imaginary part. M and N are given by the following expressions:

$$\begin{aligned} N &= n\left(\cosh^2 R + \sinh^2 R\right) + \sinh^2 R, \\ M &= -(2n+1)\cosh R \sinh R \exp i\varphi, \end{aligned} \tag{23}$$

where R is the squeezing parameter of the bath, n is the average photon number of the bath and φ is the squeezing phase:

$$n = \frac{1}{2}\left(\coth\frac{1}{2T} - 1\right), \tag{24}$$

and T is the temperature of the bath

4 Results

We are interested in investigating the behaviour of the geometric quantum discord for two bosonic modes, while interacting with a thermal squeezed bath, so we compute D_H as a function of time and some different parameters.

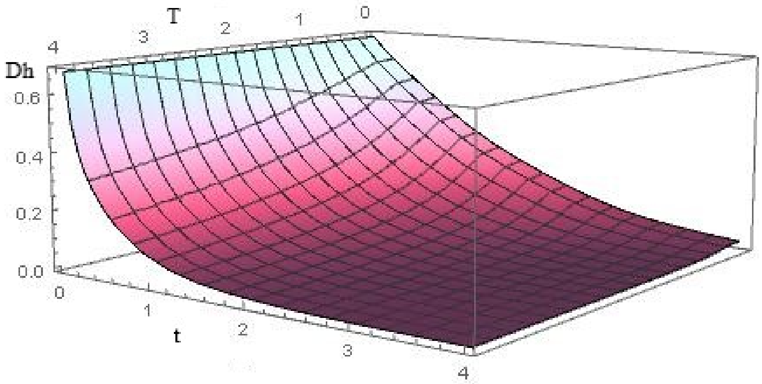

Figure 1: Hellinger discord versus time and temperature of the bath for two resonant bosonic modes, $\omega_1 = \omega_2 = 1$, average photon numbers $n_1 = 2$, $n_2 = 1$, squeezing parameters $R = 0.1$ and $r = 1$, squeezing phase $\varphi = 2$, dissipation coefficient $\lambda = 0.5$.

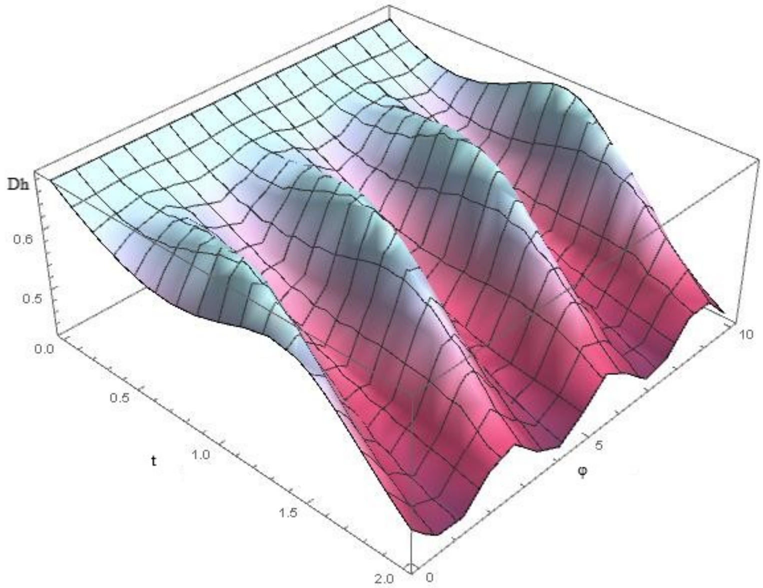

Figure 2: Hellinger discord versus time and squeezing phase for two resonant bosonic modes, $\omega_1 = \omega_2 = 1$, average photon numbers $n_1 = 2$, $n_2 = 1$, squeezing parameters $R = 0.1$ and $r = 1$, temperature $T = 1$, dissipation coefficient $\lambda = 0.1$.

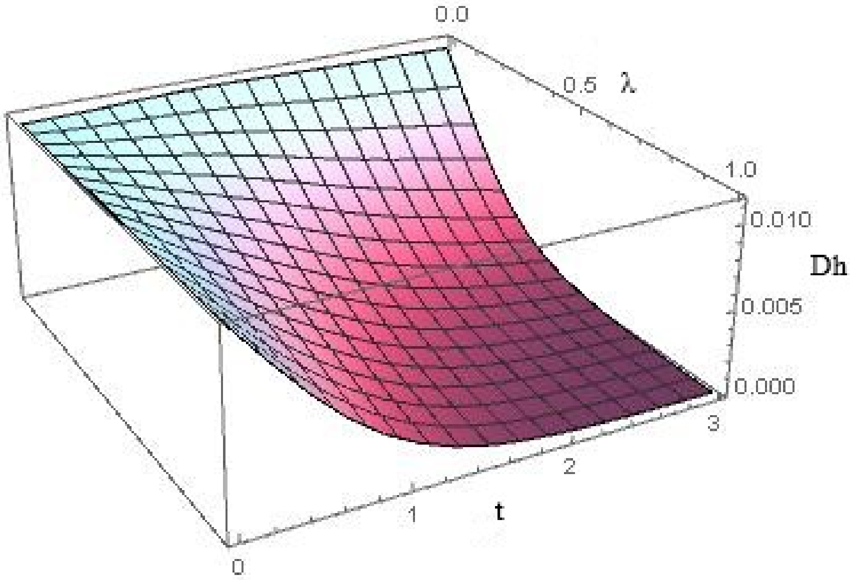

Figure 3: Hellinger discord versus time and dissipation coefficient for two resonant bosonic modes, $\omega_1 = \omega_2 = 1$, average photon numbers $n_1 = 2$, $n_2 = 1$, squeezing parameters $R = 0.1$ and $r = 0.1$, squeezing phase $\varphi = 1$, temperature $T = 1$.

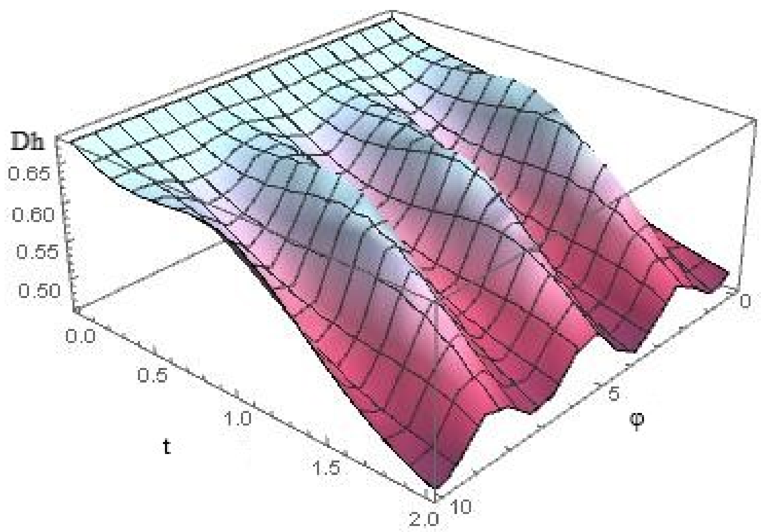

Figure 4: Hellinger discord versus time and squeezing phase for two resonant bosonic modes, in interaction with two bath with temperatures $T_1 = 1$ and $T_2 = 0.5$, $\omega_1 = \omega_2 = 1$, average photon numbers $n_1 = 2$, $n_2 = 1$, squeezing parameters $R = 0.1$ and $r = 1$, dissipation coefficient $\lambda = 0.1$.

We notice that the geometric quantum discord asymptotically decreases to zero, in time, under the influence of the parameters of the bath, in general in a non-monotonic way.

5 Conclusions

In the theory of open quantum systems, based on completely positive dynamical semigroups, we used the geometric approach to calculate the total amount of non-classical correlations, represented by quantum discord, in a composed system. This description takes into consideration the distance between the state of the system and the closest classical-quantum state, which has zero discord. We used the Hellinger metric in order to describe the time evolution of geometric quantum discord for two uncoupled, resonant bosonic modes in interaction with a common reservoir. We show that D_H takes values from 0 to 1, decreasing asymptotically under the influence of the squeezed thermal bath. We also considered the case in which each of the modes was interacting with its own bath, and described the behavior of quantum discord as a function of time and squeezing phase. The main conclusion is that the interaction with the environment leads to a loss of quantum correlations due to the dissipation and decoherence.

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