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On an inflation in holographic cosmology with inverse cosh potential

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Abstract

We consider a model of inflation based on dynamics of D3-brane located at the boundary of an asymptotic AdS_5 bulk. The matter on the brane is described by the Dirac-Born-Infeld (DBI) Lagrangian. We solve numerically the system of dynamical equations in case of the inverse cosh potential for different initial conditions. Observational parameters of inflation ($n_{\rm s}$ and r) are calculated numerically. Obtained results are compared to the results of the Planck 2018 mission.

Keywords: DBI Lagrangian; tachyon inflation; holographic cosmology.

1 Introduction

The inflation theory has been accepted as the best approach to solving some problems in the standard Big Bang cosmology (flatness problem, horizon problem, etc.). The theory proposes a period of extremely rapid (exponential) expansion of the universe during the early stage of evolution of the universe. It predicts that during inflation (which takes about 10^{-34} s) the radius of the universe increased, about $e^{60} \approx 10^{26}$ times. Although inflationary cosmology has successfully complemented the Standard Cosmological Model, the process of inflation in particular its origin, is still largely unknown.

Recent years brought us a lot of evidence of CMB from WMAP and Planck observations [?, ?]. To test inflationary cosmological models we need to compare results computed from a model to the measured values of the observational parameters, such as scalar spectral index (n_s) and tensor-to-scalar ratio (r).

The popular class of inflationary models is based on tachyon scalar field. Dynamics of tachyon scalar field θ (with dimension of length) is determined by the DBI type Lagrangian [?, ?, ?, ?]

$$\mathcal{L} = -\ell^{-4} V(\theta/\ell)\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}\,, \tag{1}$$

where V is a potential of a tachyon field with properties

$$V(0) = \text{const}, \quad \frac{dV}{d\theta}(\theta > 0) = V_{,\theta}(\theta > 0) < 0, \quad V(|\theta| \to \infty) \to 0, \tag{2}$$

and ℓ is an appropriate length scale.

One among classes of possible inflationary models is derived from braneworld cosmology. A braneworld universe is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk. One of the simplest models is Randall-Sundrum (RS) model [?, ?]. Another interesting class of extended gravity models analysed for cosmological inflation is based on non-Riemannian spacetime volume-forms. The volume-forms define generally covariant integration measures over differentiable manifolds M, which is not necessarily Riemannian ones. In this case no metric is a priori needed (see [?], [?] and references therein).

The RS models are based on two branes with opposite tensions which are placed at some distance in 5 dimensional space. In the original RS model an observer resides on the brane with negative tension, distance to the second brane corresponds to the Netwonian gravitational constant. In the second Randall-Sundrum model (RSII) an observer is placed on the positive tension brane, and the second brane is pushed to infinity. It was shown that a dynamics of inflaton field in the RSII model is closely related to the tachyon inflation [?].

Among several classes of possible inflationary models we consider one derived from braneworld cosmology. The holographic braneworld cosmology is based on the effective four-dimensional Einstein equations on the holographic boundary in the framework of anti de Sitter/conformal field theory (AdS/CFT) correspondence [?, ?]. The model is based on the holographic braneworld scenario with an effective tachyon field on a D3-brane located at the holographic boundary of an asymptotic AdS_5 bulk. There are a variety of relevant tachyonic potentials in this scenario. In this paper we study the inverse cosh potential in the form $V(\theta) \sim 1/cosh(\theta)$.

The remainder of the paper is organized as follows. In the next section, Sec. 2, we introduce the tachyon dynamics in the holographic braneworld. In Sec. 3 some observational cosmological parameters are studied for this model. The numerical calculation and the results obtained for inverse cosh potential are presented in Sec. 4. In Sec. 5, we give the concluding remarks.

2 Inflation in the holographic braneworld

The holographic Friedmann equations are derived from the effective four-dimensional Einstein's equations on the boundary of AdS_5 bulk [?]

$$R_{\mu\nu} - \frac{1}{2} R g^{(0)}_{\mu\nu} = 8\pi G_{\mathrm{N}} \left(\langle T^{\mathrm{CFT}}_{\mu\nu} \rangle + T_{\mu\nu} \right), \tag{3}$$

where $g^{(0)}_{\mu\nu}$ is a metric at the boundary, $\langle T^{\mathrm{CFT}}_{\mu\nu} \rangle$ is the expectation value of the energy-momentum tensor of the dual conformal theory and $T_{\mu\nu}$ is the energy-momentum tensor associated with matter on the brane. For a spatially flat FLRW boundary geometry with the line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2), \tag{4}$$

the holographic Friedmann equations have the form [?]

$$h^2 - \frac{1}{4} h^4 = \frac{\kappa^2}{3} \ell^4 \rho, \tag{5}$$

$$\dot{h}\left(1-\frac{1}{2}h^2\right) = -\frac{\kappa^2}{2}\ell^3(p+\rho), \tag{6}$$

where ℓ is the AdS curvature radius, $h \equiv \ell H$ is a dimensionless expansion rate and κ is the fundamental dimensionless coupling [?]

$$\kappa^2 = \frac{8\pi G_{\mathrm{N}}}{\ell^2}. \tag{7}$$

It is worth to notice that overdot denotes a derivative with respect to dimensionless time variable $\tau = t/\ell$. The solution of the first Friedmann equation, which describes the evolution of the homogenous universe consistent with prediction of standard cosmology, has the form

$$h^2 = 2\left(1-\sqrt{1-\frac{\kappa^2}{3}\ell^4\rho}\right), \tag{8}$$

which imposes the restriction to the range of the Hubble expansion rate $0 \le h^2 \le 2$ [?, ?].

The tachyon matter on the brane can be treated as an ideal fluid with the components of the energy-momentum tensor

$$T^\mu_\nu = \mathrm{diag}(\rho, -p, -p, -p). \tag{9}$$

Pressure p and energy density ρ of the tachyon fluid are given by [?]

$$\rho \equiv \mathcal{L} = \frac{\ell^{-4}V}{\sqrt{1-\dot{\theta}^2}}, \quad p \equiv \mathcal{H} = -\ell^{-4}V\sqrt{1-\dot{\theta}^2}, \tag{10}$$

where $\mathcal{H}$ is the corresponding Hamiltonian. The dynamics of the model can be described by two first order differential equations derived from the Hamilton equations

$$\dot{\theta} = \frac{\eta}{\sqrt{1+\eta^2}}, \tag{11}$$

$$\dot{\eta} = -\frac{3h\eta}{\ell} - \frac{V_{,\theta}}{V}\left(\sqrt{1+\eta^2} + \frac{\eta^2}{\sqrt{1+\eta^2}}\right), \tag{12}$$

where η is the new field

$$\eta = \ell^{-4}V^{-1}\sqrt{g_{\mu\nu}\pi^\mu\pi^\nu}, \tag{13}$$

related to the conjugate momentum $\pi^\mu = \partial\mathcal{L}/\partial\theta_{,\mu}$.

In this paper we will focus our study to the potential

$$V(\theta) = \frac{V_0}{\cosh(\omega\theta/\ell)}, \tag{14}$$

where V_0 and ω are free dimensionless parameters. This potential has already been discussed in the inflation models in the standard cosmology [?, ?]. In all equations the parameter V_0 appears only in the form of the product $\kappa^2 V_0$ and it allows us to rescale the constant κ in such a way to include the free parameter V_0, i.e. $\kappa^2 V_0 \to \kappa^2$. In this case the potential can be written in a form with only one free parameter, i.e.

$$V(\theta) = \frac{1}{\cosh(\omega\theta/\ell)}. \tag{15}$$

3 The observational parameters

To be in position to study observational parameters of inlation and compare the computed values of those parameters with observational constrains from the Planck collaboration [?] it is useful to define the slow-roll parameters ε_i. It is the most convenient to introduce the slow-roll parameters by the relation [?]

$$\varepsilon_0 = \frac{h_*}{h}, \qquad \varepsilon_{i+1} = \frac{d \ln |\varepsilon_i|}{dN}, \quad i \geq 0, \tag{16}$$

where h_* is the Hubble expansion rate at some chosen time and N is the number of e-folds defined by

$$N = \int_{t_{\rm CMB}}^{t_{\rm f}} h dt. \tag{17}$$

The first three parameters are given by

$$\varepsilon_1 \equiv -\frac{\dot{h}}{h^2}, \quad \varepsilon_2 \equiv \frac{\dot{\varepsilon_1}}{h\varepsilon_1}, \quad \varepsilon_3 \equiv \frac{\dot{\varepsilon_2}}{h\varepsilon_2}. \tag{18}$$

Inflation starts at time $t_{\rm CMB}$ and ends at time $t_{\rm f}$, when any of ε_i exceeds one. Comparing expressions for the slow-roll parameters for a general form of potential

$$\varepsilon_1 \simeq \frac{4-h^2}{12h^2(2-h^2)} \left(\frac{\ell V_{,\theta}}{V}\right)^2, \tag{19}$$

$$\varepsilon_2 \simeq 2\varepsilon_1 \left(1 - \frac{2h^2}{(2-h^2)(4-h^2)}\right) + \frac{2\ell^2}{3h^2} \left[\left(\frac{V_{,\theta}}{V}\right)^2 - \frac{V_{,\theta\theta}}{V}\right], \tag{20}$$

it follows that $\varepsilon_2 \simeq 2\varepsilon_1$ [?].

Observational parameters, spectral index ($n_{\rm s}$) and tensor to scalar ratio (r), are given by the expressions

$$n_{\rm s} - 1 = \frac{d \ln \mathcal{P}_{\rm S}}{d \ln q}, \tag{21}$$

$$r = \frac{\mathcal{P}_{\rm T}}{\mathcal{P}_{\rm S}}, \tag{22}$$

where the power spectra of the scalar perturbations $\mathcal{P}_{\rm S}$ and the power spectra of tensor perturbations $\mathcal{P}_{\rm T}$ are evaluated at the horizon crossing

$$q c_{\rm s} = ah, \tag{23}$$

where q is the (comoving) wave number and $c_{\rm s}$ is the adiabatic sound speed

$$c_{\rm s}^2 = \left.\frac{\partial p}{\partial \rho}\right|_\theta = 1 - \frac{4(2-h^2)}{3(4-h^2)}\varepsilon_1. \tag{24}$$

In Ref. [?] an approximate expression for the scalar and the tensor power spectra in holographic braneworld (in the slow roll approximation) is derived. The Lagrangian given by equation (??) belongs to the class of k-essence inflation models [?]. By introducing effective values of pressure and density and adapting the procedure from Ref. [?] for calculation of the spectra in the models with a standard k-essence action, one gets the expression for observational parameters.

Another, more general approach for calculating the scalar and the tensor power spectra is used in Ref. [?]. Based on that result, for the Lagrangian given by the expression (??), observational parameters get the form

$$r = 16\varepsilon_1 \left(1 + C\varepsilon_2 - \frac{2(2-h^2)}{3(4-h^2)}\varepsilon_1\right), \tag{25}$$

$$n_{\mathrm{s}} = 1 - 2\varepsilon_1 - 2\varepsilon_2 - \left(2 - \frac{8h^2}{3(4-h^2)^2}\right)\varepsilon_1^2 - \left(3 + 2C - \frac{2(2-h^2)}{3(4-h^2)}\right)\varepsilon_1\varepsilon_2 - C\varepsilon_2\varepsilon_3, \tag{26}$$

where $C = -2 + \ln 2 + \gamma \simeq 0.72$ and γ is the Euler constant.

Figure 1: r versus n_{S} diagram. r versus n_s diagram with observational constraints from Ref. [?]. The dots represent the theoretical predictions for observational parameters obtained by solving the equations of motion (??) and (??) numerically for randomly chosen N, ω and θ_0 in the intervals $60 \leq N \leq 90$, $0 < \omega < 0.25$ and $0 < \theta_0 < 20$.

4 Numerical calculations and results

Following a similar procedure as in Ref. [?], the system of dynamical equations (??) and (??) can be solved numerically for given initial conditions. In order to carry out calculation the values of the function h and θ at initial time ($t = 0$) must be fixed. As pointed out in Ref. [?], initial condition $\dot{\eta}_{\mathrm{i}} = 0$ yields solutions consistent with slow-roll regime, therefore, the solutions obtained using this condition are physically relevant for inflation. In this case the value η_{i} can be obtained from the the expression

$$\eta_{\mathrm{i}} = -\frac{(\ell V_{,\theta}/V)_{\mathrm{i}}}{\sqrt{9h_{\mathrm{i}}^2 - 4(\ell V_{,\theta}/V)_{\mathrm{i}}^2 + 3\sqrt{9h_{\mathrm{i}}^4 - 4h_{\mathrm{i}}^2(\ell V_{,\theta}/V)_{\mathrm{i}}^2}}}, \tag{27}$$

which follows from the equation (??).

It was shown in Ref. [?] that for the model with the exponential potential dependence on the parameter κ can be eliminated. However, for the model with the potential (??) the parameter κ cannot be eliminated. Therefore, in this model, the value of the parameter κ must be set. It is appropriate to calculate the value of η_{i} using the equation (??) from arbitrarily chosen values h_{i} (with the restriction $0 \leq h_{\mathrm{i}}^2 \leq 2$) and θ_{i}. Then, the parameter κ can be fixed using expression

$$\kappa^2 = \frac{3}{V(\theta_{\mathrm{i}})\sqrt{1+\eta_{\mathrm{i}}^2}}\left(1 - \left(1 - \frac{h_{\mathrm{i}}^2}{2}\right)^2\right), \tag{28}$$

which can be derived from the equation (??).

In addition, the more natural approach to set the values of free parameters and initial conditions is to fix the value of κ instead of θ_{i}. In this case, after substituting η_{i} from the equation (??) to (??), one can numerically solve the equation (??) for θ_{i}.

The parameters are calculated using the procedure from the Ref. [?]. Equation (??) can be rewritten in the form

$$\dot{N} = h. \tag{29}$$

Figure 2: r versus $n_{\rm S}$ diagram. r versus n_s diagram with observational constraints from Ref. [?]. As in Fig. ?? the dots represent the theoretical predictions of the values of observational parameters for randomly chosen N, h_i, ω and κ in the the following intervals $60 \leq N \leq 90$, $0 < \omega < 0.25$ and $0 < \kappa < 10$.

The system of equations (??) and (??), supplemented by the equation (??) for chosen N, $h_{\rm i}$ and κ, is solved. The values of the slow-roll parameters are found from the equation (??) by numerical differentiation. We use the criteria for the end of inflation $\varepsilon_2(t_{\rm f}) = 1$ to determine the value of the field at the end of inflation. Due to the slow-roll approximation the calculated value of e-folds will be smaller than its set value at the beginning of inflation. Although the initial conditions are given for $t = 0$, due to the difference in the e-fold number, this is not the time when inflation begins. The value of $t_{\rm CMB}$ in equation (??) is determined from

$$N(t_{\rm f}) - N(t_{\rm i}) = N. \tag{30}$$

The calculated results for $n_{\rm s}$ and r superimposed on the observational constraints taken from the Planck Collaboration 2018 [?] are presented in Figures ?? - ?? for different sets of the values of free parameters.

Figure 3: r versus $n_{\rm S}$ diagram. r versus n_s diagram with observational constraints from Ref. [?]. As in previous figures the dots represent the theoretical predictions of the values of the observational parameters for randomly chosen N, h_i, ω and κ. All intervals for the free parameters are the same as in Figure ??, except the parameter κ for which is now restricted to the interval $0 < \kappa < 1.7$.

To solve the system of equations (??) and (??) the values for θ_0 were set as random values in the given range and corresponding values of parameter κ were calculated from the equation (??). In the Figure ?? the calculated values of parameters n_s and r are confronted with observational constrains from the Planck collaboration [?]. However, it was already mentioned that it is more natural to set the value of free parameter κ as a random value, and calculate the corresponding initial conditions for solving the system of equations (??)-(??). The results in this case are shown in the Figure ??.

If we compare the results for calculated observational parameters r and n_s defined by equations (??) and (??) and shown in Figure ?? to those ones in Figure ?? it is easy to notice that there is more dispersion in data points on the Figure ??. The higher density of data points is present in the unsuitable part of the (n_s, r) plane. However, in this case we can limit the value of free parameter κ to lower and more suitable ranges. If we keep all other parameters the same and limit random values of κ to the interval $0 < \kappa < 1.7$, the upper branch disappears and we get the results which are in a good agreement with the observational constraints (Figure ??).

5 Conclusions

We considered a model of tachyon inflation based on a holographic braneworld scenario with a brane located at the boundary of the $\rm AdS_5$ bulk and simulated observational parameters of inflation for the potential (**??**). The agreement of our model with the Planck observational data is good, especially for a higher number of e-folds and lower values of fundamental dimensionless coupling constant κ.

Preliminary results are promising ones and it represents good opportunity for further (analytical) research of this and other similar tachyonic potentials.

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