Physics AUC, vol. 27, 69-78 (2017) **PHYSICS AUC**

Study of elastic wave propagation in a short rod by ultrasound method

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Abstract

Elastic waves are an effective tool for studying the stress-strain state of the material. The velocity of sound is the main parameter for acoustic non-destructive testing. The purpose of the paper is to determine and analyze the velocity of an elastic wave in a short rod. An ultrasonic flaw detector was used to measure the wave velocity in rods. An increase in the velocity of sound in short rods is found after the experiment. Dispersion curves of the problem are used for comparison results with experimental ones. Analysis of the dispersion curves showed that rise of the wave velocity is possible. Similar results were obtained by the numerical solution of the problem. It is concluded that the wave velocity in the short rods is higher than the velocity of sound. These results will allow us to precise a distance to the defect, when we carrying out the acoustic testing of the rods.

Keywords: elastic waves; waves in rods; ultrasonic methods; wave equation; sound speed.

Introduction

Elastic waves are a highly effective tool for studying the stress-strain state, structure and properties of the material. The waves propagate to great depths, and do not distort the processes occurring in the material. The feature makes it possible to use the wave propagation in many branches.

Products of industry are very different. It is necessary to develop control methods. The most common methods of testing are non-destructive testing methods [6, 8]. The methods provide a high speed of testing, reliability of results. Also the methods are relatively cheap.

Acoustic methods are based on recording the parameters of elastic oscillations in controlled objects (Ohtsu, 2015). It allows us to detect internal and surface defects. One of the methods is ultrasonic flaw detection (Liu et al., 2013; Moallemi & Shahbazpanahi, 2014). Also we can determine the wave velocity. It may be done by recording the arrival time of the wave to the receiver of the flaw detector.

There are a lot of simplifications in solving problems associated with the wave propagation along a rod. In particular, the wave velocity in the rod is determined from the solution in a one-dimensional problem (Miklowitz, 2015). The velocity is used for problems in an axisymmetric formulation. The assumption gives satisfactory solutions, but does not show the real wave pattern.

Thus the problem of the elastic wave propagation in a finite length rod is not sufficiently investigated. Therefore, it is important to study the factors that affect the change in the wave velocity. The purpose of the paper is to determine the propagation velocity of an elastic wave in short rods.

Since the wave velocity is increase in short rod, we can use the fact for the acoustic testing. For example, the phenomenon allows us to precise the distance to the defect in small area near the rod boundary. It is important significance for non-destructive testing.

Literature Review

Analytical studies of the propagation of waves have been carried out for a long time within the theory of elasticity (Novatzkii, 1975). Theoretical basis for the wave propagation is also described in modern books (Bayanov & Gulidov, 2011; Miklowitz, 2015). An important task of such studies is the solution of the wave equation, for examples (Güven, 2014; Motamed, Nobile, Tempone, 2015; Vega, 2017). The equation can be solved in different formulations (Deng et al., 2017; Lowe et al., 2016). The study of longitudinal waves is carried out not only for cylindrical homogeneous rods, but also for composite rods (Ohtsu, 2015), which may have a different cross-section (Gan, Wei, Yang, 2014; 2016).

An important problem is also the derivation of the characteristic equation. The equation establishes the relationship between the wave velocity and the frequency. The solution of the characteristic equation allows us to find the wave velocity along the rod.

One-dimensional wave equations have an analytical solution. The three-dimensional formulation of the problem (Dedè, Jäggli, Quarteroni, 2015), complicates the solution. In this case, numerical methods (Liu et al., 2013) are used to find the solution of the wave equations (Lowe et al., 2016).

Fomin, Gulidov, Sapozhnikov et al. (1999) consider the problems of the rebound of various bodies from solid barriers in a two-dimensional formulation. There are the systems of equations, the description of the difference numerical solution of such problems in the book. The results of problems on the impact of cylindrical and conical rods are described. In the latter case, the phenomenon of repeated rebound was found. A stroboscopic light source based on a ruby laser was developed for experimental observation of the processes. The scheme of the experimental device and the results of the observations are given.

A rather wide area of application of elastic waves is non-destructive testing. Acoustic control by ultrasound is described in detail in the book (Vybornov, 1985). Ultrasonic testing is used to find local defects in various materials (Le Jeune et al., 2016 Moallemi & Shahbazpanahi, 2014). The method is used in construction (Krause & Dackermann, 2015) to control concrete structures (Mandal, Tinjum, Edil, 2016; Ohtsu, 2015). Composite materials are also tested by ultrasonic flaw detection (Gholizadeh, 2016; Jolly et al., 2015). Also it finds application in aircraft (Katunin, Dragan, Dziendzikowski, 2015).

The velocity of sound is used to determine the distance to the defect. But this velocity can be different in short rods. To increase the accuracy of the testing it is necessary to carry out additional studies of the wave velocity.

Materials and Methods

For the work, the ultrasonic testing method is used to measure the velocity of sound in cylindrical rods. The most suitable method for measuring the wave velocity is the shadow method. The path of the ultrasonic pulse from the source to the receiver is shorter than in the echo method. Therefore, the shadow method will give more exact results.

UD4-T ultrasonic flaw detector was used for carrying out the experiment. Its kit includes various converters. The flaw detector is allowed us to use both the echo method and the shadow method. The device has such features as the visibility of the defect in the metal, direct measurement of the equivalent defect area, the ability to measure the acoustic properties of materials. When the results are on the display, the device allows us to "freeze" the image and represent the signal.

The flaw detector has the following technical characteristics:

1. The range of operating frequencies is 0.2 ... 10 MHz;
2. The range of measured depths is 0.5 ... 5000 mm;
3. The error in measuring the coordinates of the defect is not more than 0.1 mm;
4. The error in time intervals is 0.025 μs.

The investigation of the rods was carried out with a piezoelectric transducer as a source of oscillations. The transducer operating at a frequency of 2.5 MHz. The second converter served as a signal receiver at the opposite end.

After passing the signal from the source to the receiver, the device displays the time of arrival of the wave. In addition, it is possible to see the oscillogram of the signal or its envelope (Fig. 1). The signal source of the ultrasonic flaw detector was placed on one end of the rod, and the receiver on the other. The time of signal passage was measured by the device.

Fig. 1. Display of ultrasonic flaw detector

If the duration of the wave propagation and the length of the rod are determined, we can calculate velocity of the signal.

The analytical study was based on the solution of the Lame equation:

$$(\lambda + \mu) grad\, div\, \mathbf{u} + \mu \Delta \mathbf{u} = \rho \ddot{\mathbf{u}}, \tag{1}$$

where λ, μ –Lame constants, $\mathbf{u}$ – displacement vector.

We expand the displacement vector into two components using the scalar and vector potentials:

$$\mathbf{u} = grad\, \Phi + rot\, \mathbf{\Psi}. \tag{2}$$

From equation (2) we obtain the system of equations

$$\Delta \Phi - \frac{1}{c_1^2} \ddot{\Phi} = 0,$$

$$\Delta \mathbf{\Psi} - \frac{1}{c_2^2} \ddot{\mathbf{\Psi}} = 0. \tag{3}$$

where c_1 and c_2 – velocity of longitudinal and transverse waves, accordingly.

To solve system (3) the method of separation of variables is applied. Detailed calculations are given in [21]. We give only final result. Particular solutions for the components of the displacement vector can be written in the form

$$\overline{u}_r(r,z,t) = e^{-i(\omega_k t - kz)} \left(\frac{dJ_0(\alpha_k r)}{dr} M_k - ikJ_1(\beta_k r) N_k \right),$$

$$\overline{u}_z(r,z,t) = e^{-i(\omega_k t - kz)} \left(ikJ_0(\alpha_k r) M_k + \left(\frac{dJ_1(\beta_k r)}{dr} + \frac{J_1(\beta_k r)}{r} \right) N_k \right), \tag{4}$$

where M_k, N_k – unknown constant, J_0 and J_1 – cylindrical Bessel functions of the first kind of zero and first order, k – wave number, ω_k – frequency, $\alpha^2 = \frac{\omega_k^2}{c_1^2} - k^2$, $\beta^2 = \frac{\omega_k^2}{c_2^2} - k^2$.

We apply the following boundary conditions for this problem: radial σ_r and shearing stresses σ_{rz} in a rod on a free cylindrical boundary $r = R$ are zero, i.e.

$$\sigma_r = \lambda \left(\frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} + \frac{u_r}{r} \right) + 2\mu \frac{\partial u_r}{\partial r} \bigg|_{r=R} = 0,$$

$$\sigma_{rz} = \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) \bigg|_{r=R} = 0. \tag{5}$$

We substitute conditions (5) in system (4) and equate to zero the determinant of the system. As a result, we obtain the characteristic equation

$$\left[(\lambda + 2\mu) \alpha_k^2 J_0'' + \frac{\lambda}{R} \alpha_k J_0' - \lambda k^2 J_0 \right] \cdot \left[\beta_k^2 J_1'' + \frac{1}{R} \beta_k J_1' + \left(k^2 - \frac{1}{R^2} \right) J_1 \right] -$$

$$- 4\mu k^2 \alpha_k \beta_k J_0' J_1' = 0. \tag{6}$$

Based on equation (6), the phase velocity of the wave is determined, and dispersion curves are constructed.

The numerical solution of the problem was carried out with KRUG24 software. The KRUG24 complex was developed at the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the Russian Academy of Sciences. It is intended for the numerical solution of two-dimensional and axisymmetric problems (Fomin, Gulidov, Sapozhnikov et al., 1999).

For visualizing the calculation results of the KRUG24 program we use AutoLISP and AutoCAD. The system allows us to create surfaces from polygonal cells.

The program consists of several subroutines (Bayanov, 2013). In the first subroutine a data file is opened. There are values of the stresses at the nodes, which was obtained from the KRUG24. After reading the file, the program forms the lists of coordinates for each of the three points of the triangular cell.

In this case, the program assumes that the x-axis is directed along the rod and the y-axis in the radial direction. Stress values are plotted along the z-axis.

In the second subroutine, triangular faces are constructed. Then an image of the stress field in the rod is formed.

In the cycle of the main subroutine, polygons in AutoCAD for each cell are created, and the image is exported to .wmf file.

As a result, a set of images with stress distribution at different times is obtained.

Results

Rods of aluminum, brass and steel were used in the experiment. The rods were 2, 20 and 40 mm long. After testing the sample, the ultrasonic flaw detector provided a signal time. By the time of the signal and the length of the rod, the velocity of the wave was calculated.

Table 1 presents the results of the ultrasonic flaw detector testing of different rods.

Material	Rod length, mm	Duration of the signal, µs	wave velocity, m / s
Steel	2	0,39	5160
	20	3,85	5194
	40	7,82	5114
Brass	2	0,48	4200
	20	5,22	3830
	40	10,33	3872
Aluminum	2	0,36	5500
	20	3,84	5209
	40	7,73	5173

Table 1. Results of ultrasound testing

Rods 40 mm in length are test. In the long rods the wave propagates with rod velocity (velocity of sound) [15, 24]. The velocity calculated from the experimental data differs a little from the velocity of sound. The study of rods with a length of 20 mm gives us the similar results. Thus the free cylindrical boundaries of the rod do not exert a strong influence on the process of wave propagation.

Calculations of the signal velocity in 2 mm rods were made. The results have shown that in some materials an increase in the wave velocity is observed. The difference in velocity values can be influenced by the free cylindrical boundary. Cause of this is a small influence of the boundaries on the process of propagation of a wave.

Now we have to compare the results with the ones determined by the numerical method (Bayanov & Gulidov, 2011). The Fig. 2 shows us a comparison of velocities for three rod materials.

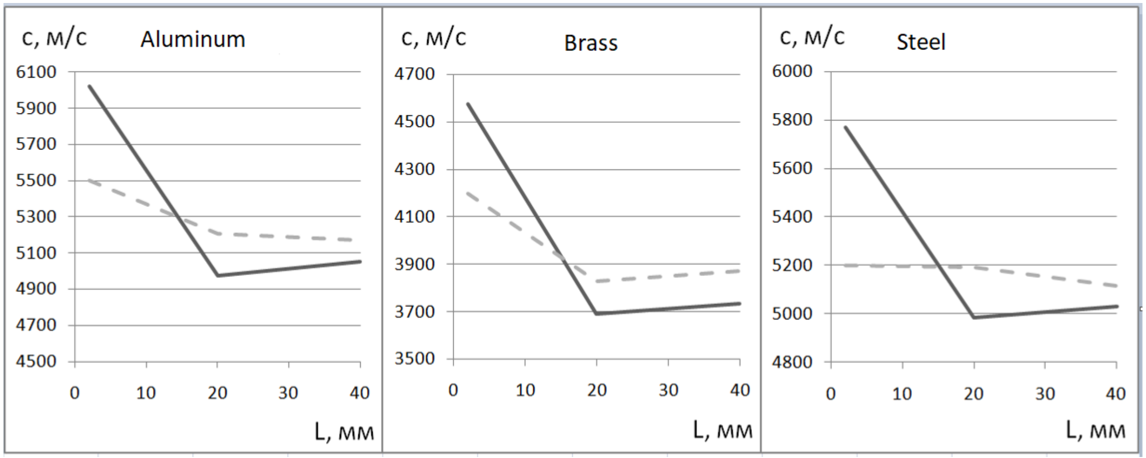


Fig. 2. Dependence of the wave velocity on the rod length, determined from experiment (– – –) and numerically (——)

On the graphs (Fig. 2), the length of the rod is plotted along the abscissa axis; the velocity of the wave is plotted along the ordinate axis. The solid line shows us the result of a numerical solution; the dashed line shows us the result of an ultrasonic testing. It can be seen that, an increase in the wave velocity at small lengths is also observed.

Let us verify analytically the value of the wave velocity in a short rod. It is necessary to find the roots of the characteristic equation (6). For simplicity, the Bessel functions are represented in the form of series. Also only the first two terms of the series are taken for the solution. In this case, the characteristic equation (6) will have two roots (Bayanov, Kurlaev, 2016). We plot the dispersion curves based on the roots.

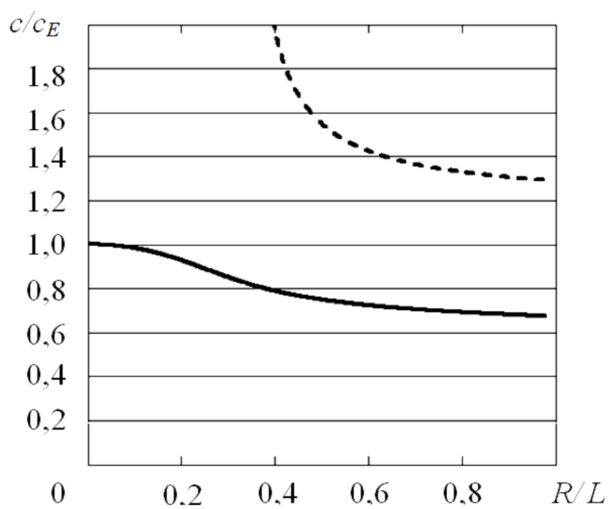


Fig. 3. Dispersion curves for an aluminum rod

The first curve in Fig. 3 (solid line) tends to the velocity of transverse waves c_2. The second curve (dashed line) exists only from a certain value of the relative wavelength. Then it tends to the velocity of longitudinal waves c_1.

If the rod radius tends to zero, then the wave velocity tends to c_E. It confirms the reliability of the results obtained. If the radius of the rod increases, then the velocities of the longitudinal and transverse waves prevail.

For qualitative analysis it is necessary to consider the stress distribution in the rod at each instant of time. We will visualize the stress distribution in the rod at certain points in time. The stress values will be taken from the output file after solving the problem in KRUG24 complex (Fomin, Gulidov, Sapozhnikov, 1999).

Let's analyze the wave during its propagation along a long rod. The problem of impact of a rod 5 cm is considered. On its example we analyze the behavior of the wave front in a rod.

The following pictures show us the result of visualization by the AutoLISP. Only a qualitative evaluation of the stresses is needed for the analysis. Therefore, the figures do not show the quantitative values of the stresses.

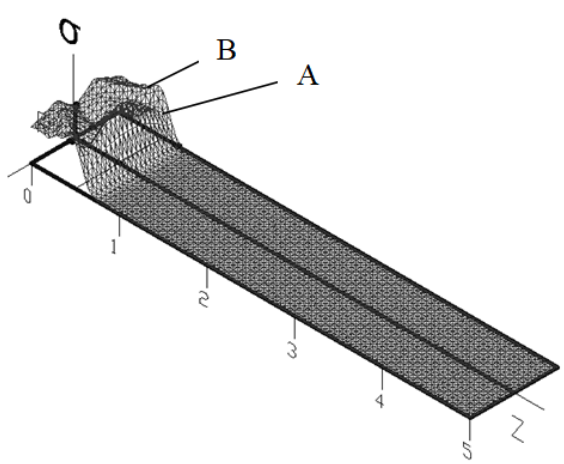


Fig. 4. Stress distribution at a time point of 1 μs

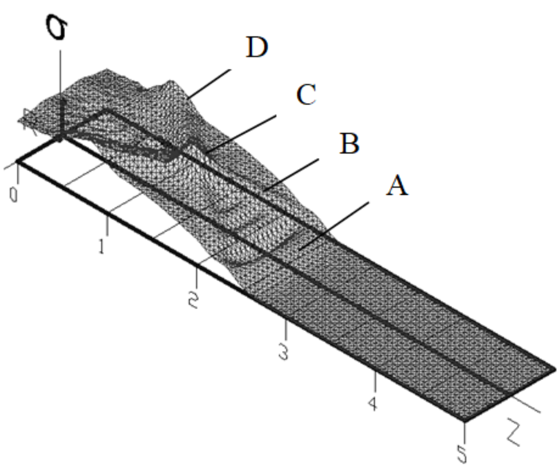


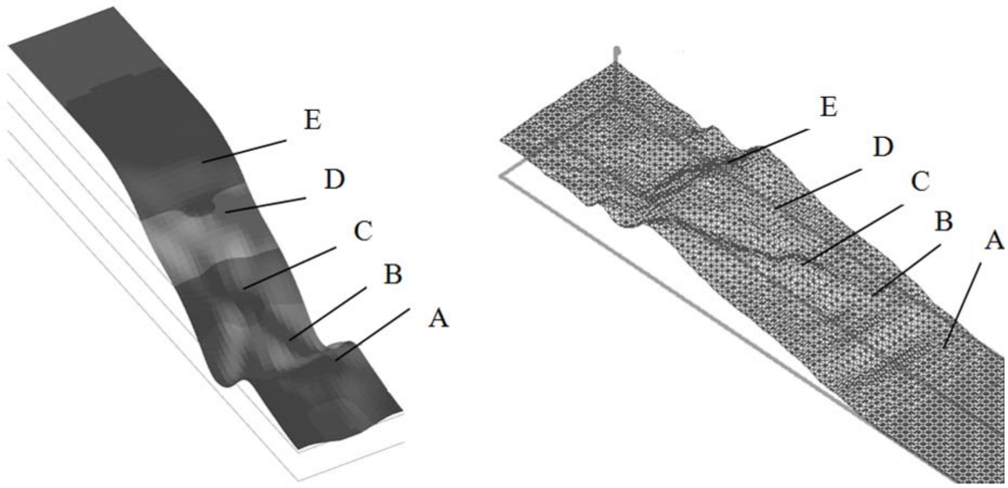
Fig. 5. Stress distribution at a time point of 4 μs

The rod collides with a barrier, and a stress wave appears. Then the wave moves along the rod (Fig. 4). The plane of wave *A* moves with the velocity of longitudinal wave c_1. There are free cylindrical surfaces in the rod. Therefore, the wave front *A* generates a discharge wave *B*. The wave moves from the free cylindrical surface to the axis of symmetry. The velocity of the wave equals to the velocity of transverse wave c_2.

When the wave *B* arrives to the axis of symmetry ($t = R / c_2$), there is a collision of unloading waves *B* (Fig. 5). Therefore, the compressive stress in the local region *C* is increased. After the wave collision, the wave *D* moves back to the outer cylindrical surface. The velocity of the wave equals to c_2. Due to the interaction of the longitudinal and transverse waves, the stress at the fronts *A* and *B* begins to decrease. In this case, all of the waves continue to move at a velocity c_1.

The results are confirmed by similar ones, which obtained by the AUTODYN.

Fig. 6 shows us the stress visualization made by AUTODYN and KRUG24 at time of 5 μs. The nature of the distribution of waves (A ... E) in a qualitative assessment is similar in both programs. As a consequence, we can conclude that the results obtained are reliable.



a b

Fig. 6. Qualitative comparison of stress distribution
a - AUTODYN, b - KRUG24

Discussion

Ultrasonic research has shown us that the elastic wave in the rod propagates with the velocity of sound. But in the short rods the wave moves a little faster. Previously conducted numerical study (Bayanov, Gulidov, 2011) gave similar results. Analysis of the dispersion curves showed that the phase velocity can be greater than the speed of sound. It is possible when the rod radius greater than the wavelength.

The visualization of the stress allowed us to consider the motion of the wave in more detail. At first, the longitudinal wave moves with velocity c_1. Due to the free cylindrical boundary of the rod, a transverse wave is appeared. It moves to the axis of the rod at a velocity c_2. Subsequently, the transverse wave affects the longitudinal wave and slows its propagation. Due to this, the wave comes to the free end of the long rod at the velocity of sound. The effect of the transverse wave at the longitudinal one is weak in the short rod. Therefore, the wave velocity is greater than the velocity of sound. These results will help us to improve the accuracy of determining the distance to the defect when we use acoustic testing of the rods.

Conclusions

It was shown experimentally that the wave velocity in short rods (the length is smaller than the diameter) is greater than the velocity of sound in the rod (by 1.1 … 1.3 times). It is incorrect to use the velocity of sound as the wave velocity for the short rod. Because analysis of dispersion curves showed us that at a rod radius exceeding the wavelength, the first mode tends to the velocity of transverse waves c_2, and the second one tends to the velocity of longitudinal waves c_1.

There is such a length of the rod (0.1 ... 0.2 diameter) at which the velocity of the wave propagation exceeds the rod velocity by approximately 1.2 times. Analysis of stress distribution suggests that the wave propagating along the rod in the forward direction has the velocity of longitudinal waves c_1 at the initial time. Then the front is affected by perturbations propagating from the free cylindrical boundaries.

When carrying out acoustic control of the rods, the results will allow us to specify the distance to the defect, which located at a small distance from the end. For further work, it is necessary to conduct an experiment with a large number of materials. Also it is necessary to establish the relationship between the value of the velocity of sound in the short rod and the constants of the materials.

Acknowledgements

The authors are grateful to the assistant professor of the Department of Gas-Dynamic Impulse Devices of the Novosibirsk State Technical University Polyakov Yu. O. for help in carrying out the experiment using an ultrasonic flaw detector.

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