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On the movement of dust particles in inhomogeneous magnetic field with radial magnetic perturbation

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Abstract

The model developed in our paper considers a stochastic magnetic field that contains a term representing the gradient of the magnetic field and a radial fluctuating term that is described by the dimensionless function $Ab_x(Y)$ that are perpendicular to the mean magnetic field B_0. We have calculated the solutions, the hodographs of velocities, the accelerations from the Newton-Lorentz equation for different values of the dimensionless Lorentz frequency Ω and the dimensionless parameters A, α_x, γ and K_B.

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1 Introduction

We have analyzed in this paper the dust particles trajectories induced by the fluctuations of the magnetic field. We have solved the Newton -Lorentz equation of dust particles for physically relevant parameter values, namely the dimensionless Larmor frequency Ω and the dimensionless parameters A, α_x, γ, Ω and K_B (see below).

The paper is organized as follows. The magnetic field model and the Lorentz equations are established in section 2. In section 3, the hodograph of velocities, the accelerations and the trajectories for the dust particles were calculated and represented. The conclusions are summarized in section 4.

2 The magnetic field model and the Lorentz equations

The expression of the inhomogeneous stochastic magnetic field that is characterized only by a radial perturbation is:

$$\mathbf{B}(X,Y,Z) = B_0 \left\{ \left[1 + XL_B^{-1}\right] \mathbf{e}_z + Ab_x\left(Y\right)\mathbf{e}_x + \mathbf{e}_y \right\} \tag{1}$$

where A is a dimensionless parameter measuring the amplitude of the magnetic field fluctuation relative to the mean magnetic field B_0 {see e.g. [3], [4], [5],[6]}. There is one linear term depending on X in the right hand side of eq.(1): the nonhomogeneous term XL_B^{-1} where L_B is the gradient scale length. We will define the term $B_0\left[1 + XL_B^{-1}\right]\mathbf{e}_z$ as the gradient $\mathbf{B}$ term. The Newton-Lorentz force is:

$$m\frac{d\mathbf{V}}{dt} = q\left(\mathbf{V} \times \mathbf{B}\right) \tag{2}$$

and the corresponding system of equations corresponding to the definition given in (1) is:

$$\frac{dV_x}{dt} = \frac{qB_0}{m} V_y \left(1 + XL_B^{-1}\right) \quad (3)$$
$$\frac{dV_y}{dt} = \frac{qB_0}{m} \left[-V_x \left(1 + XL_B^{-1}\right) + V_z Ab_x\right] \quad (4)$$
$$\frac{dV_z}{dt} = \frac{qB_0}{m} \left(V_x - V_y Ab_x\right) \quad (5)$$

We use the following dimensionless quantities:

$$\frac{\mathbf{V}}{v_0} = \mathbf{v} \quad , \quad \frac{t}{t_0} = \tau, \quad \frac{\mathbf{X}}{L} = \mathbf{x} \quad (6)$$

where $v_0 \equiv v_{th} \simeq 10^3 ms^{-1}$, $t_0 \equiv t_s \simeq 10^4$ sec and L is of order of meter. The dimensionless system of equations is the following:

$$\frac{dv_x}{d\tau} = \Omega v_y \left(1 + K_B x\right)$$
$$\frac{dv_y}{d\tau} = \Omega \left[-v_x \left(1 + K_B x\right) + v_z Ab_x\right]$$
$$\frac{dv_z}{d\tau} = \Omega \left(v_x - v_y Ab_x\right) \quad (7)$$

In the system (7) the following dimensionless parameters are introduced:

$$\text{Dimensionless frequency } \Omega = \frac{qB_0 t_0}{m} \quad (8)$$

$$\text{The inhomogeneous parameter } K_B = \frac{L}{L_B} \quad (9)$$

We choose $Ab_x = A \sin\left(\alpha_x y + \gamma\right)$ where A is dimensionless amplitude of the radial fluctuation and α_x is proportional to L^{-1}. We will consider that the masses of the dust particles are in the range from $[10^{-11},\ 10^{-10}]$ kg and the electric charges are in the range from $[10^{-14},\ \ 10^{-13}]$ C [1], [2]. The order of magnitude of the magnetic field is considered to be of order 10 T . The thermal velocity v_{th} is of order 10^3 m/s and the stopping time $t_0 = t_s$ is of order $10^4 s$ if the dimension of the dust grain is $10^{-2} m$. Ω is considered to be of order $[1, 100]$ and L_B is of order of L.

3 The trajectories, velocities, hodographs of velocities and accelerations

In Figure 1 we visualized the trajectories (left up), velcocities (right up), hodograph of velocities (left down) and accelerations (right down) for fixed values of the parameters $A = 1$, $\alpha_x = 1$, $\gamma = 0, \Omega = 1$ and $K_B = 1$. The helix trajectory is obvious with a relatively small pitch.

In Figure 2 was represented the trajectories (up) and the hodographs (down) of velocities for $A = 1,\ \alpha_x = 1, \gamma = 0,\ K_B = 1,\ \Omega = \{1$ (blue), 5 (red), 10 (green)$\}$. The volume occupied by the 3-dimensional trajectory the smaller the greater Ω is. The smallest gyration radius is then for $\Omega = 10$ (the green curve). The volume of the 3-dimensional

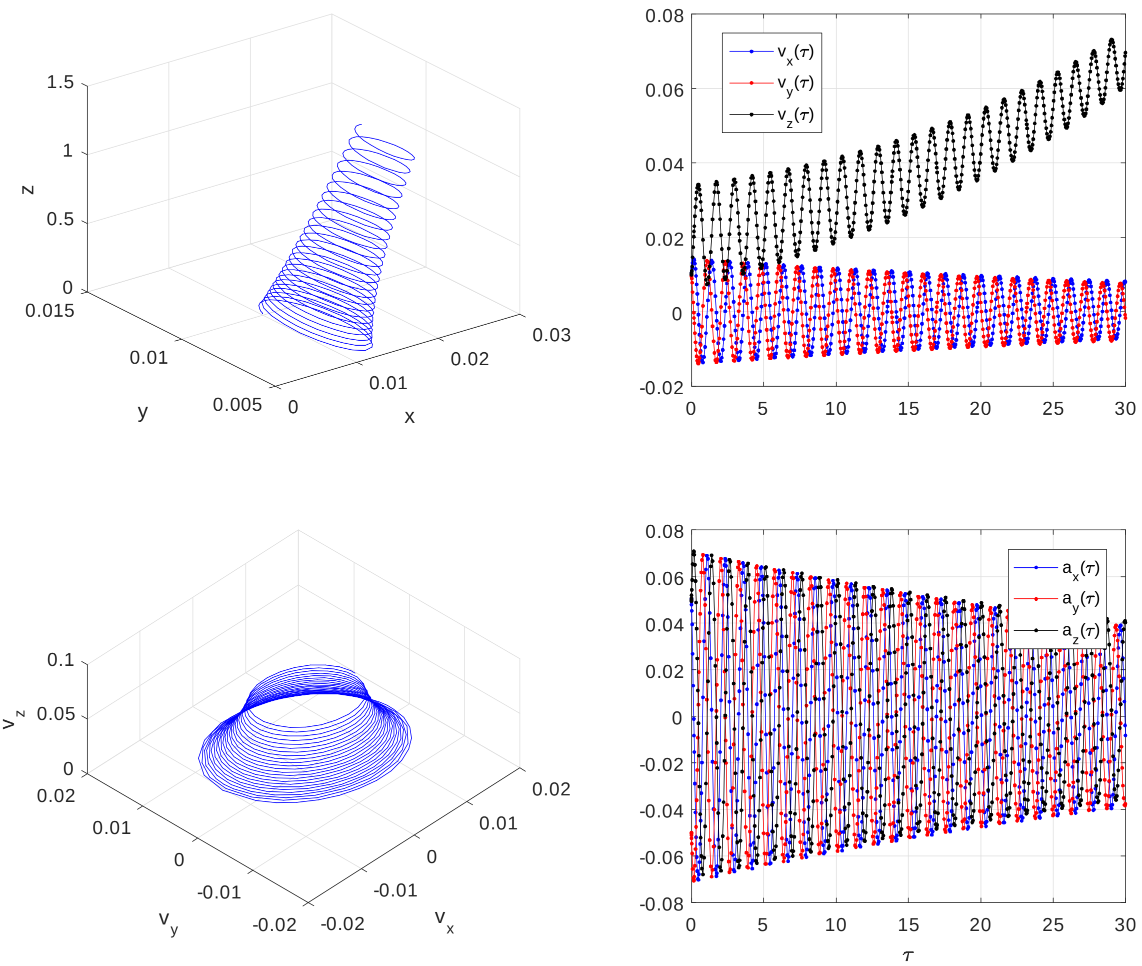

Figure 1: $A = 1$, $\alpha_x = 1$, $\gamma = 0$, $K_B = 1$, $\Omega = 1$

trajectory diminishes if the parameter K_B increases (see this feature comparing the Figures 2 and 3).In Figure 3 was represented the trajectories (up) and the hodographs (down) of velocities for $A = 1$, $\alpha_x = 1$, $\gamma = 0$, $K_B = 10$, $\Omega = \{1$ (blue), 5 (red), 10 (green)$\}$. In figure (4) was represented the trajectories (up) and the hodographs (down) of velocities for $A = 1$, $\gamma = 0, K_B = 10, \Omega = 10, \alpha_x = 1$(green), $\alpha_x = 3$(magenta), $\alpha_x = 2$(black). Varying the parameter α_x we notice that the gyration radius is diminished but the elongation of the trajectory increases: the greater is in the case of $\alpha_x = 3$(magenta).In Figure 5 was visualized the trajectories (up) and the hodographs of velocities (down) for $A = 1$, $\alpha_x = 1$, $K_B = 1$, $\Omega = 5$, $\gamma = 0.1$ (red), $\gamma = 0$ (blue). If the phase is $\gamma = 0.1$ or 0 there are no rotations for the ions.

4 Conclusion

In this paper we obtained first results concerning the movement (trajectories, hodographs of velocities, accelerations) for a dust particle for different values of the dimensionless quantities such as: Lorentz frequency Ω, the parameters A, α_x, γ and K_B.

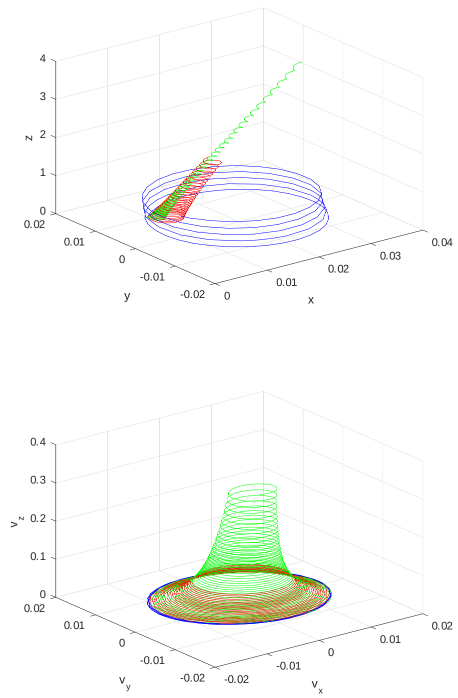

Figure 2: $A = 1$, $\alpha_x = 1$, $\gamma = 0$, $K_B = 1$, $\Omega = \{1$ (blue), 5 (red), 10 (green)$\}$

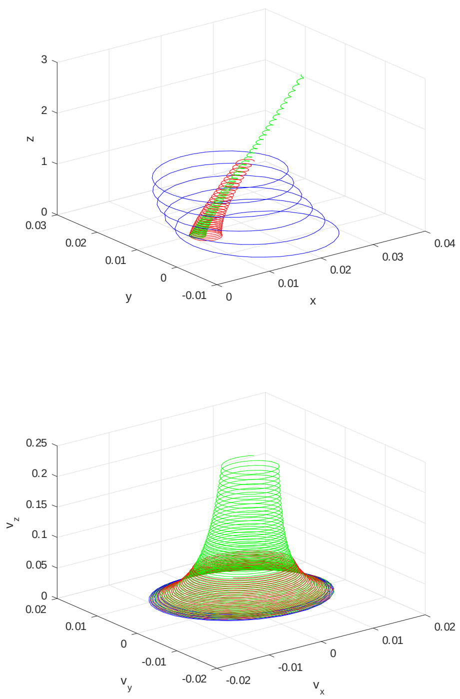

Figure 3: $A = 1$, $\alpha_x = 1$, $\gamma = 0$, $K_B = 10$, $\Omega = \{1$ (blue), 5 (red), 10 (green)$\}$

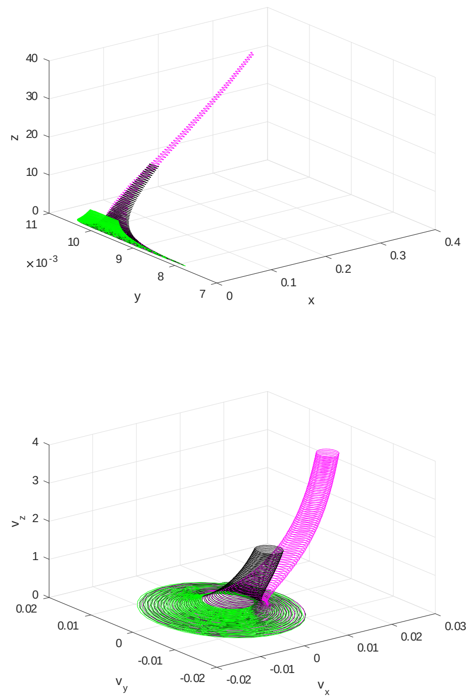

Figure 4: $A = 1,\ \alpha = 1, \gamma = 0, K_B = 10, \Omega = 1$(blue), 5(red), 10(green)

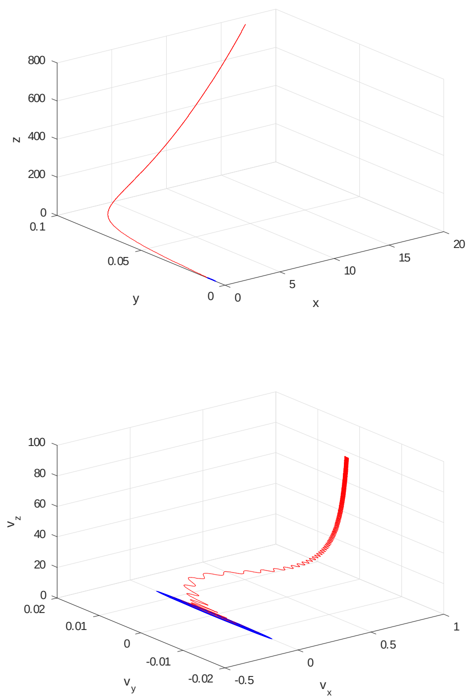

Figure 5: $A = 1,\ \alpha_x = 1,\ K_B = 1,\ \Omega = 5,\ \gamma = 0.1$ (red), $\gamma = 0$ (blue)

We can conclude that the movement is influenced by α_x, Ω and γ and a small influence is given by K_B.

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