Trajectories of dust particle in a stochastic magnetic configuration

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Abstract

The dust particles movement induced by a stochastic magnetic field with radial fluctuation is studied. The solutions of the Newton -Lorentz equation of dust particles are obtained for physically relevant parameters values; we obtained also the trajectories, the hodographs of the velocities and accelerations for different values of the dimensionless Larmor frequency and specific parameters.

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1 Introduction

We have analyzed in this paper the dust particles trajectories induced by the fluctuations of the magnetic field. We have solved the Newton -Lorentz equation of dust particles for physically relevant parameters values, namely the dimensionless Larmor frequency Ω and the dimensionless parameters A, B, p_z and p_y (see below their definitions).

The paper is organized as follows. The equations of motion for the dust particle in a stochastic magnetic field are established in section 2. In section 3, the hodographs of the velocities, the accelerations and the trajectories for the dust particles were obtained for different values of the parameters. The conclusions are summarized in section 4.

2 Equations of motion for the dust particle

The electric field is considered to be irrelevant in our analysis and only the magnetic field with fluctuations in a plane perpendicular to the mean magnetic field is used. The Newton-Lorentz force is:

$$m\frac{d\mathbf{V}}{dt} = q\left(\mathbf{V}\times\mathbf{B}\right) \tag{1}$$

where the stochastic magnetic field is given by the expression:

$$\mathbf{B}\left(\mathbf{X}\right) = B_0\left[\mathbf{e}_z + \beta b_x\left(\mathbf{X}\right)\mathbf{e}_x\right] \tag{2}$$

where $\mathbf{X}\equiv(X,\ Y,\ Z)$ and β is a dimensionless parameter measuring the amplitude of the radial magnetic field fluctuation relative to the mean magnetic field $B_0\mathbf{e}_z$. The scalar equations corresponding to eqs. (1) combined with the eq.(2) are:

$$\frac{dV_x}{dt} = \frac{qB_0}{m}V_y \tag{3}$$

$$\frac{dV_y}{dt} = \frac{qB_0}{m}\left(-V_x + V_z\beta b_x\right) \tag{4}$$
$$\frac{dV_z}{dt} = -\frac{qB_0}{m}V_y\beta b_x \tag{5}$$

We use the following dimensionless quantities:

$$\frac{\mathbf{V}}{v_0} = \mathbf{v} \quad , \quad \frac{t}{t_0} = \tau, \quad \frac{\mathbf{X}}{L} = \mathbf{x} \tag{6}$$

where the thermal velocity $v_0 \equiv v_{th}$ is of order $10^3\ m/s$ and the stopping time $t_0 = t_s$ is of order $10^4 s$ if the dimension of the dust grain is $10^{-2}m.$ and L is of order of meter. We will consider in our paper that the poloidal magnetic field fluctuation, βb_y is $\equiv 0$ and the radial magnetic field fluctuation has the following expression:

$$\beta b_x = A\sin zp_z + B\cos yp_y \tag{7}$$

The expression given in (7) is similar with the dimensionless radial velocity of the Arnold-Beltrami-Childress flow (see e.g. [5]). Here A, B are two real dimensionless parameters and $p_z = p_y \equiv 2\pi L = 1$. The dimensionless equations that we obtain are:

$$\begin{aligned}
\frac{dv_x}{d\tau} &= \Omega v_y \\
\frac{dv_y}{d\tau} &= \Omega\left[-v_x + v_z\left(A\sin zp_z + B\cos yp_y\right)\right] \\
\frac{dv_z}{d\tau} &= -\Omega\left[v_y\left(A\sin zp_z + B\cos yp_y\right)\right]
\end{aligned} \tag{8}$$

where $\Omega \equiv \frac{qB_0t_0}{m}$ that is considered to be of order $[1, 100]$. We will consider that the masses of the dust particles are in the range $[10^{-11},\ 10^{-10}]\ kg$ and the electric charges are in the range $[10^{-14},\ \ 10^{-13}]\ C$ [1]. The order of magnitude of the mean magnetic field is considered to be of order $10\ T$.

3 The trajectories, velocities and accelerations for the dust particle

In this section we represented the solutions and the trajectories for the dust particle for different values of the Lorentz frequency Ω and the parameters $A, B, p_z = p_y = 1$. In figures (1)-(4) we visualised the trajectories, velocities, hodographs of velocities and accelerations for different values of the parameters. For $\Omega = 2$ the trajectory is a helix with a relatively small pitch. The pitch the smaller the greater Ω is. We developed a numerical Matlab code in order to evaluate the solutions of the sistem of equations. The number of oscillations of the solutions increases if the Larmor frequency increases as we can observe from figures (1-4).

More informations on the dynamics will be obtained from the analysis of the running and asymptotic diffusion coefficients but this issue is left for a future paper.

4 Conclusions

The dust particles motion was studied and the solutions, the trajectories, the velocities and the accelerations for the dust particle for different values of the Lorentz dimensionless frequency Ω and the parameters $A, B, p_z = p_y = 1$ were calculated.

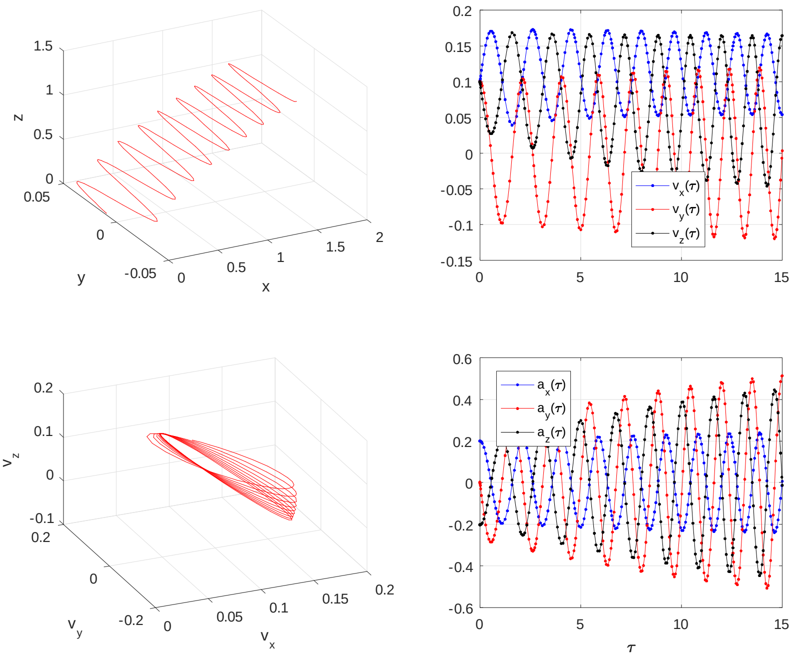

Figure 1: Trajectory (up left side), velocities (up right side), hodograph of velocities (down left side) and the accelerations (down right side) for $A = B = 1, \Omega = 2$.

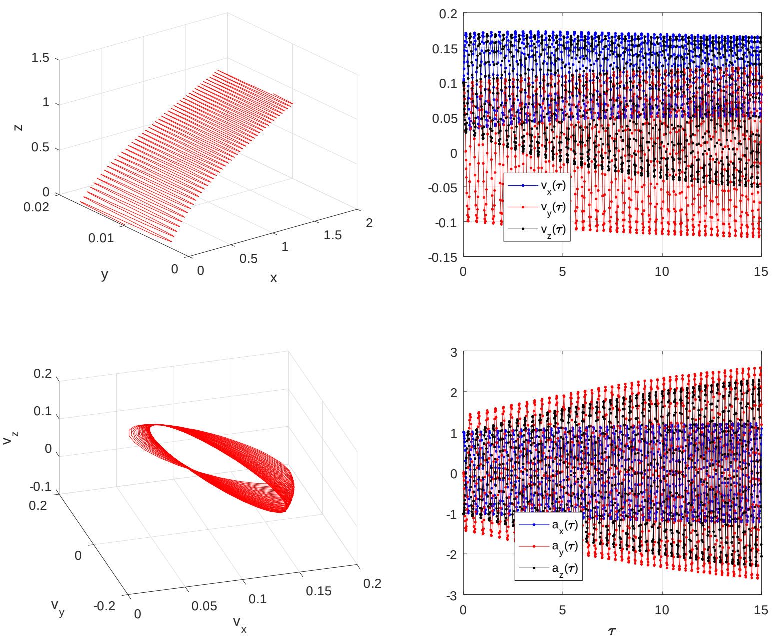

Figure 2: Trajectory (up left side), velocities (up right side), hodograph of velocities (down left side) and the accelerations (down right side) for $A = B = 1, \Omega = 10$

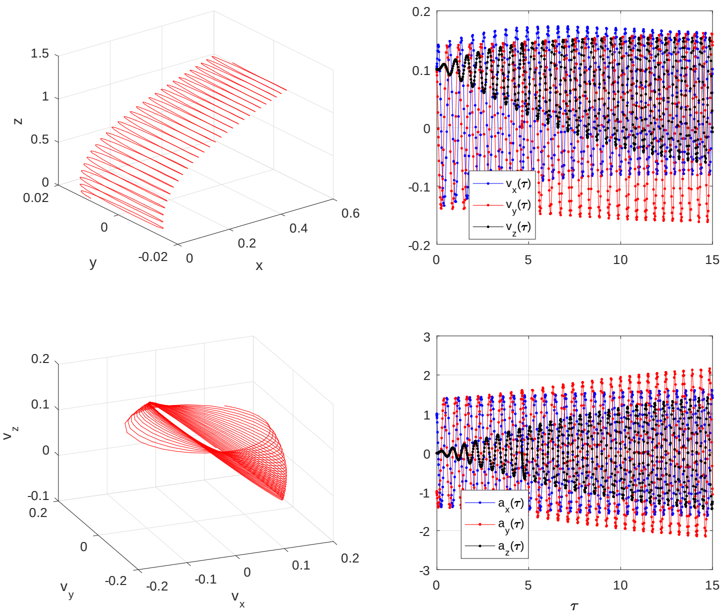

Figure 3: Trajectory (up left side), velocities (up right side), hodograph of velocities (down left side) and the accelerations (down right side) for $A = 1, B = 0, \Omega = 10$

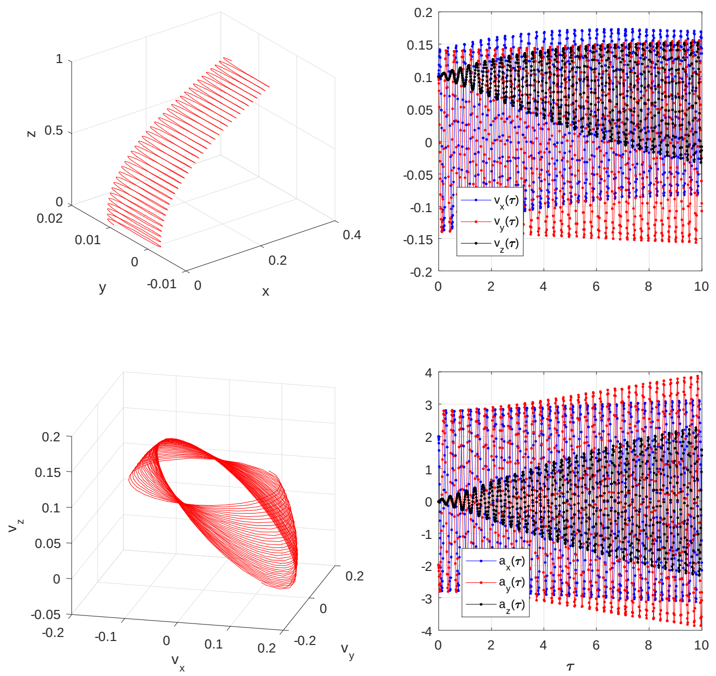

Figure 4: Trajectory (up left side), velocities (up right side), hodograph of velocities (down left side) and the accelerations (down right side) for $A = 1, B = 0, \Omega = 20$

Acknowledgments

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